CS 4700: Foundations of Artificial Intelligence **(Due: 1/31/20, 11:59pm)**

Homework #1: Background Check

55 Points Total

Instructor: Haym Hirsh *Name:* Student name, *Netid:* NetId

Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- Please include your name and NetIDs on the first page. We recommend typesetting your submission in LaTeX, and an Overleaf template is linked here.
- Homeworks must be submitted via Gradescope by the due date and time.
- Late homeworks will **not** be accepted for HW1.
- All sources of material outside the course must be cited. The University Academic Code of Conduct will be strictly enforced.

Problem 1: Probability (15 points)

A (5 points): Consider a real-valued random variable X that follows a uniform distribution between a and b. What is the mean and standard deviation of X?

B (5 points): Consider a two-player dice game. On each round:

- Player 1 rolls a single 6-sided die and adds 1 to the result.
- Player 2 rolls 2 6-sided dice and takes the maximum of the two.
- Whoever has the lower number pays the other \$1. (In case of a tie no money is paid in either direction.)

Each player starts with \$100. After playing 100 times, what is the expected value of how much money each player winds up with?

C (5 points): Imagine that you have a bag containing 100 balls of which 20 are green and 80 are red. 90% of the green balls have a stripe, whereas 30% of the red balls are without a stripe. If you pull out one ball and it has a stripe, what is the probability that it is green?

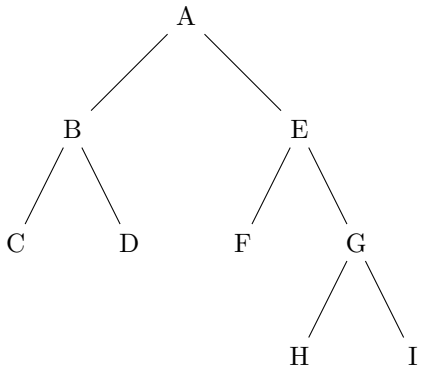
Problem 2: Calculus (10 points)

A (5 points): Consider the function $f(x) = \frac{1}{1+e^{-\overline{w}\cdot\overline{x}}}$ where $\overline{w} = (w_0, ..., w_n)$ and $\overline{x} = (x_0, ..., x_n)$. What is $\frac{\partial f}{\partial w_i}(\overline{x})$?

B (5 points): For what value(s) of x is $f(x, y) = 2x^2 + 4x + y^2$ is $f(x, y)$ minimized?

Problem 3: BST Traversals (8 points)

Consider the following binary tree:



A (2 points): What is the in-order traversal of this tree?

B (2 points): What is the post-order traversal of this tree?

C (2 points): What is the breadth-first traversal order of this tree?

D (2 points): What is the depth-first traversal order of this tree?

Problem 4: Complexity of BST (12 points)

Consider a binary search tree T containing n nodes.

A (4 points): What is the worst-case number of comparisons that are made to answer whether a particular item is in T? Give the exact formula.

B (2 points): Give the "big O" notation answer for part a.

C (4 points): What is the worst-case number of comparisons that are made to answer whether a particular item is in T if T is balanced? Give the exact formula.

D (2 points): Give the "big O" notation answer for part c.

Problem 5: First Order Logic (10 points)

Answer True or False for the following statements:

A (2 points): False $\vDash$ True

B (2 points): True $\vDash$ False

C (2 points): $[(A \wedge B) \Rightarrow C] \vDash [(A \Rightarrow C) \vee (B \Rightarrow C)]$

D (2 points): $[(A \vee B) \wedge (\neg C \vee \neg D \vee E) \vDash (A \vee B)$

E (2 points): $[(A \vee B) \wedge \neg(A \implies B)]$ is satisfiable