# CS 4700: Foundations of Artificial Intelligence

Spring 2020
Prof. Haym Hirsh

Lecture 6
February 3, 2020

# Reminder: Technology Policy

No technology except for first four rows of left and right sides

# Jupyter Notebooks

"Primer": Friday 5pm in Gates G01

# “Informed” Search

To formulate a problem:

- States: S
- Operators: Ops
- Initial state
- Goal condition: goal(s)
- Heuristic Evaluation Function f(s): $S \rightarrow \mathbb{R}$ (usually $\geq 0$)
    - f is an estimate of the merit of s
    - Typically $f(s_1) < f(s_2)$ means $s_1$ is “better” than $s_2$

# A* Search

Best-first search using

$$f(s) = g(s) + h(s)$$

$g(s)$ = sum of costs from initial state to s

$h(s)$ = estimate of cost from s to nearest goal

# A* Search

Initial call: A*(initialstate,ops,{},{})

```
A*(s,ops,open,closed) =
        If goal(s) Then return(s);
        Else If not(s ∈ closed)
          Then  successors ← {}; add(s,closed);
                For each o ∈ ops that applies to s add apply(o,s) to successors
                For each s ∈ successors
                        If s ∈ open and g(s) of current path is less, update g(s) in open
                        Else If s ∈ closed and g(s) of current path is less, add s to open w/ new g(s)
                        Else If s ∉ open and s ∉ closed, add s to open
        If not(empty(open))
                s' ← argmin_{s ∈ open}(f(s));
                open ← remove(s',open);
                A*(s',ops,open,closed)
`       Else return(FAIL)
```

# Properties of A* Search

- If
  - search space is a *finite* graph and
  - all operator costs are *positive*
- Then
  - A* is guaranteed to terminate and
  - if there is a solution, A* will find a solution (not necessarily an optimal one)

# Properties of A* Search

- If
  - search space is an *infinite* graph (but branching factor is finite) and
  - all operator costs are positive and are never less than some number Ɛ

    (in other words, they cannot get arbitrarily close to 0)
- Then
  - if there is a solution, A* will terminate with a solution (not necessarily an optimal one)

    (no guarantee of termination if there is no solution)

# Properties of A* Search

- If, in addition,
  - h(s) is admissible (for all states s, $0 \leq h(s) \leq h^*(s)$)
- Then
  - If A* terminates with a solution it will be optimal

# Properties of A* Search

- If, in addition,
  - h(s) is consistent (for all states s, h(s) ≤ h(apply(a,s)) + cost(apply(a,s))
- Then
  - h(s) is admissible,
  - the first path found to any state is guaranteed to have the lowest cost
    (do not need to check for this in the algorithm), and
  - A* is "optimal" - no other algorithm using the same h(s) and the same tie-breaking rules will expand fewer nodes than A*

# Properties of A* Search

- If
  - the search space is a tree,
  - there is a single goal state, and
  - for all states s, $|h^*(s) - h(s)| = O(\log(h^*(s)))$
    (the error of h(s) is never more than a logarithmic factor of $h^*(s)$)
- Then
  - A* runs in time polynomial in b (branching factor)

# Properties of A* Search

And many others

(Extremely widely used, so well-understood)

# A* Variants

Weighted A*:

- If
  - h(s) is admissible and
  - A* is used with $h'(s) = c \times h(s)$ where $c > 1$
- Then
  - Any goal state that A* terminates with will have cost no more than c times the cost of an optimal solution

# A* Variants

IDA*:

- Use cost-bounded depth-first search with h(initial state) as the bound
- Any time a successor is greater than the bound don’t expand it

  - But store the lowest cost C of any such state that you reach that exceeds the cost bound

- If you terminate without a goal state run cost-bounded depth-first search with depth bound C

(= Depth-first search emulation of A* search)

# A* Variants

SMA*:

- A* search with a memory bound
- If you would generate a node but don’t have space to add it to Open, remove from open the node s on Open with greatest f(s) but keep track of its parent s’ and the cost of the removed node f(s)
- If you reach a node on Open whose cost is worse than this value, you re-expand s’

# A* Variants

And many others

(Extremely widely used, so explored)

# Search Methods Thus Far

DFS

BFS

IDS

Best-First/A*

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DFS

BFS

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Best-First/A*

Focus is on optimality

# What if We’re OK with Suboptimal Solutions?

# What if We're OK with Suboptimal Solutions?

(A* variants, but what else?)

# Idea 1: Beam Search

Best-first search, but only keep the k best on Open

k is called the “beam width”

# Search Algorithm Template

Initial call: Search(initialstate,ops,{},{})

```
Search(s,ops,open,closed) =
        If goal(s) Then return(s);
        Else If not(s ∈ closed)
          Then
                successors ← {}; add(s,closed);
                For each o ∈ ops that applies to s
                        add apply(o,s) to successors
                open ← add successors to open;
        If not(empty(open))
                s' ← select(open);
                open ← remove(s',open);
                search(s',ops,open,closed)
`          Else return(FAIL)
```

# Search Algorithm Template

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```

# Beam Search

Initial call: BeamSearch(initialstate,ops,{},{},width)

```
BeamSearch(s,ops,open,closed) =
        If goal(s) Then return(s);
        Else If not(s ∈ closed)
          Then
                successors ← {}; add(s,closed);
                For each o ∈ ops that applies to s
                        add apply(o,s) to successors
                open ← add successors to open;
        open ← top-k_f(open,width)
        If not(empty(open))
                s' ← best_f(open);
                open ← remove(s',open);
                 BeamSearch(s',ops,open,closed)
`          Else return(FAIL)
```

# Beam Search

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        open ← best-k_f(open,width)
        If not(empty(open))
                s' ← best_f(open);
                open ← remove(s',open);
                 BeamSearch(s',ops,open,closed)
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```

best-$k_f$(x,k) = the k best items on x according to f

# Beam Search

Lose the guarantees, gain a bounded memory size, simple algorithm

# Idea 2: Hill Climbing

Loosely, beam search with width 1

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Loosely, beam search with width 1

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hence the name hill *climbing*)

(Just to confuse things, it includes gradient descent,
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(Just to confuse things even further textbook example minimizes f)

# Hill Climbing Example: 8 Queens

- Initial state = random placement of 8 queens, 1 per column
- Operators = pick a column and move its queen
- f(s) = # of attacked queens
- Want f(s) = 0

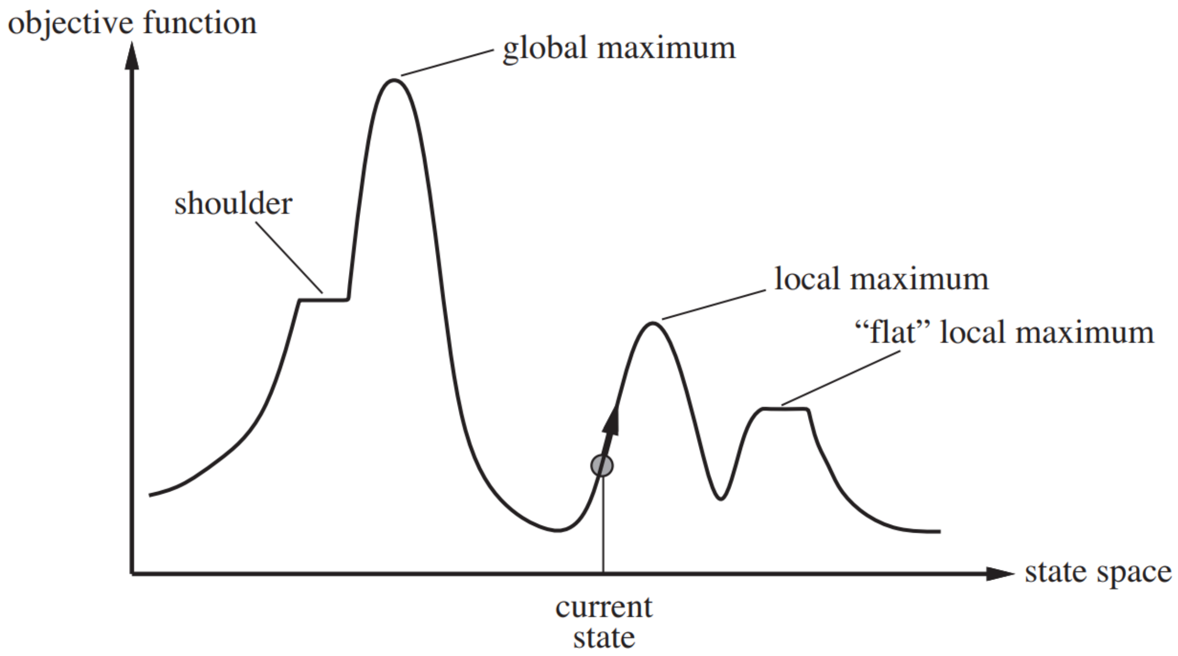
# Hill Climbing

```
hillclimbing(s):
    current ← s;
    loop
        new ← lowest-valued successor of s;
        if f(new) < f(s)
            then current ← new
            else return(current)
```

If goal is to find a maximum valued state, switch this to largest-valued and >

# Problems for Hill Climbing

- Local optima
- Plateau problem: no direction looks good (flat vs shoulder)
- Ridges: increases not aligned with axes



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This is *not* ≤

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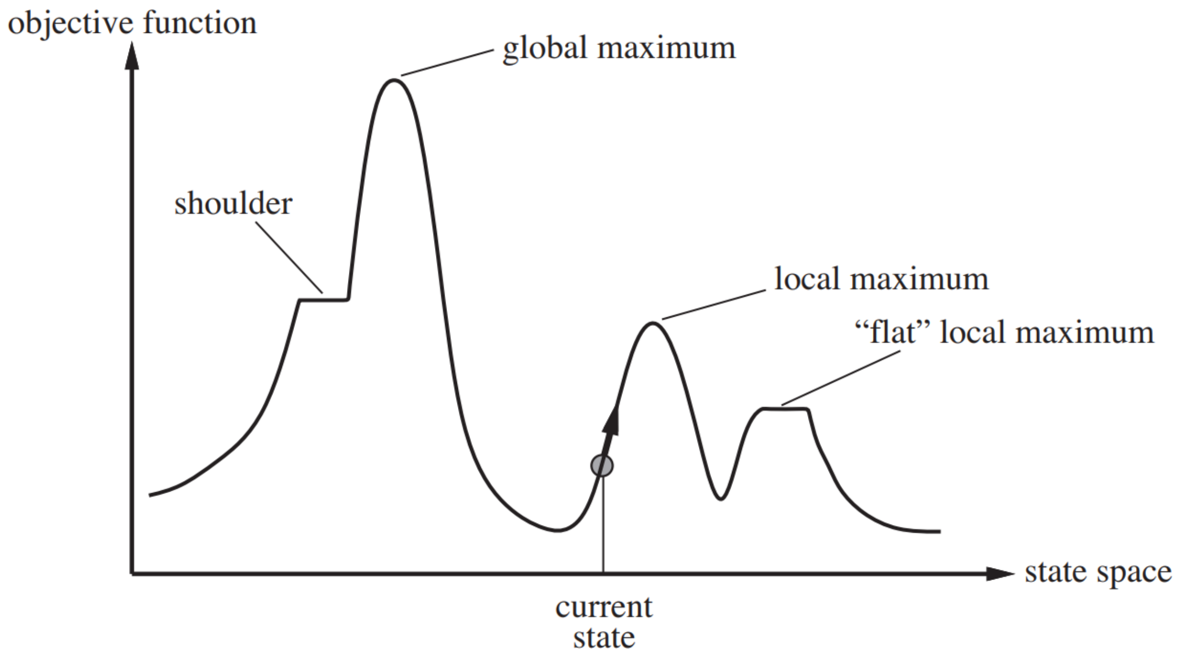

# Hill Climbing Variants

- Stochastic hill climbing:
  - Pick successor of a state probabilistically “proportional” to f values of successors
  - Can use a weighting scheme where early on you do this, but as you progress you become more and more likely to pick the best successor

  - Slower than pure hill-climbing, but can find better solutions, such as due to ridges

# Hill Climbing Variants

- Sideways moves:
  - Allow the algorithm to pick a successor with equal value if there is none with a better value
  - Do this at most some bounded number of times in a row

  - Good for plateaus

# Hill Climbing Variants

- First-choice hill-climbing:
  - Generate successors, stop and move ahead with the first successor that’s better than the current state
  - Good for problems with high branching factor

# Hill Climbing Variants

- Random restart:
  - If initial state is random or there are often ties that are broken randomly you can rerun hill climbing with different starting states

  - Good for local optima

# Hill Climbing Variants

- Combinations of the above
- Usually thought of as a tool kit and you try various options

# Simulated Annealing

Stochastic Hill Climbing Search
with a small, decreasing probability
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Stochastic Hill Climbing Search
with a small, decreasing probability
of doing a bad move

Intuition: To avoid getting stuck in local optima,
let yourself wander a little, less so as time progresses

Vocabulary: the farther into the search you go the lower the “temperature”

# Sample Simulated Annealing Algorithm

SA(s,ops):

current ← s; T ← initial T value; [For example, T=1]
loop
op ← random element of ops;
new ← apply(op,current);
delta ← f(new) – f(current);
if delta < 0 then current ← new
else with probability $e^{-\frac{\text{delta}}{T}}$ current ← new;
update T [For example, T= $\frac{1}{\text{iteration\#}}$]
until <stopping criterion> [For example, some max # of iterations]

# Genetic Algorithms and Evolutionary Computation

Totally different approach to search inspired by molecular genetics

A form of beam search

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- States: Assume have a structured representation
  - Example: N Queens – (position queen 1, …, position queen N) [n-tuples]

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  - Example: a bunch of boards with N random queens

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- Initial state: a set (“population”) of random states
  - Example: a bunch of boards with N random queens
- Operators: Apply generically, not (in its purist form) domain specific
  - Mutate: Perturb a state
    - Example: Change a queen position by 1
  - Crossover: Take two states and combine elements of both
    - Example: Take two N Queens boards, take k from one and N-k from the other

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- f(s): “fitness function”

# Genetic Algorithms and Evolutionary Computation

Algorithm sketch:

- Create an initial population of individuals (states) [population size]
- On each generation (iteration) create a new population by a combination of
  - Crossover:
    - Take two elements of the population biased by fitness function
    - Create a new individual (state) by taking pieces of each
  - Mutation:
    - Take an element of the population biased by fitness function
    - Create a new individual (state) by perturbing it

# Genetic Algorithms and Evolutionary Computation

There are MANY variants

# Genetic Algorithms and Evolutionary Computation

There are MANY variants

One that's distinctive enough to get its own mention:

Genetic programming:

- States are programs in a structured language
- Crossover and mutation create new programs from old ones