Lecture 19: Cameras
Announcements

• Prelim next Thu
  – Everything till Lecture 17 (Monday)

• My office hours
  – Wed 2:30-3:00->Tuesday 2:30-3:00
A Tale of Two Coordinate Systems

Two important coordinate systems:
1. *World* coordinate system
2. *Camera* coordinate system
Camera parameters

A camera is described by several parameters

- Translation $T$ of the optical center from the origin of world coords
- Rotation $R$ of the image plane
- focal length $f$, principal point $(x'_c, y'_c)$, pixel size $(s_x, s_y)$
- blue parameters are called “extrinsics,” red are “intrinsics”

Projection equation

$$
X = egin{bmatrix}
    s_x & 0 & x'_c \\
    s_y & 0 & y'_c \\
    s & 1 & 1
\end{bmatrix}
\begin{bmatrix}
    X \\
    Y \\
    Z
\end{bmatrix}
= \Pi X
$$

- The projection matrix models the cumulative effect of all parameters
- Useful to decompose into a series of operations

$$
\Pi =
\begin{bmatrix}
    -f s_x & 0 & x'_c \\
    0 & -f s_y & y'_c \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    1 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 \\
    0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
    R_{3x3} & 0_{3x1} & \mathbf{I}_{3x3} & T_{3x1}
\end{bmatrix}
$$

- The definitions of these parameters are not completely standardized
  - especially intrinsics—varies from one book to another
Projection matrix

(\text{in \textit{homogeneous image coordinates}})
Extrinsics

• How do we get the camera to “canonical form”?
  – (Center of projection at the origin, x-axis points right, y-axis points up, z-axis points backwards)
Affine change of coordinates

• Camera coordinate frame
  – point plus basis

• Need to change representation of point from one basis to another

• Canonical: origin (0,0) w/ axes e1, e2
Another way of thinking about this

- Change of coordinates
Coordinate frame summary

- Frame = point plus basis
- Frame matrix (uv-to-e1e2) is
  \[
  F = \begin{bmatrix}
  u & v & 0 \\
  0 & 0 & 1 \\
  \end{bmatrix}
  \]
- Move points to and from frame by multiplying with \( F \)
  \[
  p_e = F p_F \quad p_F = F^{-1} p_e
  \]
Extrinsics

• How to go between camera and world?
  – (Center of projection at the origin, x-axis points right, y-axis points up, z-axis points backwards)

Step 1: Translate by -c
Extrinsics

• How to go between camera and world?
  – (Center of projection at the origin, x-axis points right, y-axis points up, z-axis points backwards)

\[
T = \begin{bmatrix}
I_{3 \times 3} & -c \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Step 1: Translate by \(-c\)
Extrinsics

• How to go between camera and world?
  – (Center of projection at the origin, x-axis points right, y-axis points up, z-axis points backwards)

Step 1: Translate by -c
Step 2: Rotate by $R$

3x3 rotation matrix
Extrinsics

• How to go between camera and world?
  – (Center of projection at the origin, x-axis points right, y-axis points up, z-axis points backwards)

For any rotation matrix $R$ acting on $\mathbb{R}^n$,
$$ R^T = R^{-1} \text{ (The rotation is an orthogonal matrix)} $$

Step 1: Translate by $-c$
Step 2: Rotate by $R$

$$ R = \begin{bmatrix} u^T \\ v^T \\ w^T \end{bmatrix} $$
Perspective projection

$$
\begin{bmatrix}
-f & 0 & 0 \\
0 & -f & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\end{bmatrix}
$$

**K** (intrinsic) (converts from 3D rays in camera coordinate system to pixel coordinates)

In general, \( \mathbf{K} = \begin{bmatrix} -f & s & c_x \\
0 & -\alpha f & c_y \\
0 & 0 & 1 \end{bmatrix} \) (upper triangular matrix)

\( \alpha \): aspect ratio (1 unless pixels are not square)

\( s \): skew (0 unless pixels are shaped like rhombi/parallelograms)

\( (c_x, c_y) \): principal point ((0,0) unless optical axis doesn’t intersect projection plane at origin)
Projection matrix

\[ \Pi = K \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R & 0 \\ 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_{3 \times 3} & -c \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

(t in book’s notation)
Focal length

• Can think of as “zoom”

• Related to field of view
Focal length in practice

24mm

50mm

135mm
Focal length = cropping
Focal length vs. viewpoint

- Telephoto makes it easier to select background (a small change in viewpoint is a big change in background.)
• Hitchcock effect or Vertigo effect
Distortion

• 2 types
  – Perspective distortion
  – Lens distortion
Perspective distortion

• Problem for architectural photography: converging verticals

Source: F. Durand
Perspective distortion

• Problem for architectural photography: converging verticals

  - Tilting the camera upwards results in converging verticals
  - Keeping the camera level, with an ordinary lens, captures only the bottom portion of the building
  - Shifting the lens upwards results in a picture of the entire subject

• Solution: view camera (lens shifted w.r.t. film)

http://en.wikipedia.org/wiki/Perspective_correction_lens

Source: F. Durand
Perspective distortion

- Problem for architectural photography: converging verticals
- Result:

Source: F. Durand
Perspective distortion

• What does a sphere project to?

Image source: F. Durand
Perspective distortion

- The exterior columns appear bigger
- The distortion is not due to lens flaws
- Problem pointed out by Da Vinci
Perspective distortion: People
Distortion due to lens

- Radial distortion of the image
  - Caused by imperfect lenses
  - Deviations are most noticeable for rays that pass through the edge of the lens
Modeling distortion

• Radial distortion model

• Apply after projection, but before camera intrinsic: \( f \) and \((xc, yc)\) translation

Project \((\hat{x}, \hat{y}, \hat{z})\)

to “normalized”
image coordinates

\[
\begin{align*}
x'_n &= \frac{\hat{x}}{\hat{z}} \\
y'_n &= \frac{\hat{y}}{\hat{z}}
\end{align*}
\]
Modeling distortion

Apply radial distortion

\[ r^2 = x_n'^2 + y_n'^2 \]
\[ x_d' = x_n'(1 + \kappa_1 r^2 + \kappa_2 r^4) \]
\[ y_d' = y_n'(1 + \kappa_1 r^2 + \kappa_2 r^4) \]

Apply focal length

Translate image center

\[ x' = f x_d' + x_c \]
\[ y' = f y_d' + y_c \]

• To model lens distortion
  – Use above projection operation instead of standard projection matrix multiplication
Correcting radial distortion

from Helmut Dersch
Other types of projection

• Lots of intriguing variants...
• (I’ll just mention a few fun ones)
360 degree field of view...

• Basic approach
  – Take a photo of a parabolic mirror with an orthographic lens (Nayar)
  – Or buy one a lens from a variety of omnicam manufacturers...
    • See http://www.cis.upenn.edu/~kostas/omni.html
Rotating sensor (or object)

Rollout Photographs © Justin Kerr
http://research.famsi.org/kerrmaya.html

Also known as “cyclographs”, “peripheral images”