Logical Agents ---
Intro Knowledge Representation
& SAT encodings

R&N: Chapter 7
A Model-Based Agent

Requires: Knowledge and Reasoning
Knowledge and Reasoning:

humans are very good at acquiring new information by combining raw knowledge, experience with reasoning.

AI-slogan: “Knowledge is power” (or “Data is power”?)

Examples:

Medical diagnosis --- physician diagnosing a patient infers what disease, based on the knowledge he/she acquired as a student, textbooks, prior cases

Common sense knowledge / reasoning ---
common everyday assumptions / inferences. e.g.,
(1) “lecture starts at four” infer pm not am;
(2) when traveling, I assume there is some way to get from the airport to the hotel.
Logical agents:
Agents with some representation of the complex knowledge about the world / its environment, and uses inference to derive new information from that knowledge combined with new inputs (e.g. via perception).

Key issues:
1- Representation of knowledge
   What form? Meaning / semantics?
2- Reasoning and inference processes
   Efficiency.
Knowledge-base Agents

Key issues:

- Representation of knowledge $\rightarrow$ knowledge base
- Reasoning processes $\rightarrow$ inference/reasoning

Knowledge base = set of sentences in a formal language representing facts about the world (*)

(*) called Knowledge Representation (KR) language
Knowledge Representation language candidate: logical language (propositional / first-order) combined with a logical inference mechanism

Why not use natural language (e.g. English)?

We want clear syntax & semantics (well-defined meaning), and, mechanism to infer new information. Soln.: Use a formal language.
More Concrete: Propositional Logic

Syntax: build sentences from atomic propositions, using connectives $\land$, $\lor$, $\neg$, $\Rightarrow$, $\Leftrightarrow$.

(and / or / not / implies / equivalence (biconditional))

E.g.: $((\neg P) \lor (Q \land R)) \Rightarrow S$
Semantics

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>\neg P</th>
<th>P \land Q</th>
<th>P \lor Q</th>
<th>P \Rightarrow Q</th>
<th>P \equiv Q</th>
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\( \neg (P \lor Q) \)

True

True

False

True

Note: \( \Rightarrow \) somewhat counterintuitive.

What’s the truth value of “5 is even implies Sam is smart”?

True!
Satisfiability
Propositional Satisfiability problem

Satifiability (SAT): Given a formula in propositional calculus, is there a model (i.e., a satisfying interpretation, an assignment to its variables) making it true?

We consider clausal form, e.g.:

\[
( a \lor \neg b \lor \neg c ) \land ( b \lor \neg c ) \land ( a \lor c )
\]

\[2^n\] possible assignments

SAT: prototypical hard combinatorial search and reasoning problem. Problem is NP-Complete. (Cook 1971)

Surprising “power” of SAT for encoding computational problems

Modern SAT Solvers use this language.
Modern Satisfiability (SAT) solvers operating on the clausal form are surprisingly *much* more efficient than other logic-based approaches (e.g., pure resolution that will be discussed later).

The SAT solvers treat the set of clauses as a set of constraints (disjunctions) on Boolean variables, i.e., a CSP problem! Current solvers are very powerful. Can handle 1 Million+ variables and several millions of clauses.

Systematic: Davis Putnam (DPLL) + series of improvements
Stochastic local search: WalkSAT (issue?)

See R&N 7.6. “Ironically,” we are back to semantic model checking, but way more clever than basic truth assignment enumeration (exponentially faster)!
Satisfiability as an Encoding Language
Variables $C_{i,k}$ node i has color k

Total # colors: K. Total # nodes: N.

At least one of K colors per node i:

$$(C_{i,1} \lor C_{i,2} \lor C_{i,3} \lor \ldots \lor C_{i,K})$$

At most one color per node i:

$$(\neg C_{i,k} \lor \neg C_{i,k'}) \quad \text{for all } k \neq k'$$

If node i and node j (=/= i) share an edge, need to have different colors:

$$(\neg C_{i,l} \lor \neg C_{j,l}) \quad \text{for all } 1 \leq l \leq K$$

Note: Translation from “problem” into SAT. Reverse of usual translation to show NP-completeness. Works also for (easy) polytime problems!
Encoding Latin Square Problems in Propositional Logic

Variables: \( n^3 \)
\( x_{ijk} \) cell \( i,j \) has color \( k \); \( i,j,k=1,2,\ldots,n \).
Each variables represents a color assigned to a cell.

Clauses: \( O(n^4) \)
- Some color must be assigned to each cell (clause of length \( n \)); \( n^2 \)
  \[ \forall_{ij} (x_{ij1} \lor x_{ij2} \ldots x_{ijn}) \]
- No color is repeated in the same row (sets of negative binary clauses); \( n(n-1)/2 \)
  \[ \forall_{ik} (\neg x_{i1k} \lor \neg x_{i2k}) \land (\neg x_{i1k} \lor \neg x_{i3k}) \ldots (\neg x_{i1k} \lor \neg x_{ink}) \]
  \[ \ldots (\neg x_{i(n-1)k} \lor \neg x_{ink}) \]
- No color is repeated in the same column (sets of negative binary clauses);
  \( n(n-1)/2 \)
  \[ \forall_{jk} (\neg x_{1jk} \lor \neg x_{2jk}) \land (\neg x_{1jk} \lor \neg x_{3jk}) \ldots (\neg x_{1jk} \lor \neg x_{njk}) \]
  \[ \ldots (\neg x_{(n-1)jk} \lor \neg x_{njk}) \]
3D Encoding or Full Encoding

This encoding is based on the cubic representation of the quasigroup: each line of the cube contains exactly one true variable;

**Variables:** $O(n^4)$

Same as 2D encoding.

**Clauses:**

- Same as the 2D encoding plus:
  - Each color must appear at least once in each row:
    $$\bigwedge_{ik} (x_{i1k} \lor x_{i2k} \ldots x_{ink})$$
  - Each color must appear at least once in each column:
    $$\bigwedge_{jk} (x_{1jk} \lor x_{2jk} \ldots x_{njk})$$
  - No two colors are assigned to the same cell:
    $$\bigwedge_{ij} (\neg x_{ij1} \lor \neg x_{ij2}) \land (\neg x_{ij1} \lor \neg x_{ij3}) \ldots (\neg x_{ij1} \lor \neg x_{ijn})$$
    $$\ldots (\neg x_{ijn} \lor \neg x_{ijn(n-1)})$$
Dimacs format

At the top of the file is a simple header.

\texttt{p cnf <variables> <clauses>}

Each variable should be assigned an integer index. Start at 1, as 0 is used to indicate the end of a clause. The positive integer a positive literal, whereas a negative integer represents a negative literal.

Example

\[-1 \ 7 \ 0 \rightarrow (\neg x1 \lor x7)\]
### Extended Latin Square 2x2

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</table>

- A cell gets at most a color
- No repetition of color in a column
- No repetition of color in a row
- A cell gets a color
- A given color goes in each column
- A given color goes in each row

#### Order 2

- A cell gets at most a color
- No repetition of color in a column
- No repetition of color in a row
- A cell gets a color
- A given color goes in each column
- A given color goes in each row

1 - cell 11 is red
2 - cell 11 is green
3 - cell 12 is red
4 - cell 12 is green
5 - cell 21 is red
6 - cell 21 is green
7 - cell 22 is red
8 - cell 22 is green
Underlying Latin Square structure characterizes many real world applications.

- Sudoku: Underlying Latin Square structure characterizes many real world applications.
- Design of Scientific Experiments
- Routing in Fiber Optic Networks
- Many more applications…
Latin Square (Order 4)

NP-Complete

Better characterization beyond worst case?

Complexity of Latin Square Completion

Percentage of unsolvable instances

Time: 150 1820 165

35% 42% 50%
SAT Solvers in the Real World
NASA Deep Space One Spacecraft: Remote Agent

- Remote Agent (remote intelligent self-repair software) (RAX), developed at NASA and JPL, was the first artificial-intelligence control system to control a spacecraft without human supervision.
- Remote Agent successfully demonstrated the ability to plan onboard activities and correctly diagnose and respond to faults in spacecraft components through its built-in REPL environment.
NASA Deep Space One: Remote Agent

- Autonomous diagnosis & repair “Remote Agent”
- Compiled systems schematic to 7,000 var SAT problem

Started: January 1996
Launch: October 15th, 1998
Experiment: May 17-21
Deep Space One

• a failed electronics unit
  – Remote Agent fixed by reactivating the unit.

• a failed sensor providing false information
  – Remote Agent recognized as unreliable and therefore correctly ignored.

• an altitude control thruster (a small engine for controlling the spacecraft's orientation) stuck in the "off" position
  – Remote Agent detected and compensated for by switching to a mode that did not rely on that thruster.
Software and hardware verification – complete methods are critical – e.g. for verifying the correctness of chip design, using SAT encodings.

Going from 50 variable in, 200 constraints to 1,000,000+ variables and 5,000,000+ constraints in the last 20 years.

Applications:
Hardware and Software Verification Planning, Protocol Design, etc.
Progress in Last 20 years

- *Significant progress since the 1990’s. How much?*
- *Problem size: We went from 100 variables, 200 constraints (early 90’s) to 1,000,000+ variables and 5,000,000+ constraints in 20 years*

- *Search space: from $10^{30}$ to $10^{300,000}$.*
  [Aside: “one can encode quite a bit in 1M variables.”]

- *Is this just Moore’s Law? It helped, but not much…*
  – 2x faster computers does not mean can solve 2x larger instances
  – search difficulty does *not* scale linearly with problem size!

  **In fact, for O($2^n$), 2x faster, how many more vars?**
  handles 1 more variable!!

  Mainly algorithmic progress. Memory growth also key.

- *Tools: 50+ competitive SAT solvers available (e.g. Minisat solver)*
- *See http://www.satcompetition.org/*
Model Checking
The Association for Computing Machinery has announced the 2008 Turing Award Winners. Edmund M. Clarke, Allen Emerson, and Joseph Sifakis received the award for their work on an automated method for finding design errors in computer hardware and software.

"Model Checking is a type of "formal verification" that analyzes the logic underlying a design, much as a mathematician uses a proof to determine that a theorem is correct. Far from hit or miss, Model Checking considers every possible state of a hardware or software design and determines if it is consistent with the designer's specifications. Clarke and Emerson originated the idea of Model Checking at Harvard in 1981. They developed a theoretical technique for determining whether an abstract model of a hardware or software design satisfies a formal specification, given as a formula in Temporal Logic, a notation for describing possible sequences of events. Moreover, when the system fails the specification, it could identify a counterexample to show the source of the problem. Numerous model checking systems have been implemented, such as Spin at Bell Labs."
A “real world” example

From “SATLIB”:

http://www.satlib.org/benchm.html

SAT-encoded bounded model checking instances
(contributed by Ofer Shtrichman)

In Bounded Model Checking (BMC) [BCCZ99], a rather newly introduced problem in formal methods, the task is to check whether a given model M (typically a hardware design) satisfies a temporal property P in all paths with length less or equal to some bound k. The BMC problem can be efficiently reduced to a propositional satisfiability problem, and in fact if the property is in the form of an invariant (invariants are the most common type of properties, and many other temporal properties can be reduced to their form. It has the form of 'it is always true that ... '), it has a structure which is similar to many AI planning problems.
Bounded Model Checking instance:

The instance bmc-ibm-6.cnf, IBM LSU 1997:

p cnf 51639 368352
-1 7 0
-1 6 0
-1 5 0
-1 -4 0
-1 3 0
-1 2 0
-1 -8 0
-9 15 0
-9 14 0
-9 13 0
-9 -12 0
-9 11 0
-9 10 0
-9 -16 0
-17 23 0
-17 22 0

i.e. ((not \(x_1\)) or \(x_7\)) and ((not \(x_1\)) or \(x_6\)) and ... etc.
clauses / constraints are getting more interesting…
4000 pages later:

!!!
a 59-cnf clause...

```
10236 -10050 0
10236 -10051 0
10236 -10235 0
10008  10009  10010  10011  10012  10013  10014
  10015  10016  10017  10018  10019  10020  10021
  10022  10023  10024  10025  10026  10027  10028
  10029  10030  10031  10032  10033  10034  10035
  10036  10037  10038  10039  10040  10041  10042
  10043  10044  10045  10046  10047  10048  10049
  10050  10051  10235 -10236  0
  10237 -10008  0
  10237 -10009  0
  10237 -10010  0
  ...
```
Finally, 15,000 pages later:

\[
\begin{align*}
-7 & 260 0 \\
7 & -260 0 \\
1072 & 1070 0 \\
-15 & -14 -13 -12 -11 -10 0 \\
-15 & -14 -13 -12 -11 10 0 \\
-15 & -14 -13 -12 11 -10 0 \\
-15 & -14 -13 -12 11 10 0 \\
-7 & -6 -5 -4 -3 -2 0 \\
-7 & -6 -5 -4 -3 2 0 \\
-7 & -6 -5 -4 3 -2 0 \\
-7 & -6 -5 -4 3 2 0 \\
185 & 0 \\
\end{align*}
\]

Note that: \( 2^{50000} \approx 3.160699437 \cdot 10^{15051} \) ... !!!

MiniSAT solver solves this instance in less than one minute.