CS 4700: Foundations of Artificial Intelligence

Spring 2020

Prof. Haym Hirsh

Lecture 10

February 12, 2020

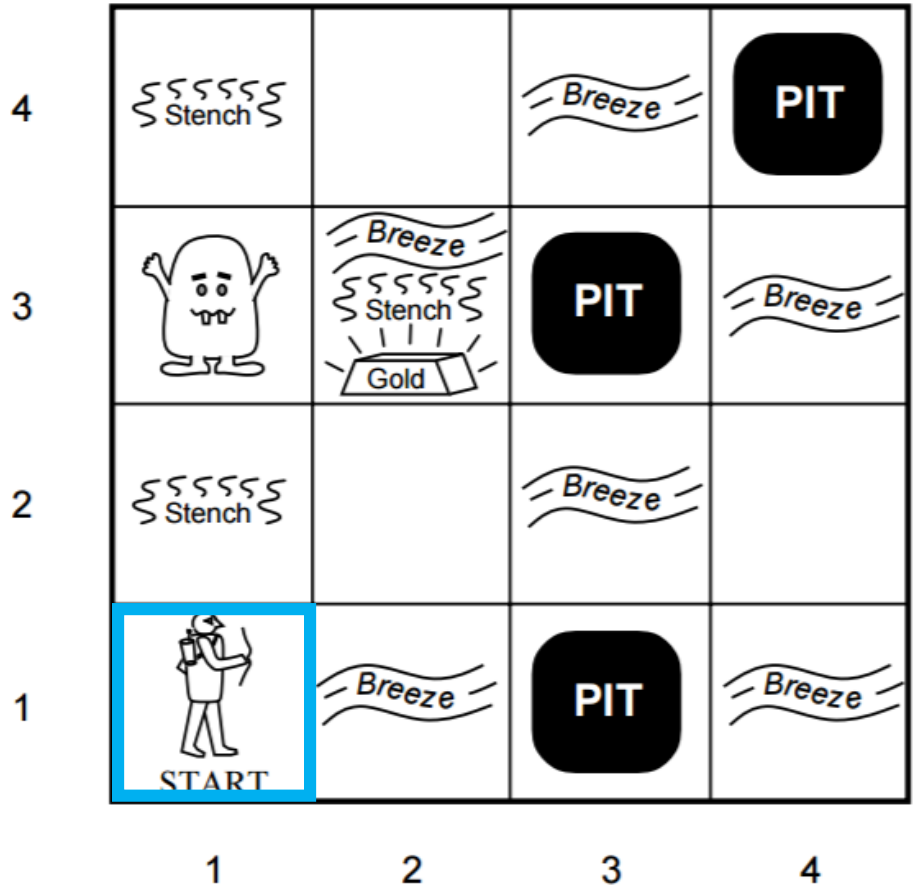
Next Topic:
Knowledge Representation and Reasoning
Textbook Chapter 7

Knowledge Representation and Reasoning

- Agents:
  - Represent what they know about the world
  - Use inference to derive new information

Sometimes called the “Logicist Approach” to AI

Wumpus World



Get points for taking gold

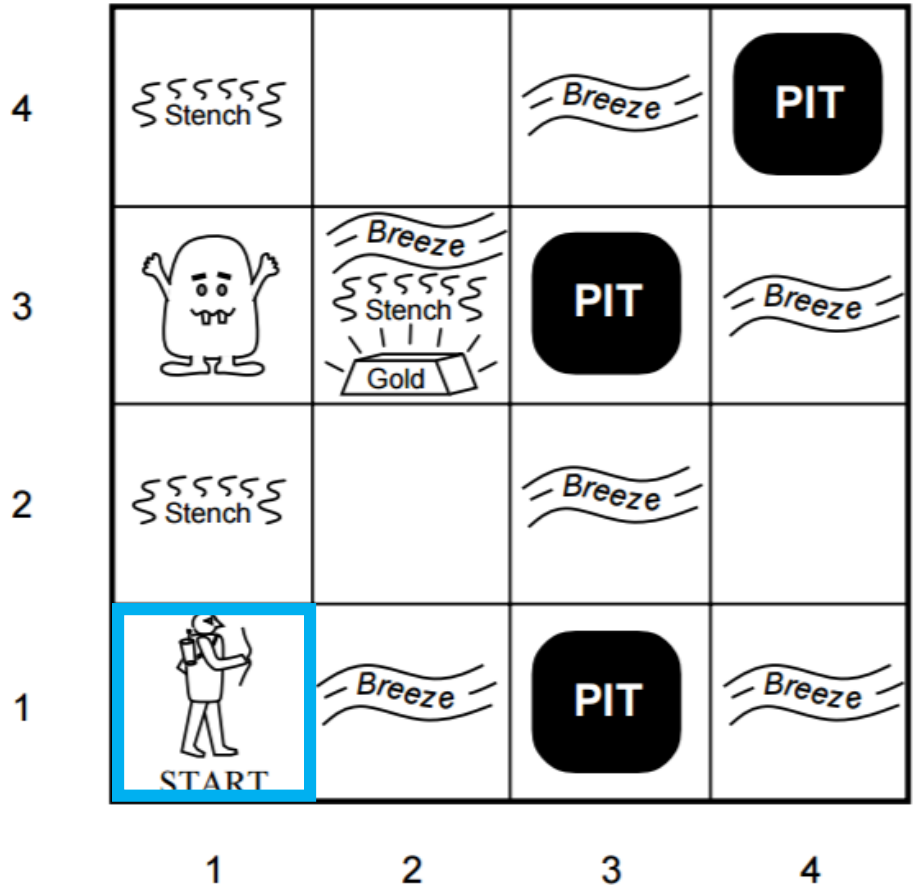
Die if in the same square as pit or wumpus

Can move Up, Down, Left, Right

Sensors:

- Stench: Wumpus is 1 away
- Breeze: Pit is 1 away
- Glitter: Gold is in current room
- States: [x, y, Stench?, Breeze?, Glitter?]
  - Initial state: [1,1,False,False,False]

Wumpus World



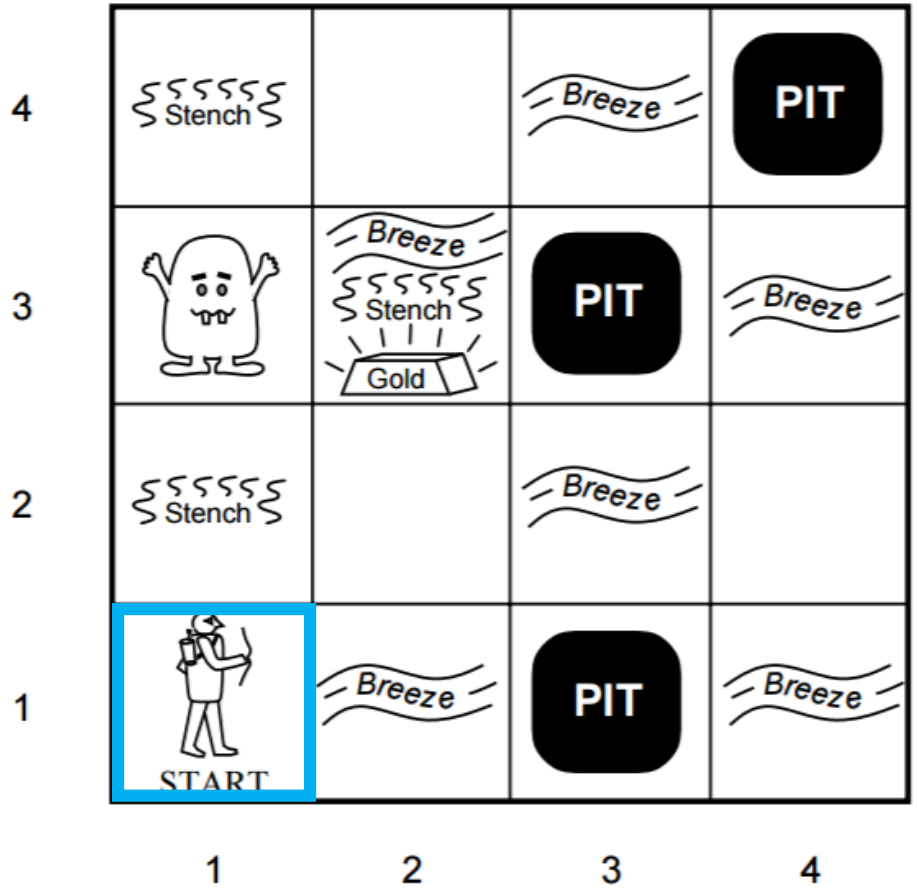
Definition of world
(not known to agent)

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2	2,2	3,2	4,2
1,1 A OK	2,1	3,1	4,1

State = [1,1,False,False,False]

What the agent knows

Wumpus World



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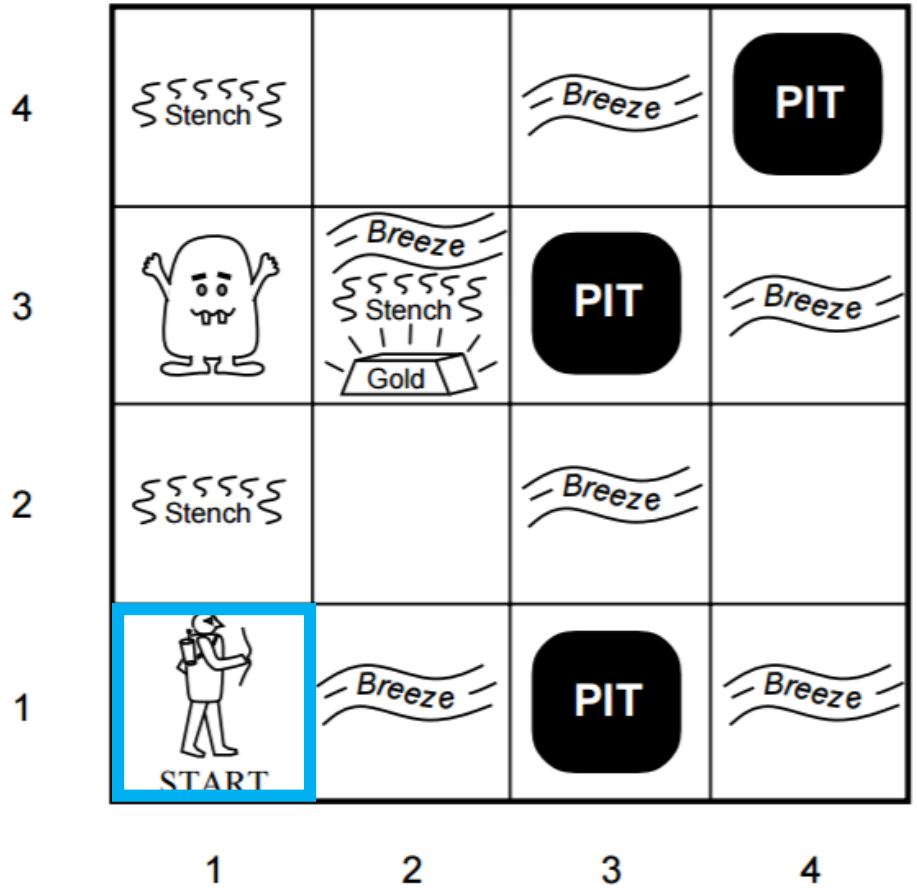
1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2	2,2	3,2	4,2
1,1 A OK	2,1	3,1	4,1

OK means “safe” (won’t die)

What the agent knows

Wumpus World



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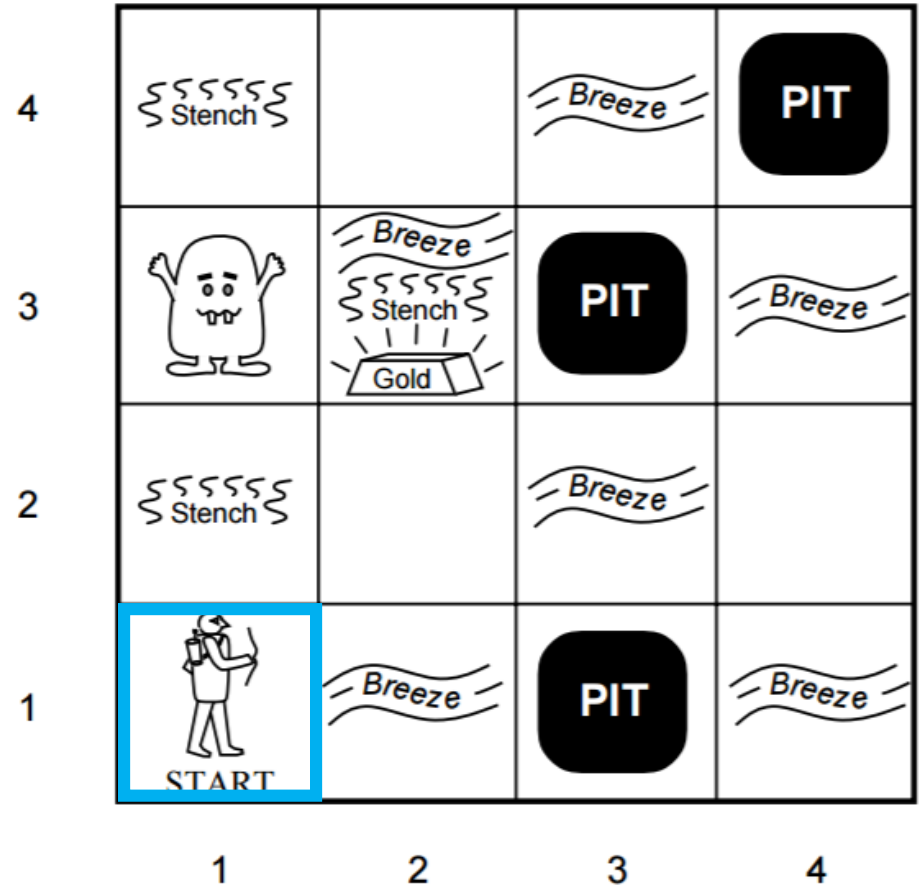

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2 OK	2,2	3,2	4,2
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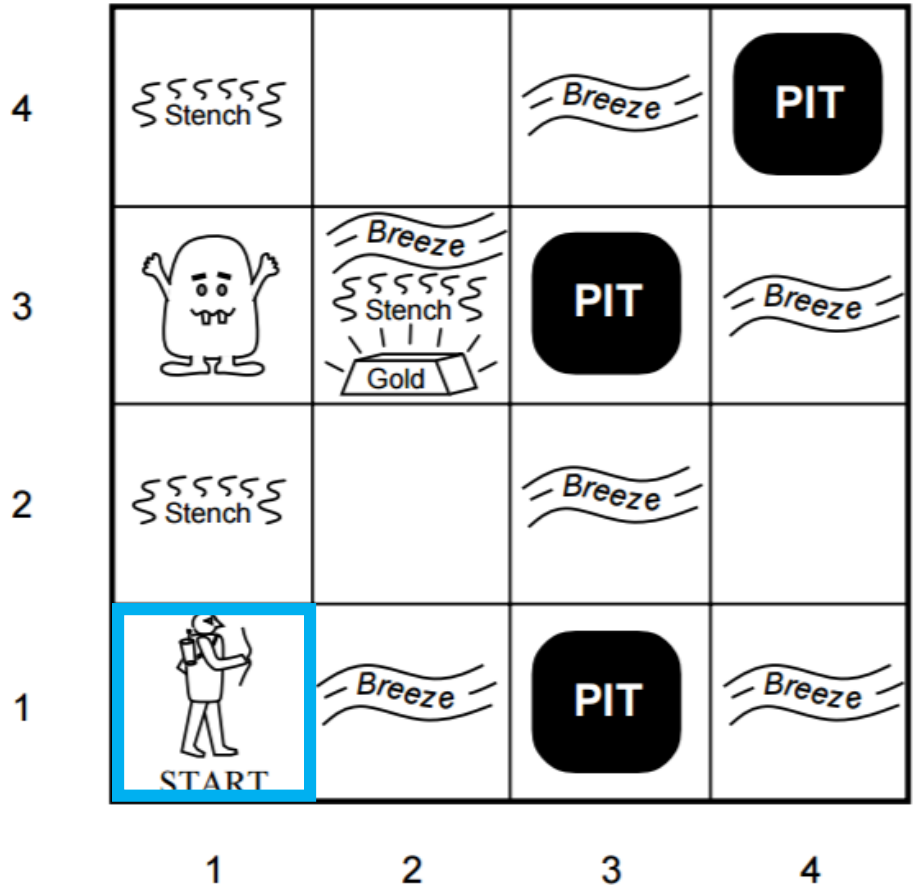
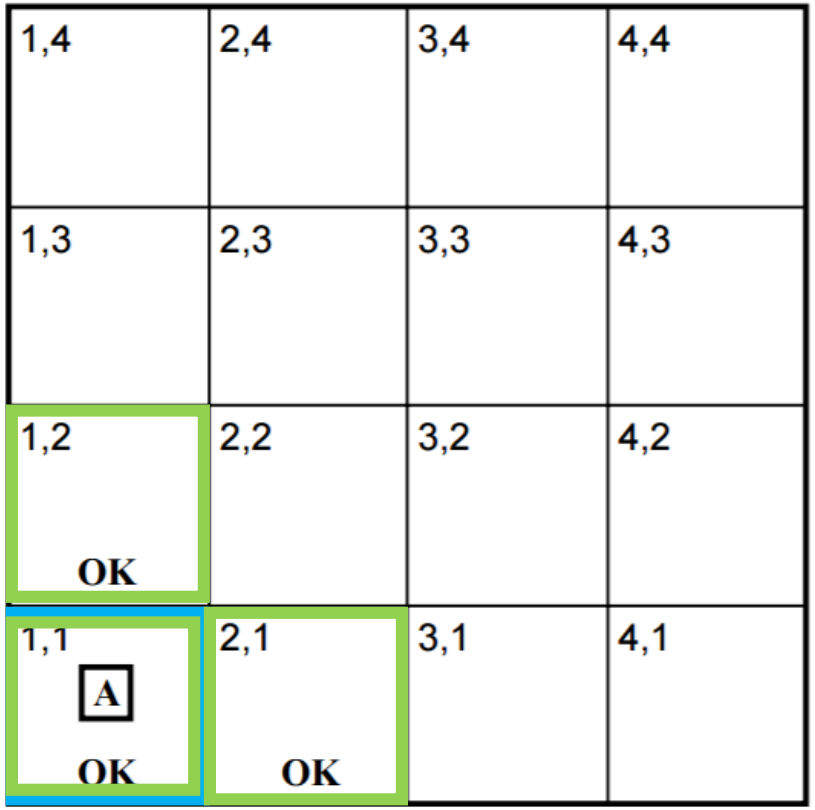
Wumpus World

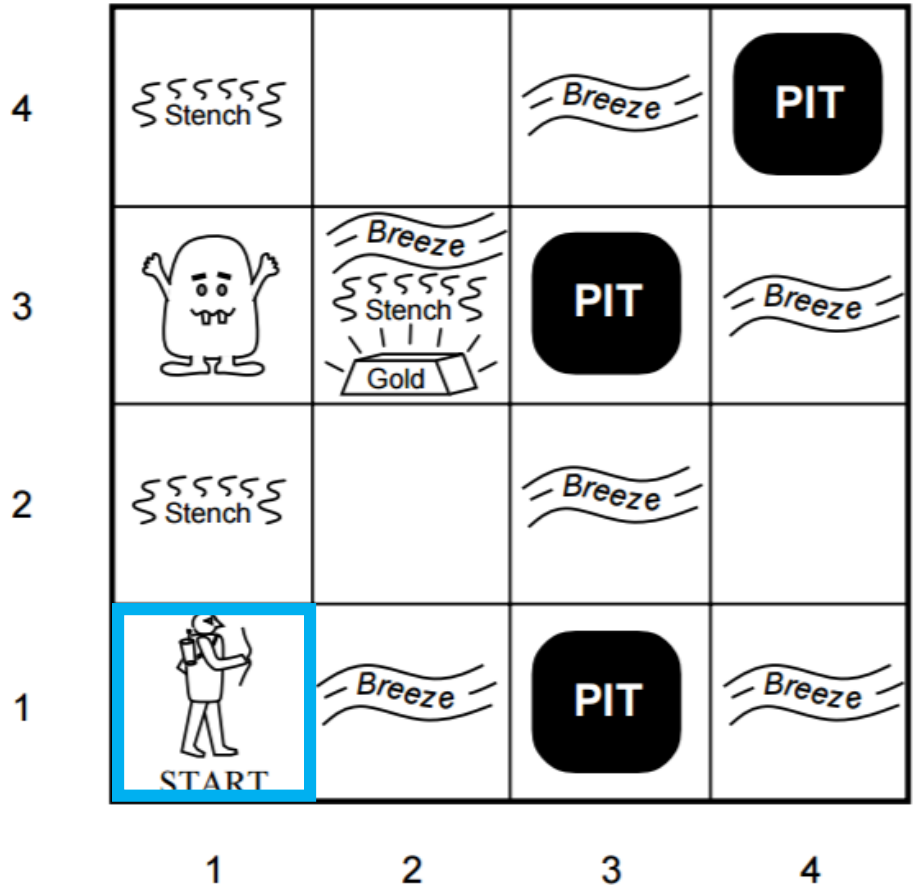




No stench => no Wumpus in adjacent squares

No breeze => no pit in adjacent squares

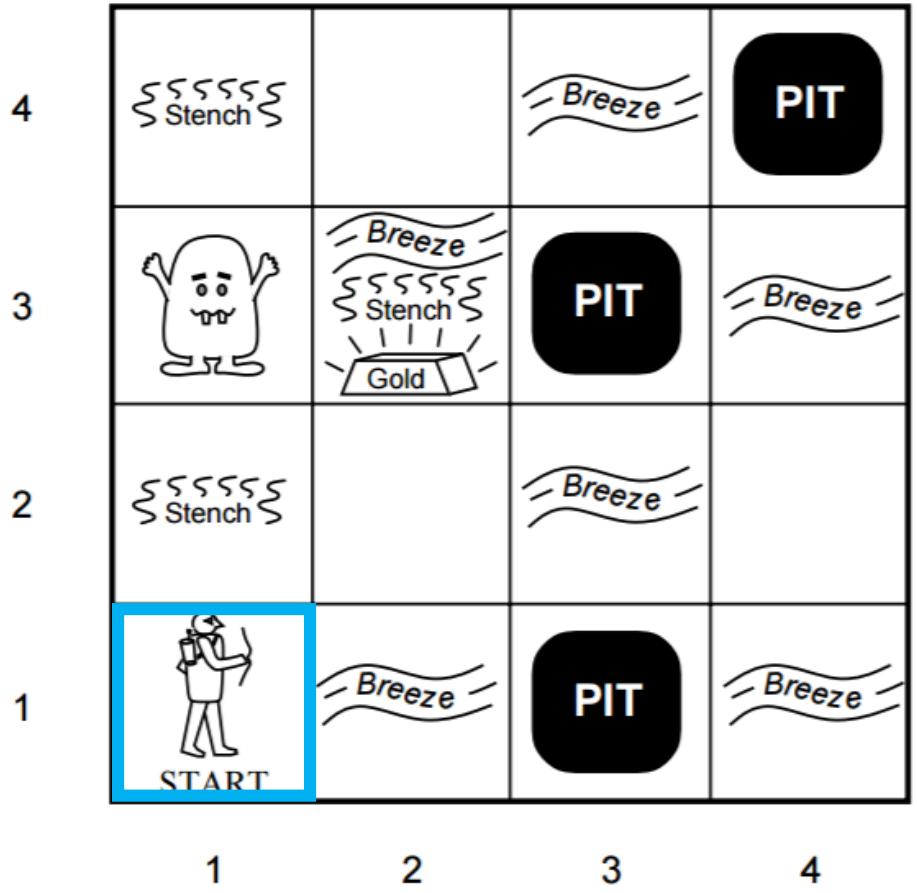
Wumpus World



1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2 OK	2,2	3,2	4,2
1,1 A OK	2,1 OK	3,1	4,1

No glitter => no gold here

Wumpus World



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1,3	2,3	3,3	4,3
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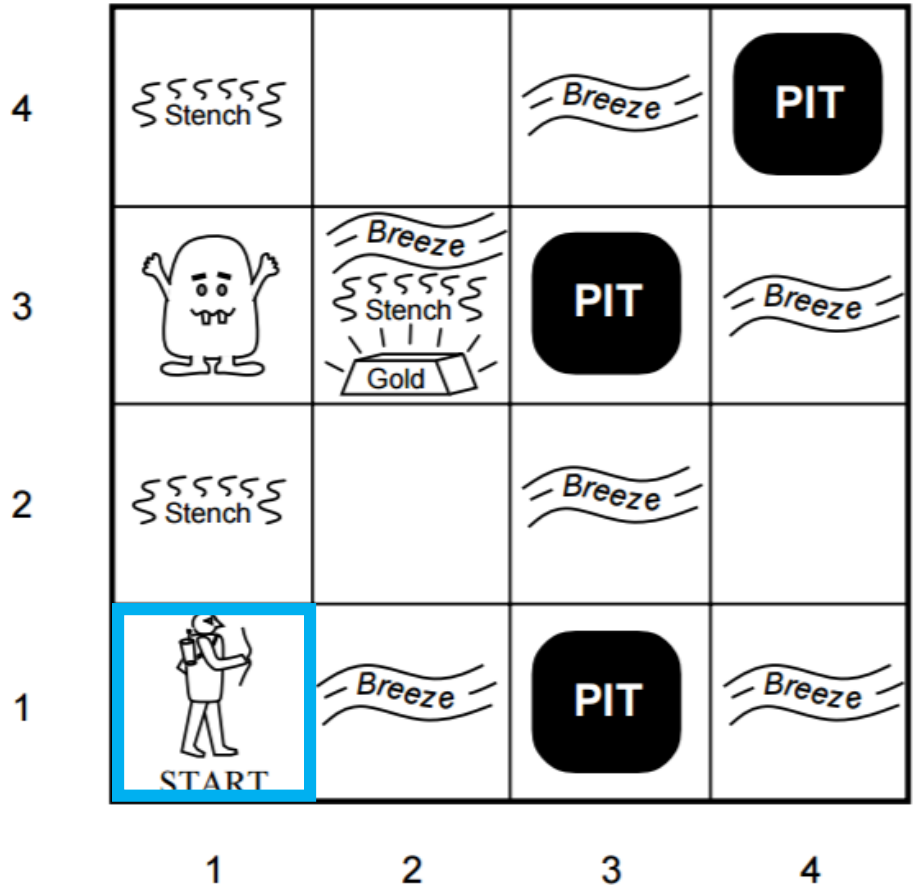
1,1 OK	2,1 A B OK	3,1	4,1

Wumpus moves Right

Wumpus World



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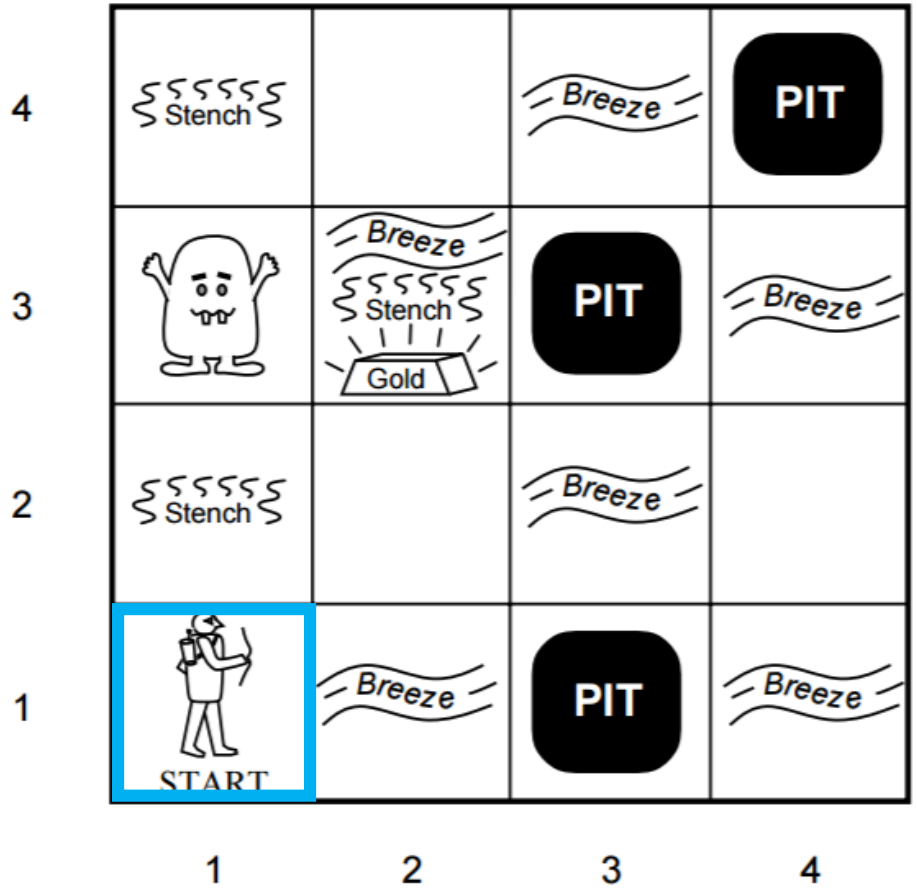
State = [2,1,False,True,False]

Wumpus World



1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2 OK	2,2 P?	3,2	4,2
1,1 OK	2,1 A B OK	3,1 P?	4,1

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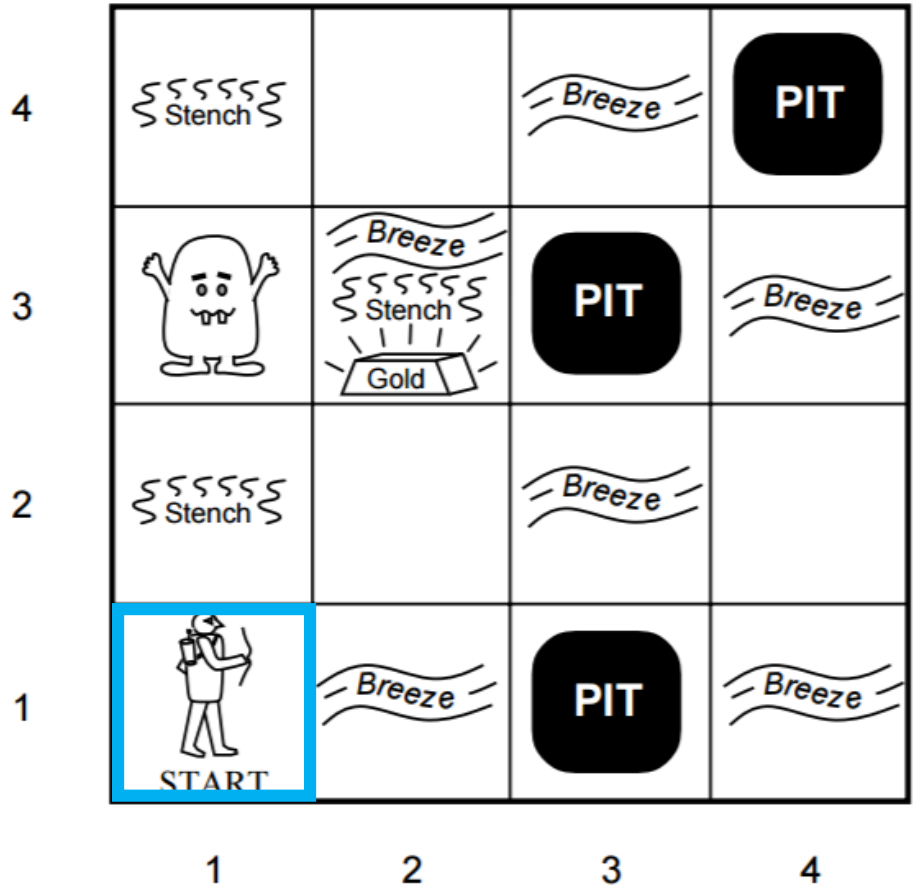

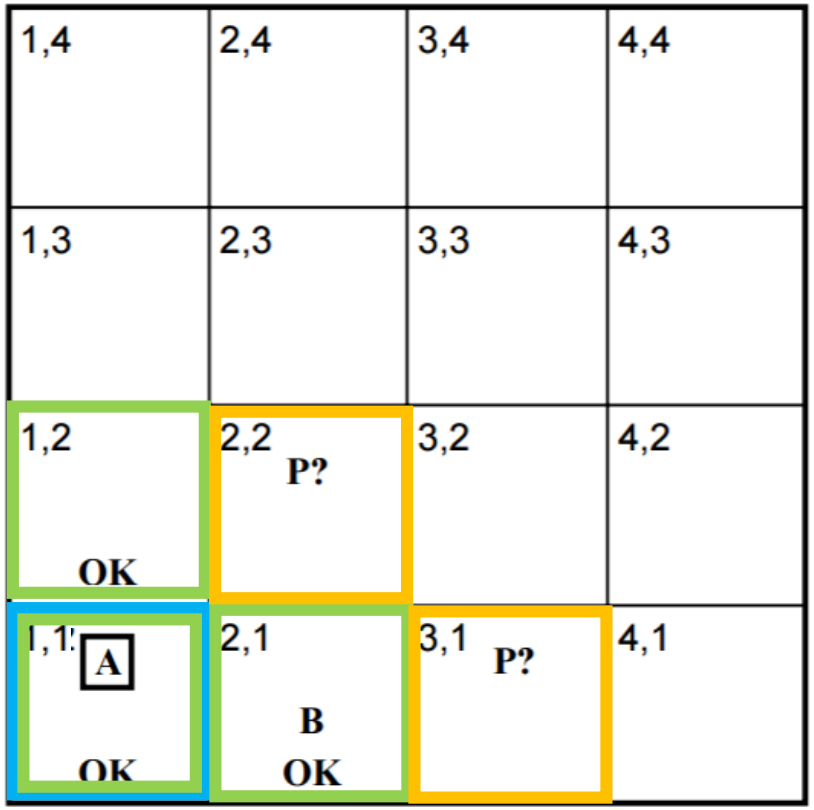

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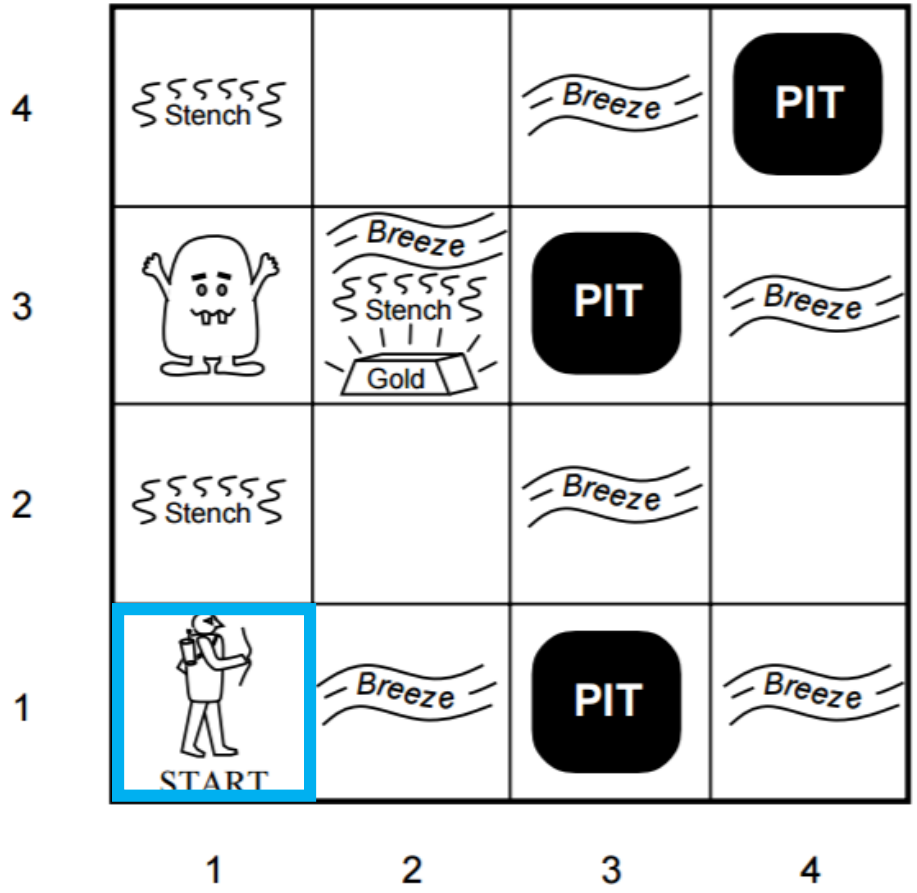

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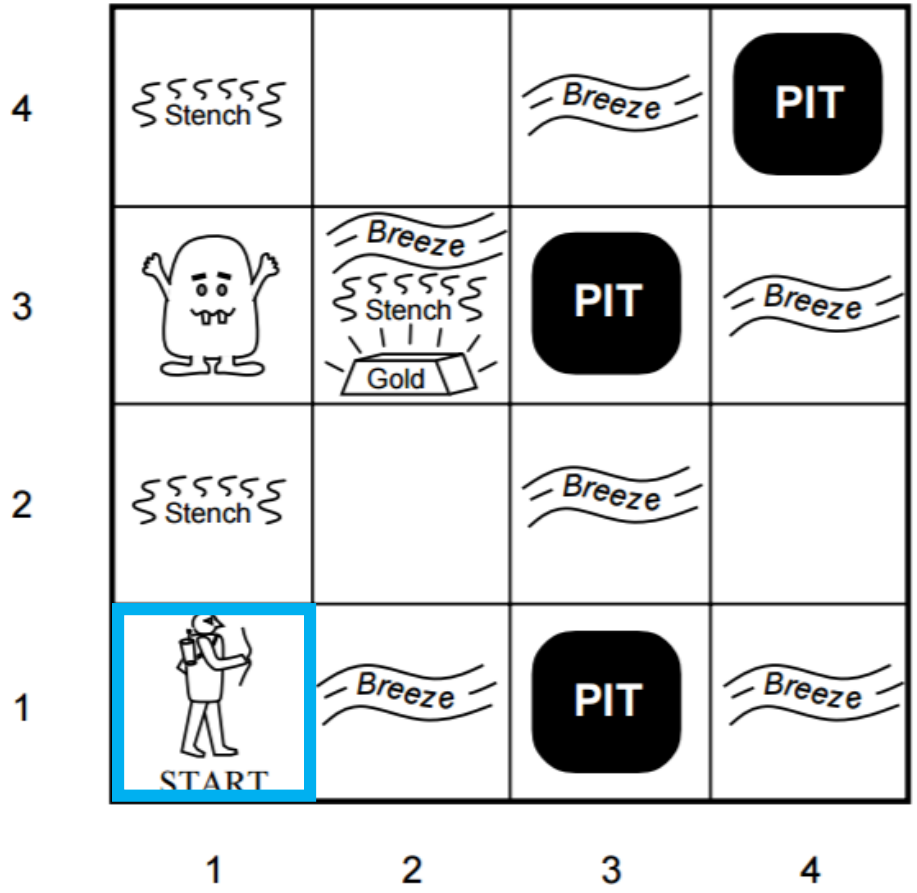

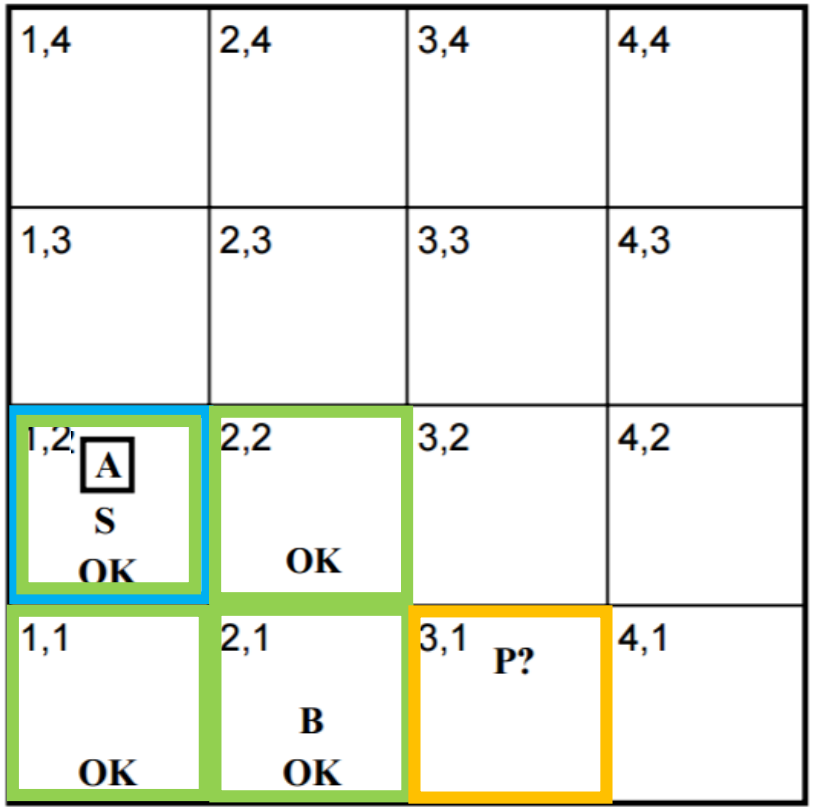

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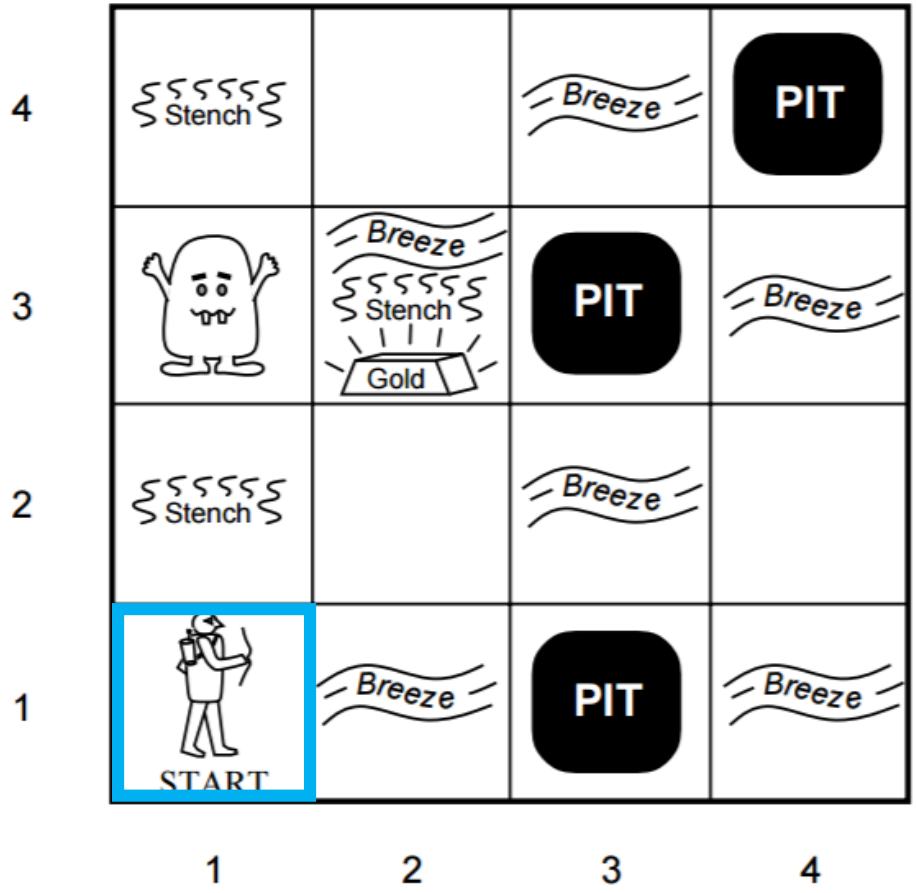

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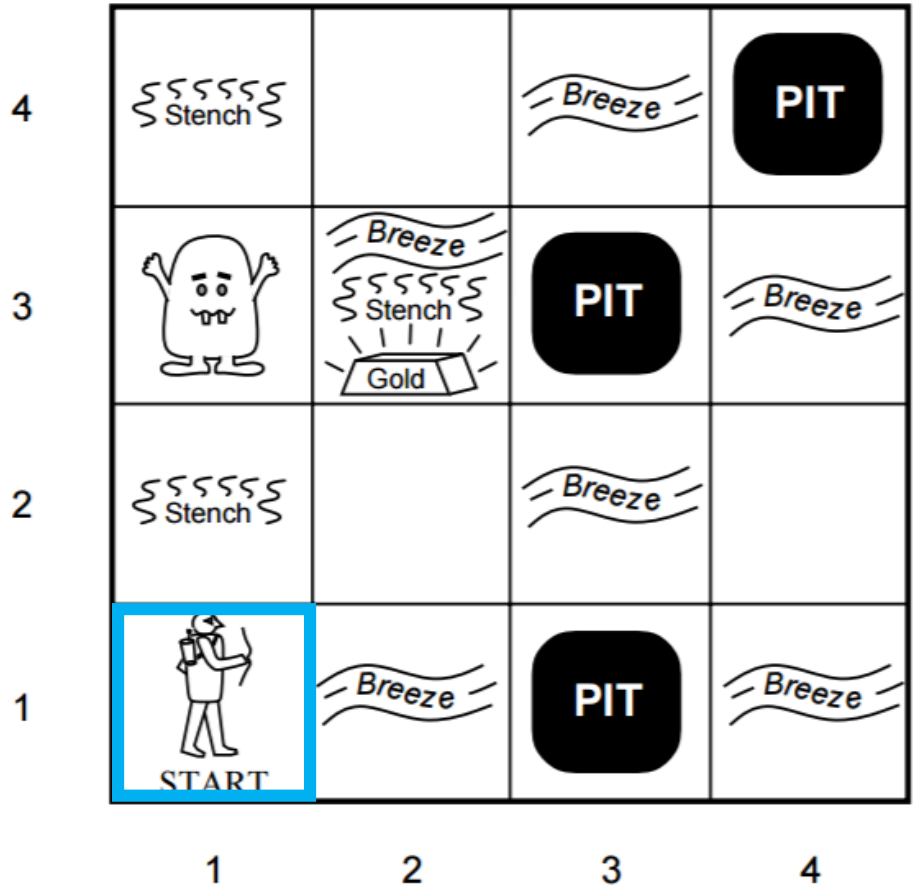

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Focus here will be on formal logic

Propositional Logic

Example:

A. If the unicorn is mythical, then it is immortal, but if it is not mythical then it is a mortal mammal.

B. If the unicorn is either immortal or a mammal, then it is horned.

C. The unicorn is magical if it is horned.

Propositional Logic

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- Is the unicorn magical?

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- Is the unicorn magical?
  - It is either immortal or a mammal (A), which means it's horned (B), which means it's magical (C)

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Starting "knowledge"

- Is the unicorn magical?
  - It is either immortal or a mammal (A), which means it's horned (B), which means it's magical (C)

"Derived" or "inferred" "knowledge"

Propositional Logic

Formulate the question in logic:

mythical	¬ mythical
immortal	¬ immortal
horned	¬ horned
magical	¬ magical

Propositional Logic

Example:

A. If the unicorn is mythical, then it is immortal, but if it is not mythical then it is a mortal mammal.

   1. $\text{mythical} \Rightarrow \text{immortal}$ 2. $\neg\ \text{mythical} \Rightarrow (\text{mortal} \wedge \text{mammal})$

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$3.\ (\text{immortal} \vee \text{mammal}) \Rightarrow \text{horned}$

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$4.\ \text{horned} \Rightarrow \text{magical}$

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Propositional Logic

1. mythical $\Rightarrow$ immortal
2. $\neg$ mythical $\Rightarrow$ (mortal $\wedge$ mammal)
3. (immortal $\vee$ mammal) $\Rightarrow$ horned
4. horned $\Rightarrow$ magical

Is the unicorn magical?

Propositional Logic

$(\text{mythical} \vee \neg \text{mythical})$

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(mythical $\vee \neg$ mythical)

(immortal $\vee \neg$ mythical) [1]

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(immortal $\vee$ mammal)

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horned [3]

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Still too informal

Propositional Logic

- Inferences are made using *inference rules*
- Popular examples:
  - Modus ponens

$$\alpha \Rightarrow \beta$$
$$\underline{\alpha \quad\quad}$$
$$\beta$$

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  - Conjuction elimination rule

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Forward pointer: We don't care about popular

Propositional Logic

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  - Conjunction elimination rule

$$\frac{\alpha \wedge \beta}{\alpha} \qquad \frac{\alpha \wedge \beta}{\beta}$$

$$\frac{\text{mortal} \wedge \text{mammal}}{\text{mammal}}$$

Forward pointer: We care if it can be automated

Elements of Formal Logic

- Syntax: What you can write down
  - “mythical” could have been “m”, or even “nuawerfbxcz”- they’re the symbols we choose, and must use consistently
- Semantics: The connection between what you write down and their meaning in the world being represented
  - Whenever the sentence A is true and the sentence B is true, then the sentence $A \wedge B$ must be true $A \wedge B$ given the meaning of A and B
  - It’s what lets us give meaning to the sentence
- Inference: Making new conclusions based on what you already know
  - If A is true and $A \Rightarrow B$ is true, then Modus Ponens allows you to conclude that B is true
  - There are statements about what syntactic structures you can create from structures you already have

Propositional Logic: Syntax

- What is legal to write down in propositional logic:

Sentence → *AtomicSentence* | *ComplexSentence*

Propositional Logic: Syntax

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Later we'll also use "()" for this

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ComplexSentence → ¬ *Sentence*
| *Sentence* ∨ *Sentence*
| *Sentence* ∧ *Sentence*
| *Sentence* ⇒ *Sentence*
| (*Sentence*)
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- Operator order of precedence: ¬, ∧, ∨, ⇒

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We will often treat ∨ and ∧ as sets when clear from context (won't worry if we say P∨Q or Q∨P)

- Operator order of precedence: ¬, ∧, ∨, ⇒ ¬ P ∨ Q vs ¬(P ∨ Q)

Propositional Logic: Semantics

Semantics is what connects symbols and sentences
to meaning in the world being represented

Symbols T and F represent “true” and “false”

Propositional Logic: Semantics

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Syntax Semantics

Propositional Logic: Semantics

Semantics for propositional logic:

Define the semantics of a complex sentence

from the semantics of its constituent parts

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Define the semantics of a complex sentence
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Truth Tables

Propositional Logic: Semantics

Truth Tables

φ	ψ	$\neg \varphi$	$\varphi \vee \psi$	$\varphi \wedge \psi$	$\varphi \Rightarrow \psi$
true	true	false	true	true	true
true	false	false	true	false	false
false	true	true	true	false	true
false	false	true	false	false	true

Propositional Logic: Semantics

Example: $\neg$ mythical $\Rightarrow$ (mortal $\wedge$ mammal)

mythical	mortal	mammal	$\neg$ mythical	mortal $\wedge$ mammal	$\neg$ mythical $\Rightarrow$ (mortal $\wedge$ mammal)
true	true	true			
true	true	false			
true	false	true			
true	false	false			
false	true	true			
false	true	false			
false	false	true			
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true	true	true			
true	true	false			
true	false	true			
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false	true	true			
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false	false	true			
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Propositional Logic: Semantics

Example: $\neg$ mythical $\Rightarrow$ (mortal $\wedge$ mammal)

mythical	mortal	mammal	$\neg$ mythical	mortal $\wedge$ mammal	$\neg$ mythical $\Rightarrow$ (mortal $\wedge$ mammal)
true	true	true	false		
true	true	false	false		
true	false	true	false		
true	false	false	false		
false	true	true	true		
false	true	false	true		
false	false	true	true		
false	false	false	true		

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true	true	true	false	true	
true	true	false	false	false	
true	false	true	false	false	
true	false	false	false	false	
false	true	true	true	true	
false	true	false	true	false	
false	false	true	true	false	
false	false	false	true	false	

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true	true	true	false	true	true
true	true	false	false	false	true
true	false	true	false	false	true
true	false	false	false	false	true
false	true	true	true	true	true
false	true	false	true	false	false
false	false	true	true	false	false
false	false	false	true	false	false