Lec 23: Single View Modeling
Projective geometry

• Readings
  • available online: http://www.cs.cmu.edu/~ph/869/papers/zisser-mundy.pdf
Ames Room
Announcements

• Prelims at 2:30 in hand back room
• Solutions on CMS tonight
• Average: 77.5, std dev 12.8
The idea for this Appendix arose from our perception of a frustrating situation faced by vision researchers. For example, one is interested in some aspect of the theory of perspective image formation such as the epipolar line. The interested party goes to the library to check out a book on projective geometry filled with hope that the necessary mathematical machinery will be directly at hand. These expectations are quickly dashed. Upon opening the book, the expectant reader finds the presentation dominated by endless observations about harmonic relations and a few chapters which explore the minutiae of Pappus’ theorem. Finally, as a last cruel twist of irony, the book ends in triumph with a rather exhilarating discourse on the conic pencil. All of the material is presented in the form of theorems defined on points, lines and conics without the use of coordinates, except perhaps for a quick pause to define barycentric coordinates just to taunt the reader. Dejected, the vision researcher throws the book aside and contents himself with some calculations using homogeneous coordinates and transformations which are covered briefly in Duda and Hart [93] or perhaps from a book on graphics [113].
Projective geometry—what’s it good for?

• Uses of projective geometry
  – Drawing
  – Measurements
  – Mathematics for projection
  – Undistorting images
  – Camera pose estimation
  – Object recognition

Paolo Uccello
Homogeneous coordinates

• \((X,Y,W)\) same as \((SX,SY,SW)\)

• Line in projective plane
  – \(ax + by + c = 0\)
  – \(aX + bY + cW = 0\)
  – \(u^Tp = p^Tu = 0\)
    • \(u = [a,b,c]^T\) and \(p = [X,Y,W]^Tp\)
    • \((x,y)\) Euclidean = \((X/W,Y/W)\)
    • \(W = 0?\) Ideal points, points at infinity \((X,Y,0)\)
• Ideal point ("point at infinity")
  – \( p \equiv (X, Y, 0) \)
Points and lines

• Intersection of two lines
  – \( \mathbf{u}_1 = (a_1, b_1, c_1), \mathbf{u}_2 = (a_2, b_2, c_2) \)
  – \( \mathbf{p} = (b_1 c_2 - b_2 c_1, a_2 c_1 - a_1 c_2, a_1 b_2 - a_2 b_1) \)
  – \( \mathbf{p} = \mathbf{u}_1 \times \mathbf{u}_2 \)
  – If \( \mathbf{u}_1 \) parallel to \( \mathbf{u}_2 \)
    • \( \mathbf{p} = (b_1 c_2 - b_2 c_1, a_2 c_1 - a_1 c_2, 0) \)

• Given two points \( \mathbf{p}_1 \) and \( \mathbf{p}_2 \)
  – Line through them \( \mathbf{u} = \mathbf{p}_1 \times \mathbf{p}_2 \)
Figure 23.11
A model for the projective plane can be constructed by rays in 3D space. The rays correspond to points in the projective plane. Two rays through the origin define a unique plane though the origin. Any plane through the origin corresponds to a projective line.
Point and line duality

• A line \( l \) is a homogeneous 3-vector

What is the line \( l \) spanned by rays \( p_1 \) and \( p_2 \)?
  • \( l \) is \( \perp \) to \( p_1 \) and \( p_2 \) \( \Rightarrow \) \( l = p_1 \times p_2 \)
  • \( l \) can be interpreted as a plane normal

What is the intersection of two lines \( l_1 \) and \( l_2 \)?
  • \( p \) is \( \perp \) to \( l_1 \) and \( l_2 \) \( \Rightarrow \) \( p = l_1 \times l_2 \)

Points and lines are dual in projective space
Figure 23.4
A perspective view of a set of parallel lines in the plane. All of the lines converge to a single vanishing point.
Vanishing points (2D)

- Image plane
- Vanishing point
- Camera center
- Line on ground plane
Vanishing points

- Properties
  - Any two parallel lines (in 3D) have the same vanishing point $v$
  - The ray from $C$ through $v$ is parallel to the lines
  - An image may have more than one vanishing point
    - in fact, every image point is a potential vanishing point
Two point perspective
Vanishing lines

- Multiple Vanishing Points
  - Any set of parallel lines on the plane define a vanishing point
  - The union of all of these vanishing points is the *horizon line*
    - also called *vanishing line*
  - Note that different planes (can) define different vanishing lines
Vanishing lines

- Multiple Vanishing Points
  - Any set of parallel lines on the plane define a vanishing point
  - The union of all of these vanishing points is the *horizon line*
    - also called *vanishing line*
  - Note that different planes (can) define different vanishing lines
Three point perspective
Computing vanishing lines

• Properties
  – \( l \) is intersection of horizontal plane through \( C \) with image plane
  – Compute \( l \) from two sets of parallel lines on ground plane
  – All points at same height as \( C \) project to \( l \)
    • points higher than \( C \) project above \( l \)
  – Provides way of comparing height of objects in the scene
Fun with vanishing points
Perspective cues
Perspective cues
Perspective cues
Perspective cues
Perspective cues
Vanishing points are useful

• Recover size
• Camera calibration
• …
Comparing heights
Measuring height

How high is the camera?