CS472 Foundations of Artificial Intelligence
Fall 1999
Assignment 2

Due Monday, October 4 at the beginning of class

1. α-β **Pruning (30 pts)**

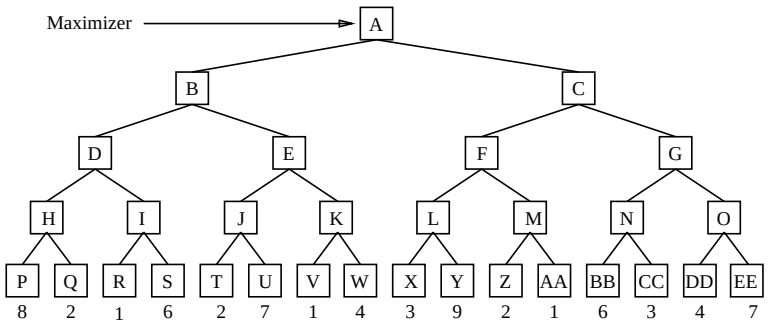

Figure 1: Game Tree for Alpha-beta Pruning

These questions refer to the game tree in Figure 1.

(a) **(5 pts.)** What is the solution? That is, which move should be made next and what is the expected value of that move?

(b) **(5 pts.)** Using alpha-beta pruning (and standard left-to-right evaluation of nodes), how many of the leaves get evaluated? Indicate all parts of the tree that are cut off. Indicate the winning path or paths. Strike out all static evaluation values that do not need to be computed.

(c) **(5 pts.)** How does the answer to (b) change if right-to-left evaluation of nodes is used?

(d) **(5 pts.)** What do the answers to (b) and (c) imply about the benefit of choosing the right evaluation order for alpha-beta search? State a heuristic which can guide the search for the best evaluation-order; this may be domain-independent or specific to a particular game like chess. What would be the savings if the "perfect" evaluation order were used?

(e) **(5 pts.)** Explain why searches in game-playing programs generally go forward from the current position instead of backward from the goal.

(f) **(5 pts.)** Describe how to modify the minimax algorithm so that it can be used in a three- or four-person game.

2. **(15 pts.)** Exercise 5.1 from Russell & Norvig.

3. **Problem Hardness (10 pts.)**

(a) **(3 pts.)** Consider randomly generated 3-SAT instances with the ratio of clauses to variables equal to 10. What can you say about the difficulty of solving such instances. Compare with the difficulty of solving randomly generated instances at the ratio of 4.3.

(b) **(3 pts.)** Consider a SAT instance encoding a problem in VLSI design. Assume the ratio of clauses to variables of that instance is 10. What can you say about the hardness of the instance?

(c) **(4 pts.)** Consider SAT instances based on randomly generated graph coloring problems. Assume that the ratio of clauses to variables is equal to 10. What can you say about the hardness of these instances?

4. **GSAT / Walksat (30 pts.)**

(a) **(10 pts.)** Consider GSAT running on a 3-SAT problem with n variables. Each variable occurs in at most c clauses. Assume that array A (size n) gives for each variable the number of additional clauses that become satisfied after "flipping" that variable. (Note: this number of additional clauses can be zero or negative). After GSAT makes a "flip", you want to *efficiently* update array A.

- Give data structures and pseudo code of the procedure to update array A.
- Give the time complexity of the procedure to update array A.

(b) **(10 pts.)** Give a set of *satisfiable* clauses (a clause is a disjunction of literals, where a literal is a Boolean variable or its negation) that would be difficult for GSAT to satisfy, starting from a random truth assignment. Your clauses may contain any number of variables but no variable may appear more than once in a clause.

(c) **(10 pts.)** In Walksat, we only flip variables that occur in clauses that are currently unsatisfied. If a formula is satisfiable, starting from a random initial assignment, can one always reach a satisfying assignment with the Walksat flipping strategy? Can one reach all satisfying assignments? Explain your answer.

5. **Adversarial Search: (15 points total)**
There's a simple game you can play with silver dollars. (A silver dollar is a coin worth \$1.) There are five (5) silver dollars on the table and you and your opponent take turns picking up 1, 2, or 3 coins until none is left. You get to keep each silver dollar you pick up. *But*, if you pick up the last coin, you have to pay \$2 to your opponent. The object of the game is to finish with the most money.

(a) (3 pts.) Consider building a game tree to solve the above problem. What would each state in the game tree represent? What is/are the operator(s)?

(b) (2 pts.) What cost function, i.e., static evaluation function, do you use to evaluate terminal nodes?

(c) (5 pts.) Show the (complete) resulting minimax tree for this problem.

(d) (2 pts.) Your opponent courteously offers to let you go first. If you accept, what is your first move? How much will you earn?

(e) (3 pts.) Clearly indicate **one** example of an alpha-beta pruning in this tree.