CS 4700: Foundations of Artificial Intelligence **(Due: 3/12/20, 11:59pm)**

Homework #3: Game Trees & Logic SOLUTIONS

85 Points Total

Instructor: Haym Hirsh *Name:* Student name, *Netid:* NetId

Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission. **These can be specific to each assignment.**

- Please include your name and NetIDs on the first page. We recommend typesetting your submission in LaTeX, and an Overleaf template is linked here. See course policy for general typesetting requirements.
- Homeworks must be submitted via Gradescope by the due date and time. This assignment requires **two individual submissions**: one to "Homework 3 - Written" and one to "Homework 3 - Python." Your submission will be marked on time if and only if both assignments are submitted by the due date and time.
- Late homeworks are accepted until **3/14/20 at 11:59pm** for a 50% penalty per course policy.
- All sources of material outside the course must be cited. The University Academic Code of Conduct will be strictly enforced.

Problem 1: Unification and Resolution (20 points)

A (6 points): A key component to first-order inference algorithms is the UNIFY algorithm. Recall that this algorithm returns a substitution that would make two logical expressions look identical. If such a substitution doesn't exist then return *failure*. Do the following unifications and give the most general unifier as your answers:

1. UNIFY$(loves(Haym, x), loves(y, AI))$ = $\{x/AI, y/Haym\}$
2. UNIFY$(loves(Haym, x), loves(x, AI))$ = $failure$
3. UNIFY$(loves(AI, x_1), loves(x_2, x_3))$= $\{x_2/AI, x_1/x_3\}$

B (4 points): Convert the following statement into conjunctive normal form (CNF)

$$\neg P_1 \vee (P_2 \Rightarrow (\neg P_3 \wedge P_4))$$

Solution: $(\neg P_1 \vee \neg P_2 \vee \neg P_3) \wedge (\neg P_1 \vee \neg P_2 \vee P_4)$

C (10 points): Given the following knowledge base:

$$\forall x\, [(A(x) \wedge B(x)) \Rightarrow C(x)] \qquad \forall x \forall y\, [C(x) \Rightarrow (D(y) \vee E(y))]$$

Do a proof by resolution to prove

$$\forall x \forall y\, [(A(x) \wedge B(x)) \Rightarrow (D(y) \vee E(y))]$$

Please show *all* steps (i.e. negation of the conclusion, CNF, Skolemization) and note the *unifiers* that were used to resolve two clauses.

Solution:

1. First we convert the starting knowledge base to CNF. For the first sentence we execute the following steps:

(a) $\forall x\,[(A(x) \land B(x)) \Rightarrow C(x)]$

(b) $\forall x\,[\neg\,(A(x) \land B(x)) \lor C(x)]$

(c) $\forall x\,[\neg A(x) \lor \neg B(x) \lor C(x)]$

We do the same for the second sentence:

(a) $\forall x \forall y\,[C(x) \Rightarrow \forall x \forall y\,(D(y) \lor E(y))]$

(b) $\forall x \forall y\,[\neg C(x) \lor D(y) \lor E(y)]$

After standardizing variables and dropping universally quantified variables we get:

$$(\neg A(x_1) \lor \neg B(x_1) \lor C(x_1)) \land (\neg C(x_2) \lor D(y_1) \lor E(y_1)]$$

.

2. Next we negate the goal and convert it into CNF via the following steps:

(a) $\neg \forall x \forall y\,[(A(x) \land B(x)) \Rightarrow (D(y) \lor E(y))]$

(b) $\neg \forall x \forall y\,[\neg(A(x) \land B(x)) \lor (D(y) \lor E(y))]$

(c) $\neg \forall x \forall y\,[\neg A(x) \lor \neg B(x) \lor D(y) \lor E(y)]$

(d) $\exists x \exists y \neg\,[\neg A(x) \lor \neg B(x) \lor D(y) \lor E(y)]$

(e) $\exists x \exists y\,[\neg\neg A(x) \land \neg\neg B(x) \land \neg D(y) \land \neg E(y)]$

(f) $\exists x \exists y\,[A(x) \land B(x) \land \neg D(y) \land \neg E(y)]$

(g) $(A(K) \land B(K) \land \neg D(L) \land \neg E(L))$ where K and L are new Skolem constants

This gives us four additional clauses, each with just a single literal:

$$A(K)$$
$$B(K)$$
$$\neg D(L)$$
$$\neg E(L)$$

3. We can now do the following resolution proof:

1) $\neg A(x_1) \lor \neg B(x_1) \lor C(x_1)$	KB
2) $\neg C(x_2) \lor D(y_1) \lor E(y_1)$	KB
3) $A(K)$	Negated Conclusion
4) $B(K)$	Negated Conclusion
5) $\neg D(L)$	Negated Conclusion
6) $\neg E(L)$	Negated Conclusion
7) $\neg B(K) \lor C(K)$	$[1, 3, x_1/K]$
8) $C(K)$	$[4, 7]$
9) $D(y_2) \lor E(y_2)$	$[2, 8, x_2]$
10) $E(L)$	$[5, 9, y_2\text{L}]$
11) ()	$[6, 10]$

Problem 2: Minimax alpha-beta pruning (10 points)

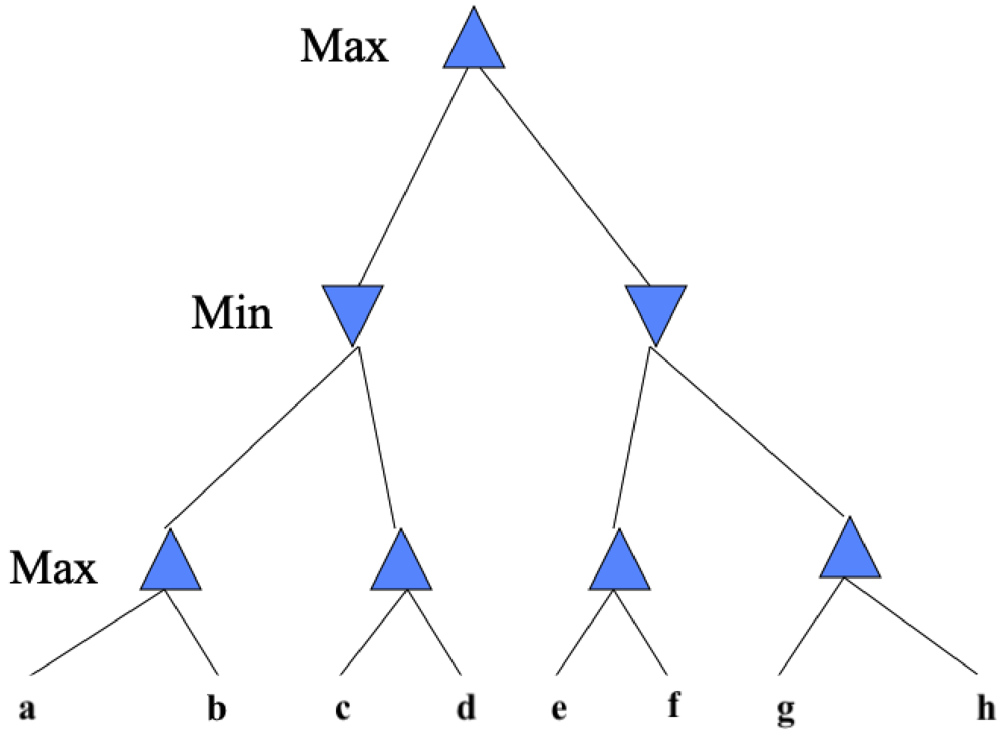

Figure 1: Min-max tree for Problem 2

Consider the above min-max tree in Figure 1, **a** to **h** represent numbers on the leaves. You should traverse the left sub-tree before the right sub-tree.

A (5 points): What criteria shall **a** to **h** hold if no pruning is possible? (Hint: the solution should be a set of inequalities)
Solution:
$c < max(a, b)$
$max(e, f) > min(max(a, b), max(c, d))$
$g < max(e, f)$

B (5 points):What is the maximum number of leaves that can be pruned? What criteria should **a** to **h** hold if maximum pruning occurs? (Hint: the solution should be a set of inequalities)
Solution:
3
$c >= max(a, b)$
$max(e, f) <= max(a, b)$

Problem 3: Minimax alpha-beta pruning, again (15 points)

Consider the min-max tree in Figure 2, **A** to **EE** are numbers on the leaf, and the numbers for leaf are shown in the last row.

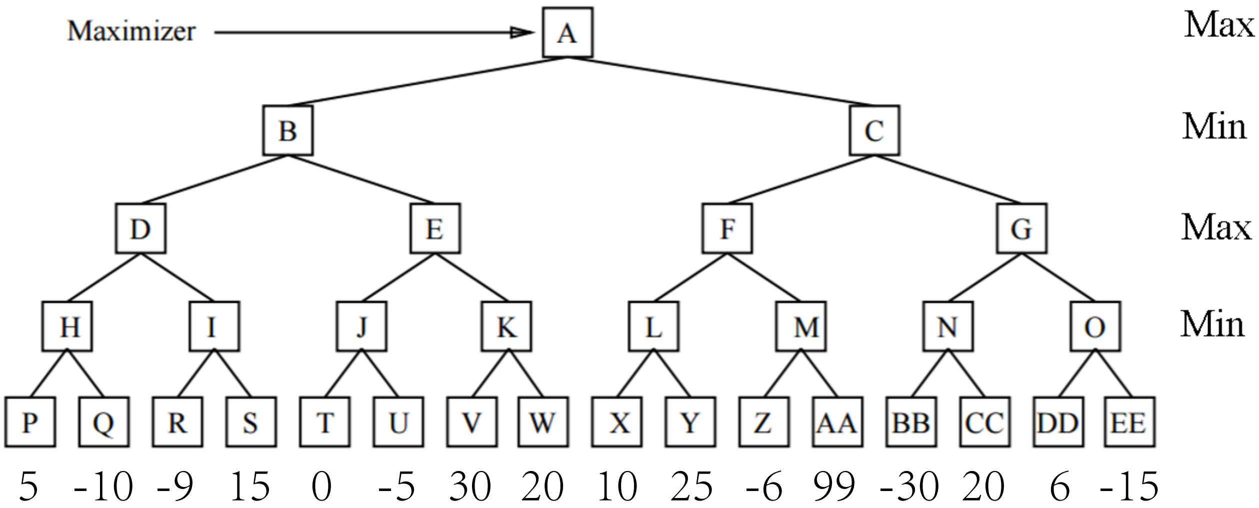

Figure 2: Min-max tree for Problem 3

A (5 points): Which move should be made at the top level (Left or Right)? What is its value? Show the steps of applying the Minimax algorithm below by giving the specific value for each node. (For example, $P = 3, Q = -10$ ...)

Solution:

Left, and the value is -9. The tree should be as figure 3.

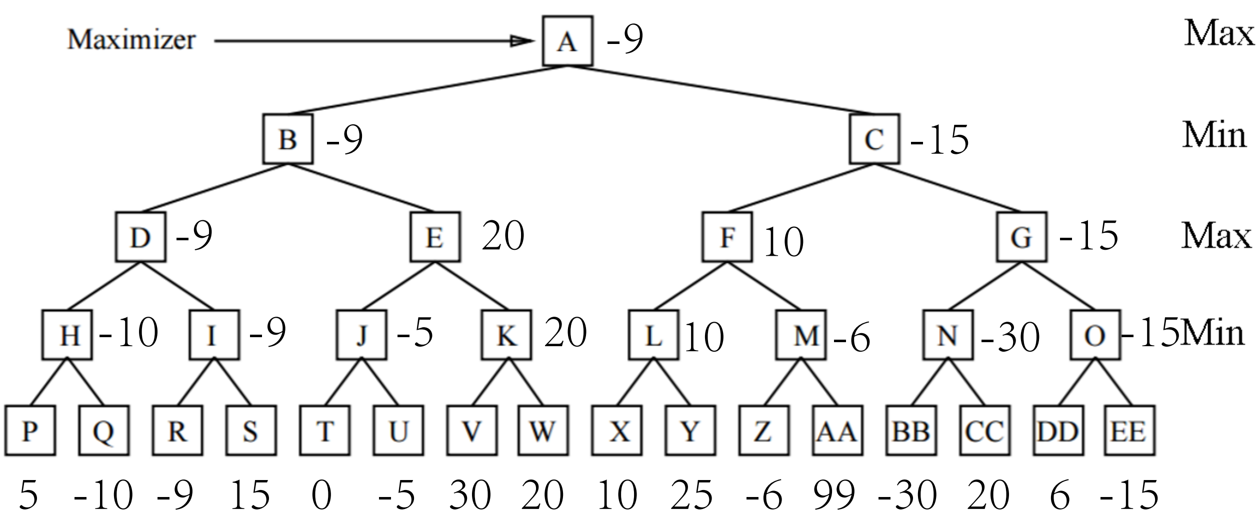

Figure 3: Min-max tree with number

For subproblems B & C, report your answers by:

1. *Writing the edge(s) that would be cut (e.g. M-to-AA)*

2. *The range of nodes to determine the pruning (e.g. range for F is $(10, \infty)$ and M is $(\infty, -6)$)*

B (5 points):Which nodes get pruned if you use alpha-beta pruning when you explore successors of states going left-to-right?
Solution:
Node: K, V, W, AA, CC
The value of α and β are $-\infty$ and -9 for the pruning of E-K.
The value of α and β are 10 and $+\infty$ for the pruning of M-AA.
The value of α and β are -9 and 10 for the pruning of N-CC.

C (5 points): Which nodes get pruned if you use alpha-beta pruning when you explore successors of states going right-to-left?
Solution:
Node: L, P, T, X, Y
The value of α and β are -9 and 20 for the pruning of H-P.
The value of α and β are 20 and $+\infty$ for the pruning of J-T.
The value of α and β are $-\infty$ and -15 for the pruning of F-L.

Problem 4: SAT Solvers (40 points)

The information for this problem can be found in a Jupyter Notebook. You can access it by using one of the following resources:

- Go to this (read only) Google Colab link: `https://colab.research.google.com/drive/1pmGpgJf6fAm5mUkrGwNrpqR9dYhMfsHJ`. Then press "Open in Playground" and then "Copy to Drive" (both in the top left corner) to make your own copy.
- Download the .ipynb file from the course website here: `https://www.cs.cornell.edu/courses/cs4700/2020sp/homeworks/SAT_exercise.ipynb`

A (8 points): See .ipynb file for question statement.
$7 \times 5 \times 15 + 7 \times 5 \times \binom{15}{2} = 4200$

B (18 points): Provide solution in .py file; submit via second Gradescope assignment.
C (14 points): Provide solution in .py file; submit via second Gradescope assignment.