CS 4700: Foundations of Artificial Intelligence

Spring 2020
Prof. Haym Hirsh

Lecture 11
February 17, 2020

Upcoming Karma Lectures

- 2/20/20 11:40am, Gates G01:
  "An Integrated Approach for Efficient Neural Network Design, Training, and Inference"
  Amir Gholaminejad, UC Berkeley
- 2/27/20 11:40am, Gates G01:
  "Foundations of Learning Systems with (Deep) Function Approximators"
  Simon Du, Princeton University
- 3/5/20 11:40am, Gates G01:
  TBA
  Kevin Ellis, MIT
- 3/12/20 11:40am, Gates G01:
  TBA
  Emma Pierson, Stanford
- 3/19/20 11:40am, Gates G01:
  TBA
  Ashia Wilson, UC Berkeley

Homework 2

- Question 1:
  - A: Give the solution path found by uniform cost search and its cost
  - B: Give the solution path found by A* search and its cost
- Question 2:
  - Diagram should show every state that wound up on Open for that algorithm
  - Number/letter the nodes whose successors are generated during the algorithm's operation (won't be all nodes)
  - You get to pick the order in which "tied" nodes are selected from Open
- Question 3:
  - What value of R would get you to that state

Homework 2 questions?

Propositional Logic: Semantics

Truth Tables

φ	ψ	$\neg \varphi$	$\varphi \vee \psi$	$\varphi \wedge \psi$	$\varphi \Rightarrow \psi$
true	true	false	true	true	true
true	false	false	true	false	false
false	true	true	true	false	true
false	false	true	false	false	true

Assignment of true/false to propositional symbols determines true/false of sentence

Propositional Logic: Semantics

Typical questions we ask:

- Given a sentence φ:
  - Tautology: Is φ always true
    All assignments of True/False to the propositional symbols make φ evaluate to True
  - Satisfiable: Is there at least one way to assign True/False to the propositional symbols so that φ is True
  - Unsatifiable: Is φ always false
    All assignments of True/False to the propositional symbols make φ evaluate to False
- Given
  - Set of sentences: KB
  - Question: β
  - Entailment: If all the sentences in KB is true, must β be true?
    Written $KB \vDash \beta$

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Propositional Logic: Semantics

- Given:
  - Set of facts: KB
  - Question: β
- Does KB *entail* β: if KB is true, must β be true?
  - $KB \vDash \beta$
- Example:

$$\{P, P \Rightarrow Q\} \vDash Q$$

Propositional Logic: Semantics

- Given:
  - Set of facts: KB
  - Question: β
- Does KB *entail* β: if KB is true, must β be true?
  - KB ⊨ β

- Example:

$$\underbrace{\{P, P \Rightarrow Q\}}_{KB} \models \underbrace{Q}_{\beta}$$

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- Example:

$\{P, P \Rightarrow Q\} \models Q$

P	Q	$P \Rightarrow Q$
true	true	true
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  - KB ⊨ β
- Example:

{P, P ⇒ Q} ⊨ Q

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true	false	false
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  - Set of facts: KB
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- Example: {mythical ⇒ immortal
  ¬ mythical ⇒ (mortal ∧ mammal)
  (immortal ∨ mammal) ⇒ horned ⊨ magical
  horned ⇒ magical}

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  - KB ⊨ β

KB

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β

⊨ magical

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- Given:
  - Set of facts: KB
  - Question: β
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KB

- Example: {mythical $\Rightarrow$ immortal
  $\neg$ mythical $\Rightarrow$ (mortal $\wedge$ mammal)
  (immortal $\vee$ mammal) $\Rightarrow$ horned
  horned $\Rightarrow$ magical}

β

$\models$ magical

Truth Table would answer this – if every "line" where all of KB is true β is also true

Propositional Logic: Semantics

MOST COMMON CONFUSION

$\varphi \Rightarrow \psi$ is a sentence IN logic

$\varphi \vDash \psi$ is a statement ABOUT sentences in logic

Propositional Logic: Semantics

MOST COMMON CONFUSION

$\varphi \Rightarrow \psi$ is a sentence IN logic

$P \Rightarrow Q$

$\varphi \vDash \psi$ is a statement ABOUT sentences in logic

$\{P, (P \Rightarrow Q)\} \vDash Q$

Propositional Logic: Semantics

MOST COMMON CONFUSION

$\varphi \Rightarrow \psi$ is a sentence IN logic

$P \Rightarrow Q$

$\varphi \vDash \psi$ is a statement ABOUT sentences in logic

$\{P, (P \Rightarrow Q)\} \vDash Q$

Gets confusing, because $\{P, (P \Rightarrow Q)\} \vDash Q$ means $(P \wedge (P \Rightarrow Q)) \Rightarrow Q$ is true

Special Forms of Entailment

- Tautology, Valid: Is φ true for all truth assignments

$$\vDash \varphi$$

Special Forms of Entailment

Example: $(P \wedge (P \Rightarrow Q)) \Rightarrow Q$

Special Forms of Entailment

- Tautology, Valid: Is φ true for all truth assignments

$$\models \varphi$$

Example: $(P \wedge (P \Rightarrow Q)) \Rightarrow Q$

- Unsatisfiable: Is φ false for all truth assignments

$$\varphi \models \text{False}$$

Special Forms of Entailment

- Tautology, Valid: Is φ true for all truth assignments

$\vDash \varphi$ Example: $(P \wedge (P \Rightarrow Q)) \Rightarrow Q$

- Unsatisfiable: Is φ false for all truth assignments

$\varphi \vDash \text{False}$ Example: $P \wedge \neg P$

Special Forms of Entailment

- Tautology, Valid: Is φ true for all truth assignments

$$\models \varphi$$

Example: $(P \wedge (P \Rightarrow Q)) \Rightarrow Q$

- Unsatisfiable: Is φ false for all truth assignments

$$\varphi \models \text{False}$$

Example: $P \wedge \neg P$

$$\varphi \models ()$$

Special Forms of Entailment

- Tautology, Valid: Is φ true for all truth assignments

  $\vDash \varphi$ Example: $(P \wedge (P \Rightarrow Q)) \Rightarrow Q$

- Unsatisfiable: Is φ false for all truth assignments

  $\varphi \vDash \text{False}$ Example: $P \wedge \neg P$

  $\varphi \vDash ()$

- Satisfiable: Does there exist a truth assignment for which φ is true

  Equivalent to: $\neg\, \varphi$ is not a tautology

Special Forms of Entailment

- Tautology, Valid: Is φ true for all truth assignments

$$\models \varphi$$

Example: $(P \wedge (P \Rightarrow Q)) \Rightarrow Q$

- Unsatisfiable: Is φ false for all truth assignments

$$\varphi \models \text{False}$$

Example: $P \wedge \neg P$

$$\varphi \models ()$$

- Satisfiable: Does there exist a truth assignment for which φ is true

Equivalent to: $\neg\, \varphi$ is not a tautology

Example: $P \wedge Q$

Propositional Logic: Semantics

KB	$\vDash$?	β
1. mythical ⇒ immortal 2. ¬mythical ⇒ (mortal ∧ mammal) 3. (immortal ∨ mammal) ⇒ horned 4. horned ⇒ magical		magical

Elements of Formal Logic

- Syntax: What you can write down
- Semantics: The connection between what you write down and their meaning in the world being represented
- Inference: Making new conclusions based on what you already know

Propositional Logic: Inference

- Given:
  - Set of facts: KB
  - Question: β
- Can β be inferred from KB using a set of inference rules I:
  Can rules in I allow you to conclude β when starting from KB?
  Does KB *imply* β using I?
  - KB $\vdash_{\mathrm{I}}$ β
  - I is usually clear from context, so usually just write KB $\vdash$ β

Propositional Logic: Semantics

KB	$\models$?	β
1. mythical $\Rightarrow$ immortal 2. $\neg$mythical $\Rightarrow$ (mortal $\wedge$ mammal) 3. (immortal $\vee$ mammal) $\Rightarrow$ horned 4. horned $\Rightarrow$ magical		magical

Propositional Logic: Inference

KB	$\vdash$?	β

1. mythical $\Rightarrow$ immortal
2. $\neg$mythical $\Rightarrow$ (mortal $\wedge$ mammal)
3. (immortal $\vee$ mammal) $\Rightarrow$ horned
4. horned $\Rightarrow$ magical

magical

Propositional Logic: Inference

- Could answer this using truth tables:
  - For every set of truth assignments for which KB is true is β also true?

- Example:

  Find all combinations of

  mythical $\in$ {true,false}
  immortal $\in$ {true,false}
  mortal $\in$ {true,false}
  mammal $\in$ {true,false}
  horned $\in$ {true,false}
  magical $\in$ {true,false}

  for which

  mythical $\Rightarrow$ immortal
  $\neg$ mythical $\Rightarrow$ (mortal $\wedge$ mammal)
  (immortal $\vee$ mammal) $\Rightarrow$ horned
  horned $\Rightarrow$ magical)

  are true, then see if magical is true in all of them

- Problem: Exponential in number of variables

Propositional Logic: Inference

- Solution: Use inference rules that let you conclude new sentences from existing sentences

- Examples: Modus Ponens, Conjunction Elimination

Propositional Logic: Syntax vs Semantics vs Inference

MOST COMMON CONFUSION

$\varphi \Rightarrow \psi$ is a statement in logic

$\varphi \vDash \psi$ is a statement about logic semantics

$\varphi \vdash \psi$ is a statement about specific logic inference rules

Propositional Logic: Syntax vs Semantics vs Inference

MOST COMMON CONFUSION

$\varphi \Rightarrow \psi$ is a statement in logic

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$\varphi \vdash \psi$ is a statement about specific logic inference rules

(CS motive: use syntactic rules mechanically to infer new entailed sentences)

Why Propositional Logic?

There exist inference rules I such that
if φ and ψ are in propositional logic:

$$\varphi \vDash \psi \text{ if an only if } \varphi \vdash_{\mathrm{I}} \psi$$

Why Propositional Logic?

There exist inference rules I such that
if φ and ψ are in propositional logic:

$\varphi \vDash \psi$ if an only if $\varphi \vdash_{\mathrm{I}} \psi$

One direction (if $\varphi \vdash_{\mathrm{I}} \psi$ then $\varphi \vDash \psi$) says:
anything you infer is true (“soundness”)

The other direction(if $\varphi \vDash \psi$ then $\varphi \vdash_{\mathrm{I}} \psi$) says:
anything true can be inferred (“completeness”)

Soundness and Completeness of Inference Rules

- Soundness: Anything that you conclude must be true

$$KB \vdash \beta \quad \text{implies} \quad KB \models \beta$$

- Completeness: If something is true it can be inferred by the rules

$$KB \models \beta \quad \text{implies} \quad KB \vdash \beta$$

Our Focus: Resolution Theorem Proving

Sound and complete for propositional logic*

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Sound and complete for propositional logic*

*requires more details

Resolution Theorem Proving

Detail 1:

$KB \models \beta$

if and only if

$KB \wedge \neg(\beta)$ is unsatisfiable

Resolution Theorem Proving

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$KB \wedge \neg (\beta) \models ()$

Resolution Theorem Proving

Detail 1:

$$KB \vDash \beta$$

if and only if

$KB \wedge \neg (\beta)$ is unsatisfiable

$$KB \wedge \neg (\beta) \vDash ()$$

Resolution is complete only for unsatisfiability

Resolution Theorem Proving

Detail 1:

$KB \models \beta$

if and only if

$KB \land \neg (\beta)$ is unsatisfiable

$KB \land \neg (\beta) \models ()$

Resolution is complete only for unsatisfiability

("Refutation complete")

Resolution Theorem Proving

Detail 2:

Requires that all sentences be written in a constrained subset of propositional logic called

“Conjuctive (or Clausal) Normal Form”

(CNF)

Resolution Theorem Proving

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Requires that all sentences be written in a constrained subset of propositional logic called

“Conjuctive (or Clausal) Normal Form”

(CNF)

Every sentence in propositional logic has an equivalent sentence in CNF
(“equivalent” means truth table is identical)

Conjunctive Normal Form (CNF)

- CNF: Conjunction of disjunction of literals

<Literal> = <PropositionalSymbol> | $\neg$ <PropositionalSymbol>
<Clause> = (<Literal$_1$> $\vee$ … $\vee$ <Literal$_k$>) | ()
<Sentence> = (<Clause$_1$> $\wedge$ … <Clause$_n$>)

Conjunctive Normal Form (CNF)

- CNF: Conjunction of disjunction of literals

  <Literal> = <PropositionalSymbol> | $\neg$ <PropositionalSymbol>
  <Clause> = (<Literal_1> $\vee$ … $\vee$ <Literal_k>) | ()
  <Sentence> = (<Clause_1> $\wedge$ … <Clause_n>)

- Any sentence φ in propositional logic has an equivalent sentence CNF(φ) in CNF

$$\varphi \vDash \text{CNF}(\varphi)$$

$$\text{CNF}(\varphi) \vDash \varphi$$

Resolution Theorem Proving

To determine

$$KB \models ?\ \beta$$

Do resolution theorem proving

$$CNF(KB) \land CNF(\neg\beta) \vdash ?\ ()$$

Resolution Theorem Proving

Algorithm:

1. Convert KB and $\neg$ (β) to CNF, conjoin them to get KB’
2. Apply resolution to KB’ until you get an empty conjunction

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KB

- Example: {mythical $\Rightarrow$ immortal
  $\neg$ mythical $\Rightarrow$ (mortal $\wedge$ mammal)
  (immortal $\vee$ mammal) $\Rightarrow$ horned
  horned $\Rightarrow$ magical}

β

$\models$? magical

KB

- Example: $[(\text{mythical} \Rightarrow \text{immortal}) \wedge (\neg\, \text{mythical} \Rightarrow (\text{mortal} \wedge \text{mammal})) \wedge ((\text{immortal} \vee \text{mammal}) \Rightarrow \text{horned}) \wedge \text{horned} \Rightarrow \text{magical}] \vDash ?\ \text{magical}$

β

Example

$$(\text{mythical} \Rightarrow \text{immortal}) \wedge (\neg\text{mythical} \Rightarrow (\text{mortal} \wedge \text{mammal})) \wedge ((\text{immortal} \vee \text{mammal}) \Rightarrow \text{horned}) \wedge (\text{horned} \Rightarrow \text{magical}) \vDash \text{magical} \ ?$$

Example

$(\text{mythical} \Rightarrow \text{immortal}) \wedge (\neg\text{mythical} \Rightarrow (\text{mortal} \wedge \text{mammal}))$
$\wedge ((\text{immortal} \vee \text{mammal}) \Rightarrow \text{horned}) \wedge (\text{horned} \Rightarrow \text{magical}) \vDash \text{magical}$?

$(\text{my} \Rightarrow \text{i}) \wedge (\neg\text{my} \Rightarrow (\text{mo} \wedge \text{ml})) \wedge ((\text{i} \vee \text{ml}) \Rightarrow \text{h}) \wedge (\text{h} \Rightarrow \text{ma}) \vDash \text{ma}$?

Compute CNF($(\text{my} \Rightarrow \text{i}) \wedge (\neg\text{my} \Rightarrow (\text{mo} \wedge \text{ml})) \wedge ((\text{i} \vee \text{ml}) \Rightarrow \text{h}) \wedge (\text{h} \Rightarrow \text{ma})$)
Compute CNF($\neg\text{ma}$)

Conversion to CNF
Step 1: Remove $\Rightarrow$

$\alpha \Rightarrow \beta$ is the same as $\neg\alpha \vee \beta$

1. Replace some $\alpha \Rightarrow \beta$ with $\neg \alpha \vee \beta$

   Repeat until no $\Rightarrow$ is left

[no more "$\Rightarrow$"]

Example

$$(my \Rightarrow i) \ \wedge \ (\neg my \Rightarrow (mo \wedge ml)) \ \wedge \ ((i \vee ml) \Rightarrow h) \wedge \ (h \Rightarrow ma)$$

Example

$(my \Rightarrow i) \wedge (\neg my \Rightarrow (mo \wedge ml)) \wedge ((i \vee ml) \Rightarrow h) \wedge (h \Rightarrow ma)$

Example

$\underline{(my \Rightarrow i)} \land \underline{(\neg my \Rightarrow (mo \land ml))} \land \underline{((i \lor ml) \Rightarrow h)} \land \underline{(h \Rightarrow ma)}$

$(\neg my \lor i) \land (\neg\neg my \lor (mo \land ml)) \land (\neg(i \lor ml) \lor h) \land (\neg h \lor ma)$

Conversion to CNF
Step 2: Push in $\neg$

$\neg(\alpha \lor \beta)$ is the same as $\neg\alpha \land \neg\beta$

$\neg(\alpha \land \beta)$ is the same as $\neg\alpha \lor \neg\beta$

(DeMorgan's Laws)

2. Replace $\neg(\alpha \lor \beta)$ with $\neg\alpha \land \neg\beta$ or Replace $\neg (\alpha \land \beta)$ with $\neg \alpha \lor \neg \beta$

Repeat until no more can be done

[no more "$\Rightarrow$"]

["$\neg$" only before propositional symbols]

Example

$$(my \Rightarrow i) \land (\neg my \Rightarrow (mo \land ml)) \land ((i \lor ml) \Rightarrow h) \land (h \Rightarrow ma)$$

Step 1 gives

$$(\neg my \lor i) \land (\neg\neg my \lor (mo \land ml)) \land (\neg(i \lor ml) \lor h) \land (\neg h \lor ma)$$

Example

$$(my \Rightarrow i) \wedge (\neg my \Rightarrow (mo \wedge ml)) \wedge ((i \vee ml) \Rightarrow h) \wedge (h \Rightarrow ma)$$

Step 1 gives

$$(\neg my \vee i) \wedge (\neg\neg my \vee (mo \wedge ml)) \wedge (\underline{\neg(i \vee ml)} \vee h) \wedge (\neg h \vee ma)$$

Example

$(my \Rightarrow i) \wedge (\neg my \Rightarrow (mo \wedge ml)) \wedge ((i \vee ml) \Rightarrow h) \wedge (h \Rightarrow ma)$

Step 1 gives

$(\neg my \vee i) \wedge (\neg\neg my \vee (mo \wedge ml)) \wedge (\underline{\neg(i \vee ml)} \vee h) \wedge (\neg h \vee ma)$

Step 2 gives

$(\neg my \vee i) \wedge (\neg\neg my \vee (mo \wedge ml)) \wedge ((\neg i \wedge \neg ml) \vee h) \wedge (\neg h \vee ma)$

Example

$(my \Rightarrow i) \land (\neg my \Rightarrow (mo \land ml)) \land ((i \lor ml) \Rightarrow h) \land (h \Rightarrow ma)$

Step 1 gives

$(\neg my \lor i) \land (\neg\neg my \lor (mo \land ml)) \land (\neg(i \lor ml) \lor h) \land (\neg h \lor ma)$

Step 2 gives

$(\neg my \lor i) \land (\neg\neg my \lor (mo \land ml)) \land ((\neg i \land \neg ml) \lor h) \land (\neg h \lor ma)$

Conversion to CNF
Step 3: Remove Double-Negation

$\neg\neg\alpha$ is the same as α

3. Replace $\neg\neg\alpha$ with α

Repeat until no more can be done

[no more "⇒"]

["¬" only before propositional symbols]

["¬" only appears as singleton]

Example

$(my \Rightarrow i) \land (\neg my \Rightarrow (mo \land ml)) \land ((i \lor ml) \Rightarrow h) \land (h \Rightarrow ma)$

Step 1 gives

$(\neg my \lor i) \land (\neg\neg my \lor (mo \land ml)) \land (\neg(i \lor ml) \lor h) \land (\neg h \lor ma)$

Step 2 gives

$(\neg my \lor i) \land (\neg\neg my \lor (mo \land ml)) \land ((\neg i \land \neg ml) \lor h) \land (\neg h \lor ma)$

Example

$(my \Rightarrow i) \;\wedge\; (\neg my \Rightarrow (mo \wedge ml)) \;\wedge\; ((i \vee ml) \Rightarrow h) \wedge\; (h \Rightarrow ma)$

Step 1 gives

$(\neg my \vee i) \wedge (\neg\neg my \vee (mo \wedge ml)) \wedge (\neg(i \vee ml) \vee h) \wedge (\neg h \vee ma)$

Step 2 gives

$(\neg my \vee i) \wedge (\underline{\neg\neg my} \vee (mo \wedge ml)) \wedge ((\neg i \wedge \neg ml) \vee h) \wedge (\neg h \vee ma)$

Example

$$(my \Rightarrow i) \land (\neg my \Rightarrow (mo \land ml)) \land ((i \lor ml) \Rightarrow h) \land (h \Rightarrow ma)$$

Step 1 gives

$$(\neg my \lor i) \land (\neg\neg my \lor (mo \land ml)) \land (\neg(i \lor ml) \lor h) \land (\neg h \lor ma)$$

Step 2 gives

$$(\neg my \lor i) \land (\underline{\neg\neg my} \lor (mo \land ml)) \land ((\neg i \land \neg ml) \lor h) \land (\neg h \lor ma)$$

Step 3 gives

$$(\neg my \lor i) \land (my \lor (mo \land ml)) \land ((\neg i \land \neg ml) \lor h) \land (\neg h \lor ma)$$

Example

$$(my \Rightarrow i) \ \wedge \ (\neg my \Rightarrow (mo \wedge ml)) \ \wedge \ ((i \vee ml) \Rightarrow h) \wedge \ (h \Rightarrow ma)$$

Step 1 gives

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Step 2 gives

$$(\neg my \vee i) \wedge (\neg\neg my \vee (mo \wedge ml)) \wedge ((\neg i \wedge \neg ml) \vee h) \wedge (\neg h \vee ma)$$

Step 3 gives

$$(\neg my \vee i) \wedge (my \vee (mo \wedge ml)) \wedge ((\neg i \wedge \neg ml) \vee h) \wedge (\neg h \vee ma)$$

Conversion to CNF
Step 4: Distribute $\vee$ over $\wedge$

$\alpha \vee (\beta \wedge \gamma)$ is the same as $(\alpha \vee \beta) \wedge (\alpha \vee \gamma)$

4. Replace $\alpha \vee (\beta \wedge \gamma)$ with $(\alpha \vee \beta) \wedge (\alpha \vee \gamma)$

Repeat until no more can be done

[no more "$\Rightarrow$"]

["$\neg$" only before propositional symbols]

["$\neg$" only appears as singleton]

[done]

Example

$(my \Rightarrow i) \land (\neg my \Rightarrow (mo \land ml)) \land ((i \lor ml) \Rightarrow h) \land (h \Rightarrow ma)$

Step 1 gives

$(\neg my \lor i) \land (\neg\neg my \lor (mo \land ml)) \land (\neg(i \lor ml) \lor h) \land (\neg h \lor ma)$

Step 2 gives

$(\neg my \lor i) \land (\neg\neg my \lor (mo \land ml)) \land ((\neg i \land \neg ml) \lor h) \land (\neg h \lor ma)$

Step 3 gives

$(\neg my \lor i) \land (my \lor (mo \land ml)) \land ((\neg i \land \neg ml) \lor h) \land (\neg h \lor ma)$

Example

$(my \Rightarrow i) \wedge (\neg my \Rightarrow (mo \wedge ml)) \wedge ((i \vee ml) \Rightarrow h) \wedge (h \Rightarrow ma)$

Step 1 gives

$(\neg my \vee i) \wedge (\neg\neg my \vee (mo \wedge ml)) \wedge (\neg(i \vee ml) \vee h) \wedge (\neg h \vee ma)$

Step 2 gives

$(\neg my \vee i) \wedge (\neg\neg my \vee (mo \wedge ml)) \wedge ((\neg i \wedge \neg ml) \vee h) \wedge (\neg h \vee ma)$

Step 3 gives

$(\neg my \vee i) \wedge (\underline{my \vee (mo \wedge ml)}) \wedge (\underline{(\neg i \wedge \neg ml) \vee h}) \wedge (\neg h \vee ma)$

Step 4 gives

$(\neg my \vee i) \wedge (my \vee mo) \wedge (my \vee ml) \wedge (\neg i \vee h) \wedge (\neg ml \vee h) \wedge (\neg h \vee ma)$

Example

$$\text{CNF}(\ (my \Rightarrow i) \ \wedge \ (\neg my \Rightarrow (mo \wedge ml)) \ \wedge \ ((i \vee ml) \Rightarrow h) \wedge \ (h \Rightarrow ma)\ )$$

=

$$(\neg my \vee i) \wedge (my \vee mo) \wedge (my \vee ml) \wedge (\neg i \vee h) \wedge (\neg ml \vee h) \wedge (\neg h \vee ma)$$

Conversion to CNF Summary

1. Remove implications:
   Replace $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$ everywhere [no more "$\Rightarrow$"]
2. Push in $\neg$
   Replace $\neg(\alpha \vee \beta)$ with $\neg\alpha \wedge \neg\beta$ ["$\neg$" only before
   Replace $\neg(\alpha \wedge \beta)$ with $\neg\alpha \vee \neg\beta$ propositional symbols]
3. Remove double negations:
   Replace $\neg\neg\alpha$ with α ["$\neg$" only singletons]
4. Distribute $\vee$ over $\wedge$:
   Rewrite $\alpha \vee (\beta \wedge \gamma)$ as $(\alpha \vee \beta) \wedge (\alpha \vee \gamma)$

Resolution Theorem Proving

To determine: KB $\models$? β

Algorithm:

1. Convert KB and $\neg$ β to CNF, conjoin them to get KB’
2. Apply resolution to KB’ until you get an empty clause

Resolution Theorem Proving

To determine: KB $\vDash$? β

Algorithm:

1. Convert KB and ¬ β to CNF, conjoin them to get KB’
2. Apply resolution to KB’ until you get an empty clause

Resolution Inference Rule

$$\frac{(\boldsymbol{p} \vee \alpha_1 \vee \ldots \vee \alpha_a) \qquad (\neg \boldsymbol{p} \vee \beta_1 \vee \ldots \vee \beta_b)}{(\alpha_1 \vee \ldots \vee \alpha_a \vee \beta_1 \vee \ldots \vee \beta_b)}$$

where each α_i and β_j are literals (symbols or their negations) and $\boldsymbol{p}$ is a propositional symbol

Resolution Inference Rule

$$\frac{(p \lor \alpha_1 \lor \ldots \lor \alpha_a) \qquad (\neg p \lor \beta_1 \lor \ldots \lor \beta_b)}{(\alpha_1 \lor \ldots \lor \alpha_a \lor \beta_1 \lor \ldots \lor \beta_b)}$$

where each α_i and β_j are literals (symbols or their negations) and p is a propositional symbol

This really means

$$\frac{(\alpha_1 \lor \ldots \lor \alpha_{i-1} \lor p \lor \alpha_{i+1} \lor \ldots \lor \alpha_a) \qquad (\beta_1 \lor \ldots \lor \beta_{j-1} \lor \neg p \lor \beta_{j+1} \lor \ldots \lor \beta_b)}{(\alpha_1 \lor \ldots \lor \alpha_{i-1} \lor \alpha_{i+1} \lor \ldots \lor \alpha_a \lor \beta_1 \lor \ldots \lor \beta_{j-1} \lor \beta_{j+1} \lor \ldots \lor \beta_b)}$$

(but this becomes more awkward notationally)

Resolution Inference Rule Example

$(P \lor Q \lor \neg R)$

$\underline{(\neg S \lor T \lor \neg Q \lor V)}$

Resolution Inference Rule Example

$(P \vee Q \vee \neg R)$

$\underline{(\neg S \vee T \vee \neg Q \vee V)}$

Resolution Inference Rule Example

$$\frac{\begin{array}{c}(P \vee Q \vee \neg R)\\(\neg S \vee T \vee \neg Q \vee V)\end{array}}{(P \vee \neg R \vee \neg S \vee T \vee V)}$$

Resolution Inference Rule Example

WARNING!

$(P \lor Q \lor \neg R)$

$\underline{(\neg P \lor \neg Q \lor V)}$

Resolution Inference Rule Example

WARNING!

$(P \lor Q \lor \neg R)$
$(\neg P \lor \neg Q \lor V)$

Resolution Inference Rule Example

WARNING!

$(P \lor Q \lor \neg R)$

$\underline{(\neg P \lor \neg Q \lor V)}$

$(Q \lor \neg R \lor \neg Q \lor V)$

Resolution Inference Rule Example

WARNING!

$(P \lor Q \lor \neg R)$

$\underline{(\neg P \lor \neg Q \lor V)}$

Resolution Inference Rule Example

WARNING!

$$(P \vee Q \vee \neg R)$$
$$\underline{(\neg P \vee \neg Q \vee V)}$$
$$(P \vee \neg R \vee \neg P \vee V)$$

Resolution Inference Rule Example

WARNING!

$$\frac{(P \lor Q \lor \neg R) \quad (\neg P \lor \neg Q \lor V)}{(P \lor \neg R \lor \neg P \lor V)}$$

OR

$$\frac{(P \lor Q \lor \neg R) \quad (\neg P \lor \neg Q \lor V)}{(Q \lor \neg R \lor \neg Q \lor V)}$$

NOT

$$\frac{(P \lor Q \lor \neg R) \quad (\neg P \lor \neg Q \lor V)}{(\neg R \lor V)}$$

Resolution Theorem Proving

To determine: KB $\models$? β

Algorithm:

1. Convert KB and $\neg$ β to CNF, conjoin them to get KB'
2. Apply resolution to KB' until you get an empty clause

Resolution Proof Example

$(my \Rightarrow i) \land (\neg my \Rightarrow (mo \land ml)) \land ((i \lor ml) \Rightarrow h) \land (h \Rightarrow ma) \vDash ma$?

Compute CNF($(my \Rightarrow i) \land (\neg my \Rightarrow (mo \land ml)) \land ((i \lor ml) \Rightarrow h) \land (h \Rightarrow ma)$)

Compute CNF($\neg ma$)

KB’ = $(\neg my \lor i) \land (my \lor mo) \land (my \lor ml) \land (\neg i \lor h) \land (\neg ml \lor h) \land (\neg h \lor ma) \land (\neg ma)$

Resolution Proof Example

$(\neg\, my \lor i) \land (my \lor mo) \land (my \lor ml) \land (\neg i \lor h) \land (\neg ml \lor h) \land (\neg\, h \lor ma) \land (\neg\, ma)$

Resolution Proof Example

$(\neg\, my \lor i)$ $(my \lor mo)$ $(my \lor ml)$ $(\neg i \lor h)$ $(\neg ml \lor h)$ $(\neg\, h \lor ma)$ $(\neg\, ma)$

Resolution Proof Example

$(\neg my \vee i)$ $(my \vee mo)$ $(my \vee ml)$ $(\neg i \vee h)$ $(\neg ml \vee h)$ $(\neg h \vee ma)$ $(\neg ma)$

Resolution Proof Example

$(\neg\, my \vee i)$ $(my \vee mo)$ $(my \vee ml)$ $(\neg i \vee h)$ $(\neg ml \vee h)$ $(\neg\, h \vee ma)$ $(\neg\, ma)$

$(i \vee ml)$

Resolution Proof Example

$(\neg\, my \lor i)$ $(my \lor mo)$ $(my \lor ml)$ $(\neg i \lor h)$ $(\neg ml \lor h)$ $(\neg\, h \lor ma)$ $(\neg\, ma)$

$(i \lor ml)$

Resolution Proof Example

$(\neg\, my \lor i)$ $(my \lor mo)$ $(my \lor ml)$ $\underline{(\neg i \lor h)}$ $(\neg ml \lor h)$ $(\neg\, h \lor ma)$ $(\neg\, ma)$

$\underline{(i \lor ml)}$

Resolution Proof Example

$(\neg\, my \lor i)$ $(my \lor mo)$ $(my \lor ml)$ <u>$(\neg i \lor h)$</u> $(\neg ml \lor h)$ $(\neg\, h \lor ma)$ $(\neg\, ma)$

<u>$(i \lor ml)$</u>

$(ml \lor h)$

Resolution Proof Example

$(\neg\, my \lor i)$ $(my \lor mo)$ $(my \lor ml)$ $(\neg i \lor h)$ $(\neg ml \lor h)$ $(\neg\, h \lor ma)$ $(\neg\, ma)$

$(i \lor ml)$

$(ml \lor h)$

Resolution Proof Example

$(\neg my \lor i)$ $(my \lor mo)$ $(my \lor ml)$ $(\neg i \lor h)$ <u>$(\neg ml \lor h)$</u> $(\neg h \lor ma)$ $(\neg ma)$

$(i \lor ml)$

<u>$(ml \lor h)$</u>

Resolution Proof Example

$(\neg\, my \vee i)$ $(my \vee mo)$ $(my \vee ml)$ $(\neg i \vee h)$ $\underline{(\neg ml \vee h)}$ $(\neg\, h \vee ma)$ $(\neg\, ma)$

$(i \vee ml)$

$\underline{(ml \vee h)}$

(h)

Resolution Proof Example

$(\neg\, my \vee i)$ $(my \vee mo)$ $(my \vee ml)$ $(\neg i \vee h)$ $(\neg ml \vee h)$ $(\neg\, h \vee ma)$ $(\neg\, ma)$

$(i \vee ml)$

$(ml \vee h)$

(h)

Resolution Proof Example

(¬ my ∨ i)   (my ∨ mo)   (my ∨ ml)   (¬i ∨ h)   (¬ml ∨ h)   (¬ h ∨ ma)   (¬ ma)

(i ∨ ml)

(ml ∨ h)

(h)

Resolution Proof Example

$(\neg\, my \vee i)$ $(my \vee mo)$ $(my \vee ml)$ $(\neg i \vee h)$ $(\neg ml \vee h)$ $(\neg\, h \vee ma)$ $(\neg\, ma)$

$(i \vee ml)$

$(ml \vee h)$

(h)

(ma)

Resolution Proof Example

$(\neg\ my \vee i)$ $(my \vee mo)$ $(my \vee ml)$ $(\neg i \vee h)$ $(\neg ml \vee h)$ $(\neg\ h \vee ma)$ $(\neg\ ma)$

$(i \vee ml)$

$(ml \vee h)$

(h)

(ma)

Resolution Proof Example

$(\neg my \lor i)$ $(my \lor mo)$ $(my \lor ml)$ $(\neg i \lor h)$ $(\neg ml \lor h)$ $(\neg h \lor ma)$ <u>$(\neg ma)$</u>

$(i \lor ml)$

$(ml \lor h)$

(h)

<u>(ma)</u>

Resolution Proof Example

$(\neg my \lor i)$ $(my \lor mo)$ $(my \lor ml)$ $(\neg i \lor h)$ $(\neg ml \lor h)$ $(\neg h \lor ma)$ <u>$(\neg ma)$</u>

$(i \lor ml)$

$(ml \lor h)$

(h)

<u>(ma)</u>

$()$

Linearized Proof

1. $(\neg my \vee i)$
2. $(my \vee mo)$
3. $(my \vee ml)$
4. $(\neg i \vee h)$
5. $(\neg ml \vee h)$
6. $(\neg h \vee ma)$

Premises

7. $\neg ma$ — Negated Goal

8.	$(i \vee ml)$	[1,3]
9.	$(ml \vee h)$	[8,4]
10.	h	[9,5]
11.	ma	[10,6]
12.	()	[11,7]

Resolution Proof Example

(¬ my ∨ i) (my ∨ mo) (my ∨ ml) (¬i ∨ h) (¬ml ∨ h) (¬ h ∨ ma) (¬ ma)

(i ∨ ml)

(ml ∨ h)

(h)

(ma)

()

Resolution Proof Example

$(\neg\, my \lor i)$ $(my \lor mo)$ $(my \lor ml)$ $(\neg i \lor h)$ $(\neg ml \lor h)$ $(\neg\, h \lor ma)$ $\underline{(\neg\, ma)}$

$(i \lor ml)$

$(ml \lor h)$

(h)

$\underline{(ma)}$

$()$

Is this the only way to do it?

Resolution Proof Example

$(\neg my \lor i)$ $(my \lor mo)$ $(my \lor ml)$ $(\neg i \lor h)$ $(\neg ml \lor h)$ $(\neg h \lor ma)$ $\underline{(\neg ma)}$

$(i \lor ml)$

$(ml \lor h)$

(h)

$\underline{(ma)}$

$()$

Is this the only way to do it?

No. But we just need *one* way, not all ways. Any proof is fine.

Resolution Proof Example

(¬ my ∨ i)   (my ∨ mo)   (my ∨ ml)   (¬i ∨ h)   (¬ml ∨ h)   (¬ h ∨ ma)   (¬ ma)

(i ∨ ml)

(ml ∨ h)

(h)

(ma)

()

Is this the only way to do it?

No. But we just need *one* way, not all ways. Any proof is fine.

Which one? Choose a search method.