Lecture 13: Cameras and geometry

Reading

• Szeliski 2.1.3-2.1.6
Image formation

• Let’s design a camera
  – Idea 1: put a piece of film in front of an object
  – Do we get a reasonable image?

Pinhole camera

• Add a barrier to block off most of the rays
  – This reduces blurring
  – The opening known as the aperture
  – How does this transform the image?
Adding a lens

• A lens focuses light onto the film
  – There is a specific distance at which objects are “in focus”
    • other points project to a “circle of confusion” in the image
  – Changing the shape of the lens changes this distance

Lytro Lightfield Camera
The human eye is a camera

- **Iris** - colored annulus with radial muscles
- **Pupil** - the hole (aperture) whose size is controlled by the iris
- What’s the “film”?  
  - photoreceptor cells (rods and cones) in the retina

Eyes in nature: eyespots to pinhole camera
Projection

Projection
Müller-Lyer Illusion

http://www.michaelbach.de/ot/sze_muelue/index.html

Modeling projection

- The coordinate system
  - We will use the pinhole model as an approximation
  - Put the optical center (Center Of Projection) at the origin
  - Put the image plane (Projection Plane) in front of the COP
    - Why?
  - The camera looks down the negative z axis
    - we need this if we want right-handed-coordinates
Modeling projection

- Projection equations
  - Compute intersection with PP of ray from \((x,y,z)\) to COP
  - Derived using similar triangles (on board)
    \[
    (x, y, z) \rightarrow \left(-\frac{d}{z}x, -\frac{d}{z}y, -d\right)
    \]
  - We get the projection by throwing out the last coordinate:
    \[
    (x, y, z) \rightarrow \left(-\frac{d}{z}x, -\frac{d}{z}y\right)
    \]

- Is this a linear transformation?
  - no—division by \(z\) is nonlinear

Homogeneous coordinates to the rescue!

\[
(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad (x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}
\]

Converting from homogeneous coordinates

\[
\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow \frac{x}{w}, \frac{y}{w} \quad \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow \frac{x}{w}, \frac{y}{w}, \frac{z}{w}
\]
Perspective Projection

Projection is a matrix multiply using homogeneous coordinates:

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1/d & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
= \begin{bmatrix}
x \\
y \\
-z/d \\
1
\end{bmatrix}
\Rightarrow \begin{bmatrix}
-x/z \\
y/z \\
-dx/z \\
-dy/z
\end{bmatrix}
\]

divide by third coordinate

This is known as **perspective projection**

- The matrix is the **projection matrix**

- (Can also represent as a 4x4 matrix – OpenGL does something like this)

Perspective Projection

- How does scaling the projection matrix change the transformation?

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1/d & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
= \begin{bmatrix}
x \\
y \\
-z/d \\
1
\end{bmatrix}
\Rightarrow \begin{bmatrix}
-x/z \\
y/z \\
-dx/z \\
-dy/z
\end{bmatrix}
\]

\[
\begin{bmatrix}
-d & 0 & 0 & 0 \\
0 & -d & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
= \begin{bmatrix}
-dx \\
y \\
-dy \\
z
\end{bmatrix}
\Rightarrow \begin{bmatrix}
-dx/z \\
-dy/z \\
-dx/z \\
-dy/z
\end{bmatrix}
\]
Orthographic projection

• Special case of perspective projection
  – Distance from the COP to the PP is infinite

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix} =
\begin{bmatrix}
x \\
y \\
1
\end{bmatrix} \Rightarrow (x, y)
\]

Variants of orthographic projection

• Scaled orthographic
  – Also called “weak perspective”

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1/d
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix} =
\begin{bmatrix}
x \\
y \\
1/d
\end{bmatrix} \Rightarrow (dx, dy)
\]

• Affine projection
  – Also called “paraperspective”

\[
\begin{bmatrix}
a & b & c & d \\
e & f & g & h \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
\]
Orthographic projection

Perspective projection
Dimensionality Reduction Machine
(3D to 2D)

3D world

2D image

Point of observation

What have we lost?
• Angles
• Distances (lengths)

Projection properties

• Many-to-one: any points along same ray map to same point in image
• Points → points
• Lines → lines (collinearity is preserved)
  – But line through focal point projects to a point
• Planes → planes (or half-planes)
  – But plane through focal point projects to line
Projection properties

- Parallel lines converge at a vanishing point
  - Each direction in space has its own vanishing point
  - But parallels parallel to the image plane remain parallel