CS4670/5670: Computer Vision

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Single-view modeling, Part 2

Projective geometry

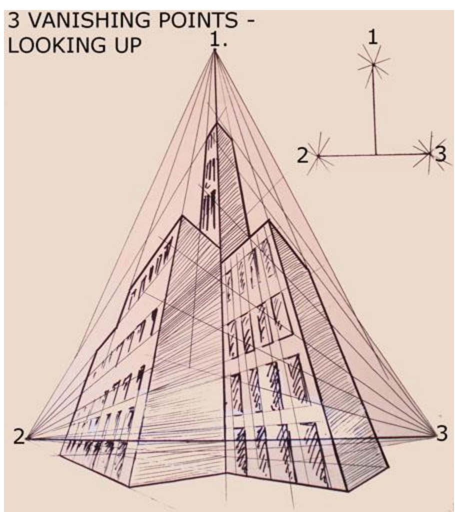
Ames Room

- Readings
  - Mundy, J.L. and Zisserman, A., Geometric Invariance in Computer Vision, Appendix: Projective Geometry for Machine Vision, MIT Press, Cambridge, MA, 1992, **(read 23.1 - 23.5, 23.10)**
    - available online: http://www.cs.cmu.edu/~ph/869/papers/zisser-mundy.pdf

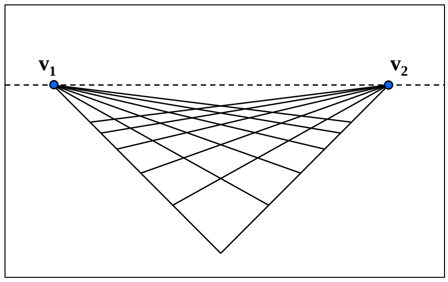
Announcements

- Midterm to be handed in Thursday by 5pm
- Please hand it back at my office, Upson 4157

Three point perspective

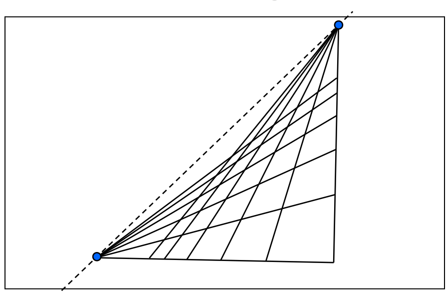


Vanishing lines



- Multiple Vanishing Points
  - Any set of parallel lines on the plane define a vanishing point
  - The union of all of these vanishing points is the *horizon line*
    - also called *vanishing line*
  - Note that different planes (can) define different vanishing lines

Vanishing lines



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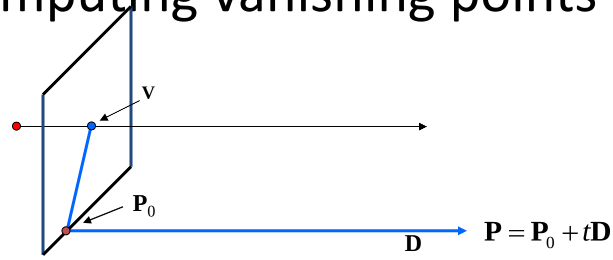
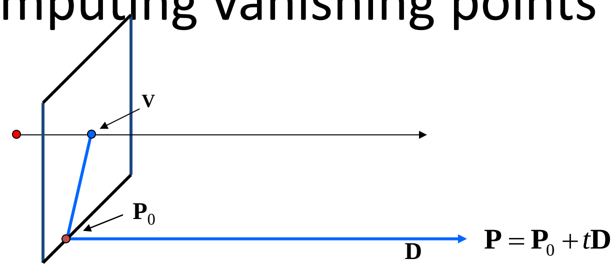
Computing vanishing points



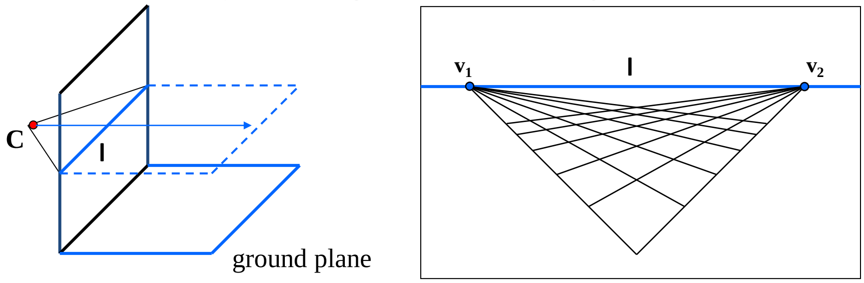
Computing vanishing points



$$\mathbf{P}_t = \begin{bmatrix} P_X + tD_X \\ P_Y + tD_Y \\ P_Z + tD_Z \\ 1 \end{bmatrix} \cong \begin{bmatrix} P_X / t + D_X \\ P_Y / t + D_Y \\ P_Z / t + D_Z \\ 1/t \end{bmatrix}$$

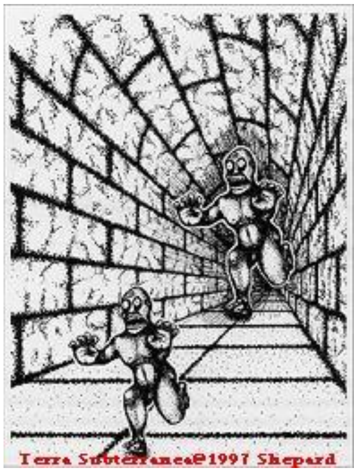
- Properties $\mathbf{v} = \mathbf{\Pi P}_\infty$
  - $\mathbf{P}_\infty$ is a point at *infinity*, **v** is its projection
  - Depends only on line *direction*
  - Parallel lines $\mathbf{P}_0$ + t**D**, $\mathbf{P}_1$ + t**D** intersect at $\mathbf{P}_\infty$

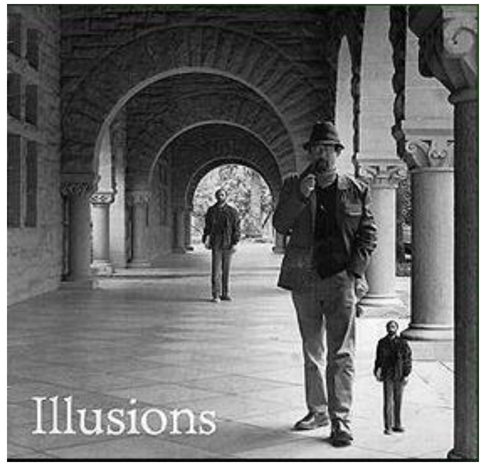
Computing vanishing lines



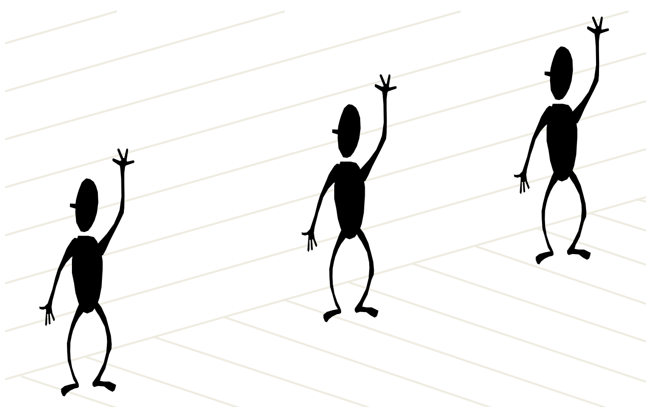
- Properties
  - **l** is intersection of horizontal plane through **C** with image plane
  - Compute **l** from two sets of parallel lines on ground plane
  - All points at same height as **C** project to **l**
    - points higher than C project above l
  - Provides way of comparing height of objects in the scene

Fun with vanishing points

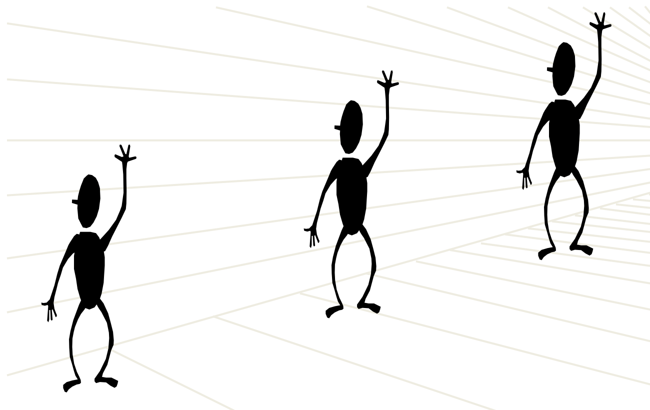




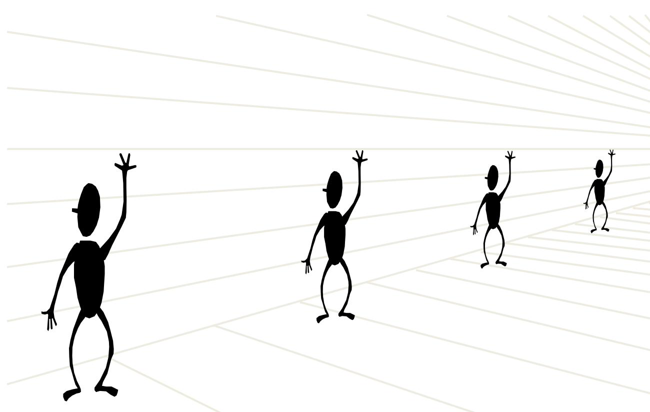
Perspective cues



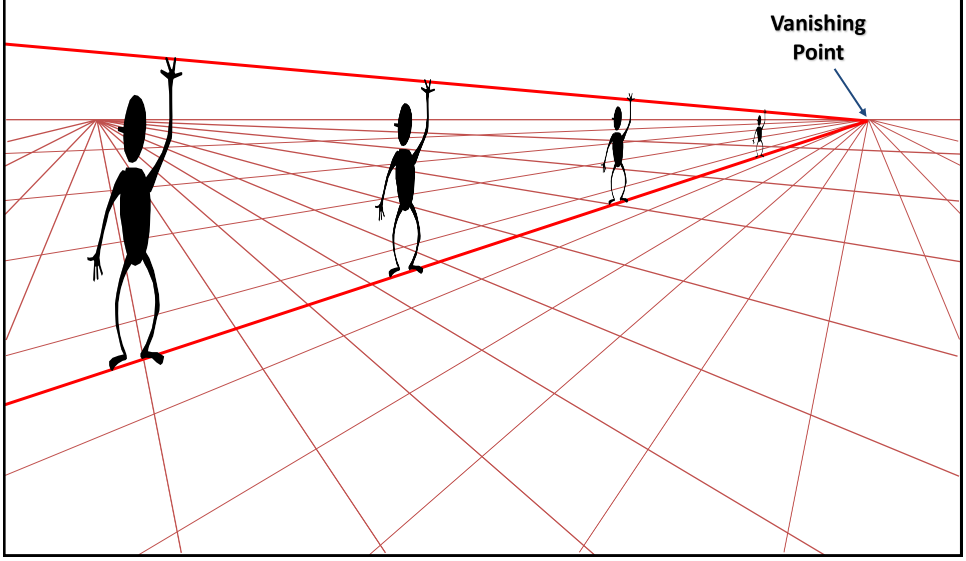
Perspective cues



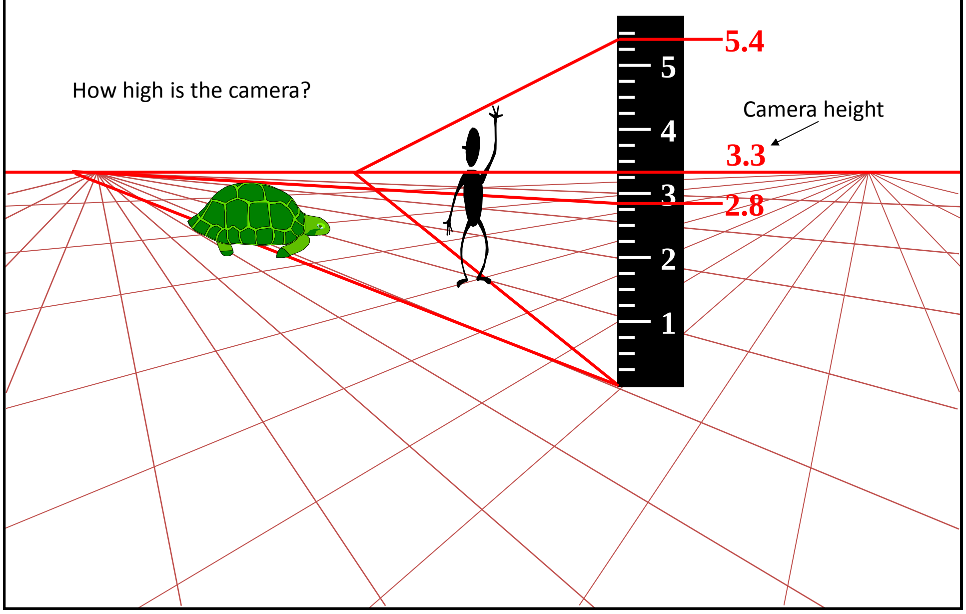
Perspective cues



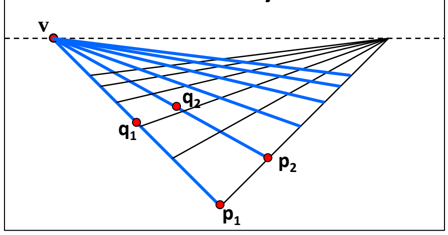
Comparing heights



Measuring height



Computing vanishing points (from lines)



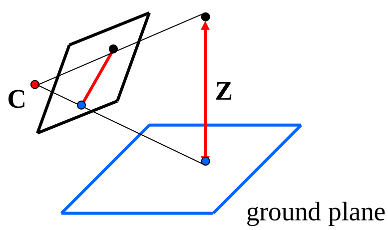
- Intersect p_1q_1 with p_2q_2

$$v = (p_1 \times q_1) \times (p_2 \times q_2)$$

Least squares version

- Better to use more than two lines and compute the "closest" point of intersection
- See notes by Bob Collins for one good way of doing this:
  - http://www-2.cs.cmu.edu/~ph/869/www/notes/vanishing.txt

Measuring height without a ruler



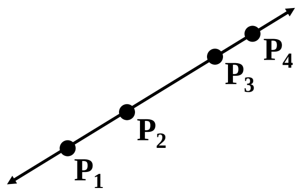
Compute Z from image measurements

- Need more than vanishing points to do this

The cross ratio

- A Projective Invariant
  - Something that does not change under projective transformations (including perspective projection)

The *cross-ratio* of 4 collinear points



$$\frac{\|\mathbf{P}_3 - \mathbf{P}_1\| \, \|\mathbf{P}_4 - \mathbf{P}_2\|}{\|\mathbf{P}_3 - \mathbf{P}_2\| \, \|\mathbf{P}_4 - \mathbf{P}_1\|}$$

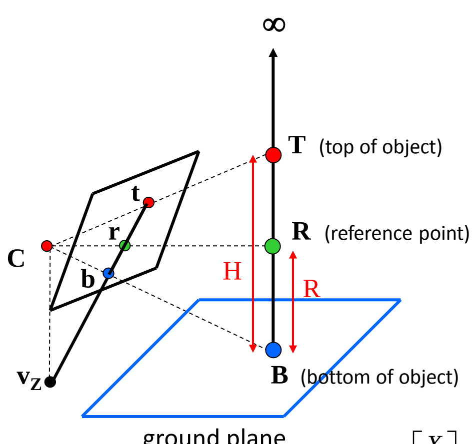
$$\mathbf{P}_i = \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

Can permute the point ordering $\quad \dfrac{\|\mathbf{P}_1 - \mathbf{P}_3\| \, \|\mathbf{P}_4 - \mathbf{P}_2\|}{\|\mathbf{P}_1 - \mathbf{P}_2\| \, \|\mathbf{P}_4 - \mathbf{P}_3\|}$

- 4! = 24 different orders (but only 6 distinct values)

This is the fundamental invariant of projective geometry

Measuring height



$$\frac{\|\mathbf{T} - \mathbf{B}\| \, \|\infty - \mathbf{R}\|}{\|\mathbf{R} - \mathbf{B}\| \, \|\infty - \mathbf{T}\|} = \frac{H}{R}$$

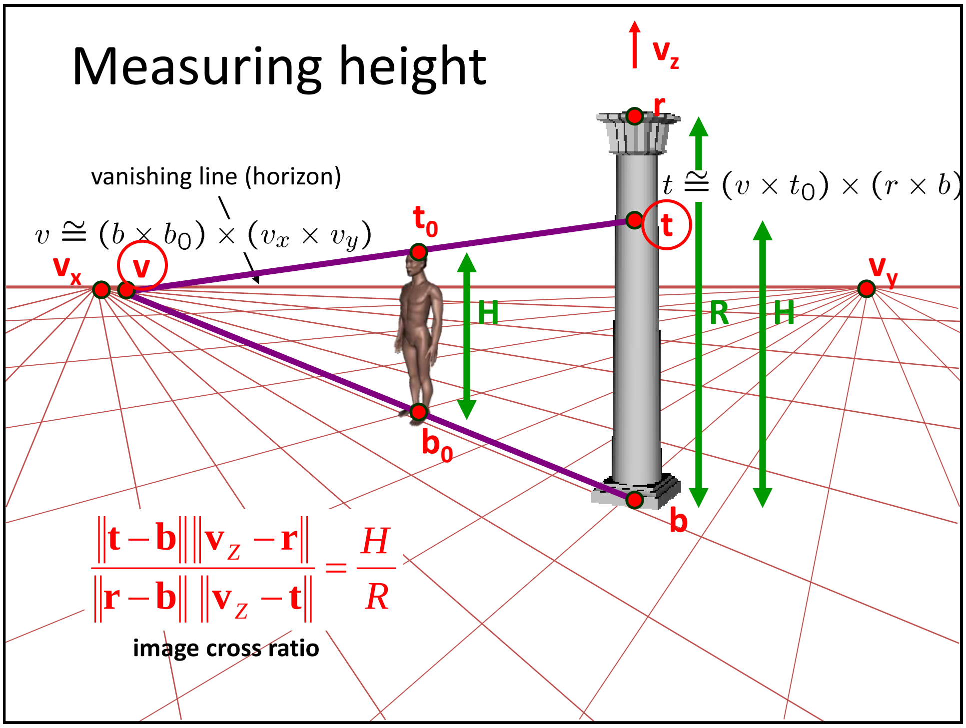
scene cross ratio

$$\frac{\|\mathbf{t} - \mathbf{b}\| \, \|\mathbf{v}_Z - \mathbf{r}\|}{\|\mathbf{r} - \mathbf{b}\| \, \|\mathbf{v}_Z - \mathbf{t}\|} = \frac{H}{R}$$

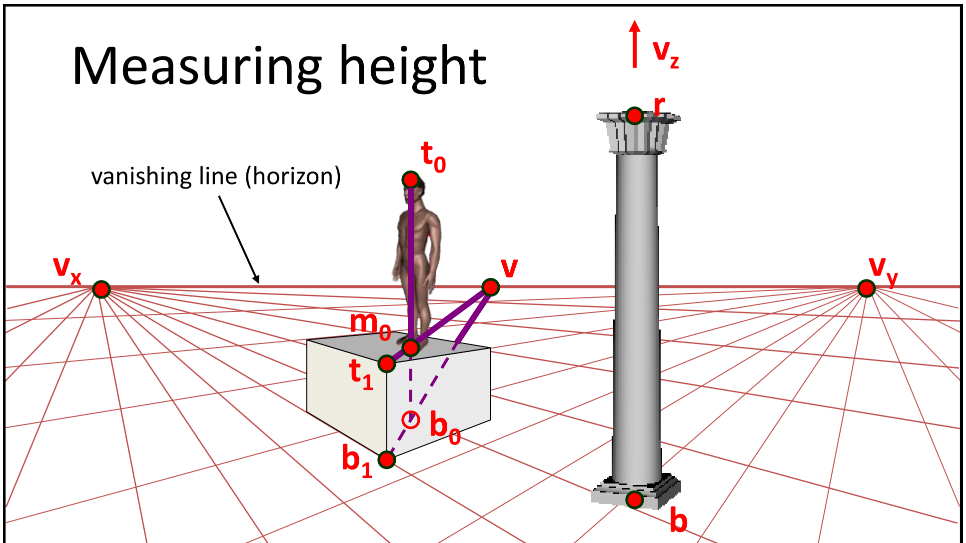
image cross ratio

scene points represented as $\mathbf{P} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$ image points as $\mathbf{p} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

Measuring height



Measuring height



What if the point on the ground plane $\mathbf{b_0}$ is not known?

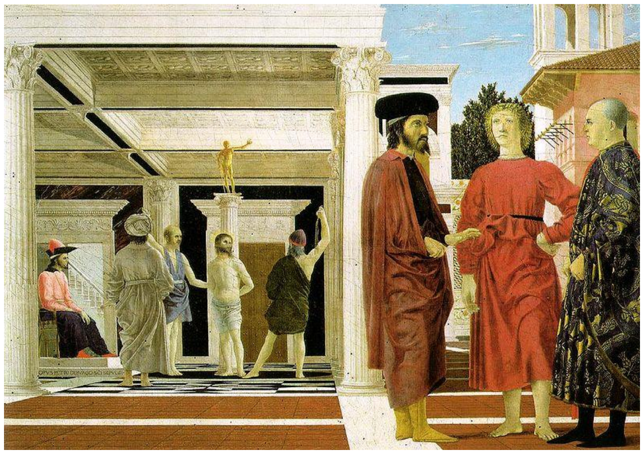
- Here the guy is standing on the box, height of box is known
- Use one side of the box to help find $\mathbf{b_0}$ as shown above

3D Modeling from a photograph

St. Jerome in his Study, H. Steenwick

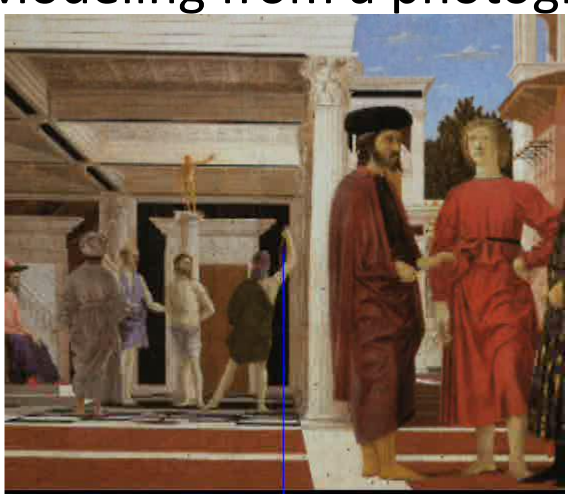
3D Modeling from a photograph

3D Modeling from a photograph



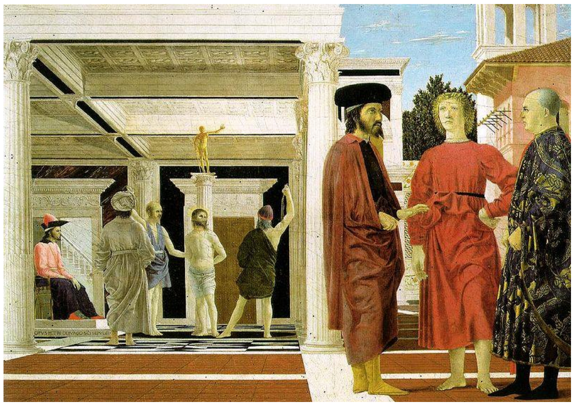
Flagellation, Piero della Francesca

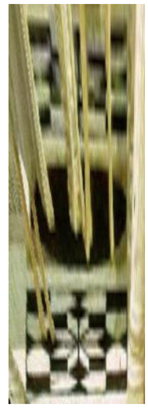
3D Modeling from a photograph



video by Antonio Criminisi

3D Modeling from a photograph





Camera calibration

- Goal: estimate the camera parameters
  - Version 1: solve for projection matrix

$$\mathbf{x} = \begin{bmatrix} wx \\ wy \\ w \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{\Pi X}$$

- Version 2: solve for camera parameters separately
  - intrinsics (focal length, principle point, pixel size)
  - extrinsics (rotation angles, translation)
  - radial distortion

Vanishing points and projection matrix

$$\mathbf{\Pi} = \begin{array}{c} \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \\ \begin{matrix} \boldsymbol{\pi}_1 & \boldsymbol{\pi}_2 & \boldsymbol{\pi}_3 & \boldsymbol{\pi}_4 \end{matrix} \end{array} = \begin{bmatrix} \boldsymbol{\pi}_1 & \boldsymbol{\pi}_2 & \boldsymbol{\pi}_3 & \boldsymbol{\pi}_4 \end{bmatrix}$$

- $\boldsymbol{\pi}_1 = \mathbf{\Pi}\begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}^T$ = $\mathbf{v}_x$ (X vanishing point)
- similarly, $\boldsymbol{\pi}_2 = \mathbf{v}_Y$, $\boldsymbol{\pi}_3 = \mathbf{v}_Z$
- $\boldsymbol{\pi}_4 = \mathbf{\Pi}\begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix}^T = \text{projection of world origin}$

$$\mathbf{\Pi} = \begin{bmatrix} \mathbf{v}_X & \mathbf{v}_Y & \mathbf{v}_Z & \mathbf{o} \end{bmatrix}$$

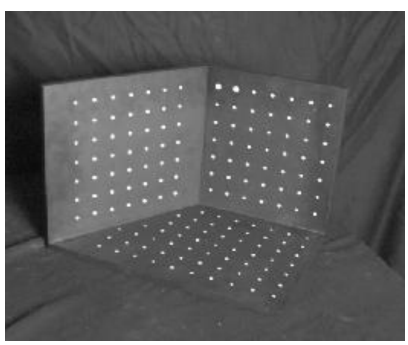
Not So Fast! We only know **v**'s up to a scale factor

$$\mathbf{\Pi} = \begin{bmatrix} a\,\mathbf{v}_X & b\mathbf{v}_Y & c\mathbf{v}_Z & \mathbf{o} \end{bmatrix}$$

- Can fully specify by providing 3 reference points

Calibration using a reference object

- Place a known object in the scene
  - identify correspondence between image and scene
  - compute mapping from scene to image



Issues

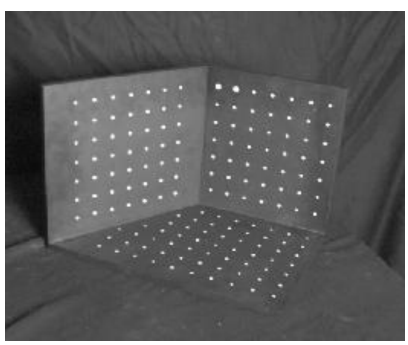
- must know geometry very accurately
- must know 3D->2D correspondence

Chromaglyphs

Courtesy of Bruce Culbertson, HP Labs
http://www.hpl.hp.com/personal/Bruce_Culbertson/ibr98/chromagl.htm

Estimating the projection matrix

- Place a known object in the scene
  - identify correspondence between image and scene
  - compute mapping from scene to image



$$\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \cong \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

Direct linear calibration

$$\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \cong \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

$$\begin{aligned} u_i &= \frac{m_{00}X_i + m_{01}Y_i + m_{02}Z_i + m_{03}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + m_{23}} \\ v_i &= \frac{m_{10}X_i + m_{11}Y_i + m_{12}Z_i + m_{13}}{m_{20}X_i + m_{21}Y_i + m_{22}Z_i + m_{23}} \end{aligned}$$

$$\begin{aligned} u_i(m_{20}X_i + m_{21}Y_i + m_{22}Z_i + m_{23}) &= m_{00}X_i + m_{01}Y_i + m_{02}Z_i + m_{03} \\ v_i(m_{20}X_i + m_{21}Y_i + m_{22}Z_i + m_{23}) &= m_{10}X_i + m_{11}Y_i + m_{12}Z_i + m_{13} \end{aligned}$$

$$\begin{bmatrix} X_i & Y_i & Z_i & 1 & 0 & 0 & 0 & 0 & -u_iX_i & -u_iY_i & -u_iZ_i & -u_i \\ 0 & 0 & 0 & 0 & X_i & Y_i & Z_i & 1 & -v_iX_i & -v_iY_i & -v_iZ_i & -v_i \end{bmatrix} \begin{bmatrix} m_{00} \\ m_{01} \\ m_{02} \\ m_{03} \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{13} \\ m_{20} \\ m_{21} \\ m_{22} \\ m_{23} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Direct linear calibration

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 & -u_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 & -v_1 \\ & & & & & & \vdots & & & & & \\ X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -u_nX_n & -u_nY_n & -u_nZ_n & -u_n \\ 0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -v_nX_n & -v_nY_n & -v_nZ_n & -v_n \end{bmatrix} \begin{bmatrix} m_{00} \\ m_{01} \\ m_{02} \\ m_{03} \\ m_{10} \\ m_{11} \\ m_{12} \\ m_{13} \\ m_{20} \\ m_{21} \\ m_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

Can solve for m_{ij} by linear least squares

- use eigenvector trick that we used for homographies

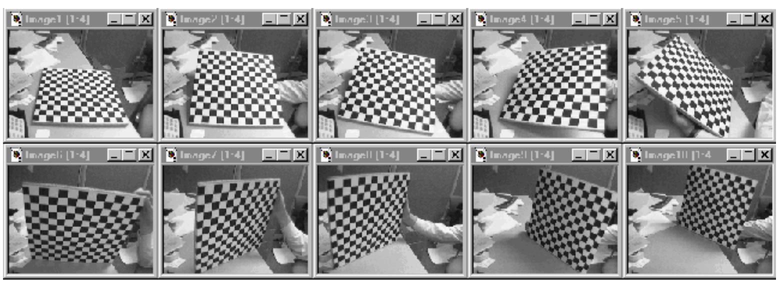
Direct linear calibration

- Advantage:
  - Very simple to formulate and solve
- Disadvantages:
  - Doesn't tell you the camera parameters
  - Doesn't model radial distortion
  - Hard to impose constraints (e.g., known f)
  - Doesn't minimize the right error function

For these reasons, *nonlinear methods* are preferred

- Define error function E between projected 3D points and image positions
  - E is nonlinear function of intrinsics, extrinsics, radial distortion
- Minimize E using nonlinear optimization techniques

Alternative: multi-plane calibration



Images courtesy Jean-Yves Bouguet, Intel Corp.

Advantage

- Only requires a plane
- Don't have to know positions/orientations
- Good code available online! (including in OpenCV)
  - Matlab version by Jean-Yves Bouget: http://www.vision.caltech.edu/bouguetj/calib_doc/index.html
  - Zhengyou Zhang's web site: http://research.microsoft.com/~zhang/Calib/

Some Related Techniques

- Image-Based Modeling and Photo Editing
  - Mok et al., SIGGRAPH 2001
  - http://graphics.csail.mit.edu/ibedit/
- Single View Modeling of Free-Form Scenes
  - Zhang et al., CVPR 2001
  - http://grail.cs.washington.edu/projects/svm/
- Tour Into The Picture
  - Anjyo et al., SIGGRAPH 1997
  - http://koigakubo.hitachi.co.jp/little/DL_TipE.html