Ungraded quiz: camera calibration and stereo

May 3, 2020

1. When performing camera calibration, we set up a system of equations \( A\|p\| = 0 \) in the parameters \( p \) that define the camera projection matrix. We then tried to minimize \( A\|p\| \) subject to \( \|p\| = 1 \). Here, we constrain \( \|p\| = 1 \) because:

(a) A camera projection matrix is valid only if its Frobenius norm is 1.
(b) The constraint makes the optimization easier to implement.
(c) The correspondences used to form \( A \) might be noisy.
(d) The equations \( A\|p\| = 0 \) are not sufficient to produce a unique matrix \( P \), and will produce a family of solutions.

(d) is correct. Because the projection equations are all expressed in homogeneous coordinates, if \( P \) is a solution, so is \( \alpha P \). To isolate a single solution, we need to add a constraint on the Frobenius norm.

2. For a particular camera, the intrinsic camera parameters are \( K = I \). Its projection matrix \( P \) is one of the following. Which is it?

(a) \[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]
(b) \[
\begin{bmatrix}
0.8 & 0.6 & 0 \\
-0.6 & 0.8 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]
(c) \[
\begin{bmatrix}
0.8 & 0.6 & 0 & 5 \\
-0.6 & 0.8 & 0 & 7 \\
0 & 0 & 1 & 1
\end{bmatrix}
\]
(d) \[
\begin{bmatrix}
3 & 0 & 0 & 0 \\
0 & 5 & 0 & 0 \\
0 & 0 & 7 & 0
\end{bmatrix}
\]

\( P \) is a \( 3 \times 4 \) matrix. If \( K \) is identity, then \( P = [R \; t] \), where \( R \) is a rotation
matrix. Thus the first $3 \times 3$ submatrix of $P$ must be a rotation matrix. This in turn means that the first 3 columns of $P$ must be (a) orthogonal to each other, and (b) unit norm. Only (c) satisfies this constraint.

3. Two cameras are looking at a scene. They have projection matrices

\[ P^{(1)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \text{and} \quad P^{(2)} = \begin{bmatrix} 0.8 & 0 & 0.6 & -4 \\ 0 & 1 & 0 & 0 \\ -0.6 & 0 & 0.8 & 3 \end{bmatrix}. \]

A 3D world point appears in the first image at the location $(2, 0)$, and in the second image at location $(-18, 0)$ (Each tuple is the $(x,y)$ coordinates). What is the 3D location of this world point?

Suppose the world point is \( \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \). From the first camera, we have:

\[
\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \equiv P^{(1)} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \quad (1)
\]

\[
\Rightarrow \lambda \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = P^{(1)} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \quad (2)
\]

\[
\Rightarrow \lambda \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad (3)
\]

\[
\Rightarrow 2\lambda = X \\
0 = Y \\
\lambda = Z
\]

\[
\Rightarrow X = 2Z \quad (4)
\]

\[
Y = 0
\]

(6)
Let us now look at the second camera:

\[
\begin{bmatrix}
-18 \\
0 \\
1
\end{bmatrix} \equiv P^{(2)} \begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix}
\]  
(7)

\[
\Rightarrow \lambda \begin{bmatrix}
-18 \\
0 \\
1
\end{bmatrix} = P^{(2)} \begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix}
\]  
(8)

\[
\Rightarrow -18\lambda = 0.8X + 0.6Z - 4
\]
\[
0 = Y
\]
\[
\lambda = -0.6X + 0.8Z + 3
\]  
(9)

Substituting \(X = 2Z\) in the first and third equations, we get:

\[
-18\lambda = 2.2Z - 4 \quad (10)
\]
\[
\lambda = -0.4Z + 3 \quad (11)
\]
\[
\Rightarrow -18(-0.4Z + 3) = 2.2Z - 4 \quad (12)
\]
\[
\Rightarrow 7.2Z - 54 = 2.2Z - 4 \quad (13)
\]
\[
\Rightarrow 5Z = 50 \quad (14)
\]
\[
\Rightarrow Z = 10 \quad (15)
\]
\[
\Rightarrow X = 20 \quad (16)
\]

Thus, the point is \((20, 0, 10)\).