# CS4670/5760: Computer Vision

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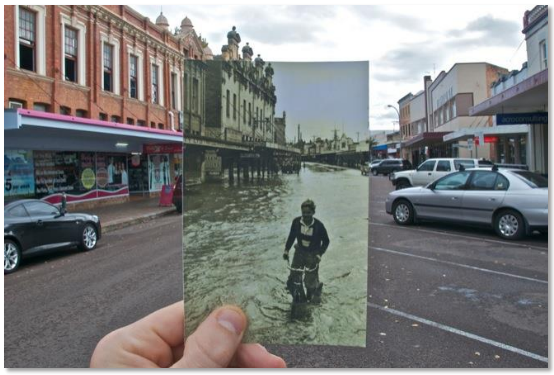
## Lecture 11: Image alignment, Part 2



http://www.wired.com/gadgetlab/2010/07/camera-software-lets-you-see-into-the-past/

## Reading

- Szeliski: Chapter 6.1

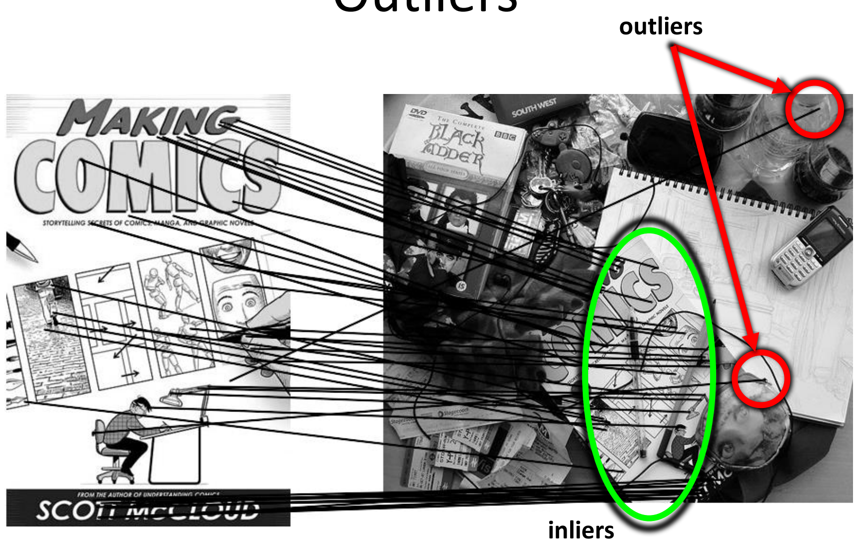
# Image Alignment Algorithm

Given images A and B

1. Compute image features for A and B
2. Match features between A and B
3. Compute homography between A and B using least squares on set of matches

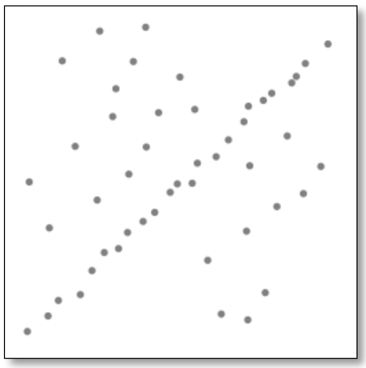
What could go wrong?

# Outliers

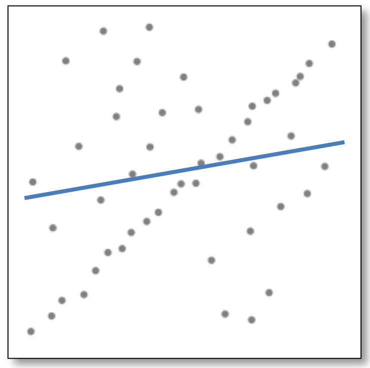


# Robustness

- Let's consider a simpler example… linear regression



Problem: Fit a line to these datapoints



Least squares fit

- How can we fix this?

# We need a better cost function…

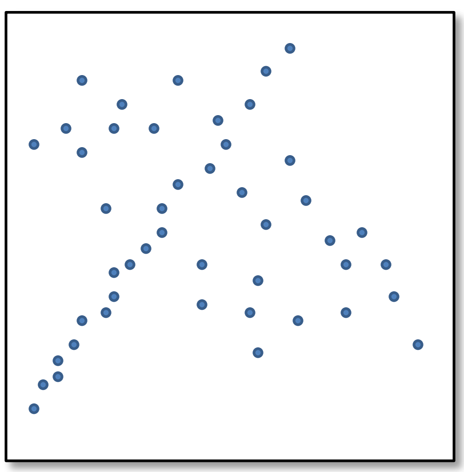
- Suggestions?

# Idea

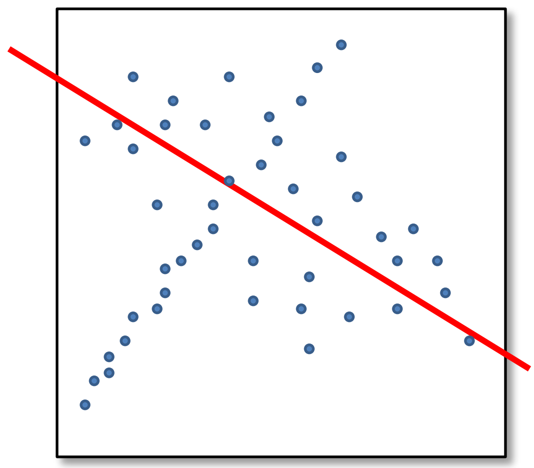
- Given a hypothesized line
- Count the number of points that "agree" with the line
  - "Agree" = within a small distance of the line
  - I.e., the **inliers** to that line

- For all possible lines, select the one with the largest number of inliers

# Counting inliers

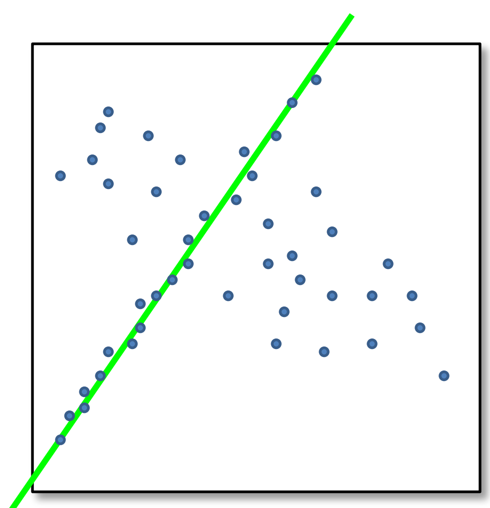


# Counting inliers



**Inliers: 3**

# Counting inliers

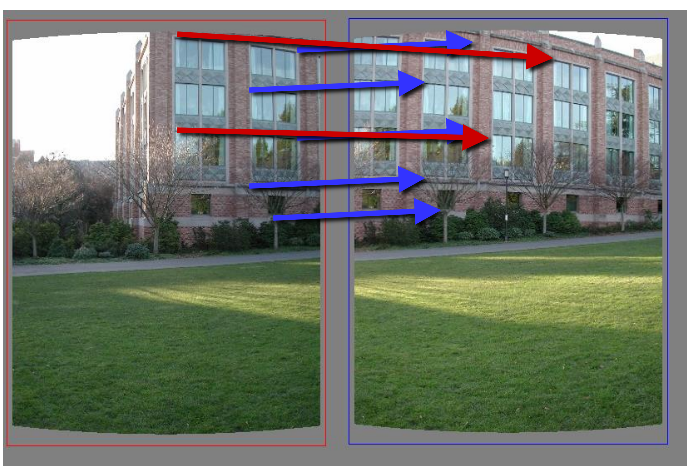


**Inliers: 20**

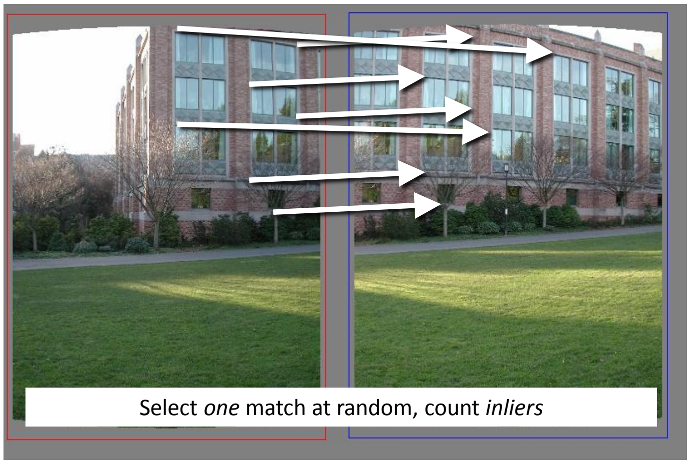
# How do we find the best line?

- Unlike least-squares, no simple closed-form solution
- Hypothesize-and-test
  - Try out many lines, keep the best one
  - Which lines?

# Translations

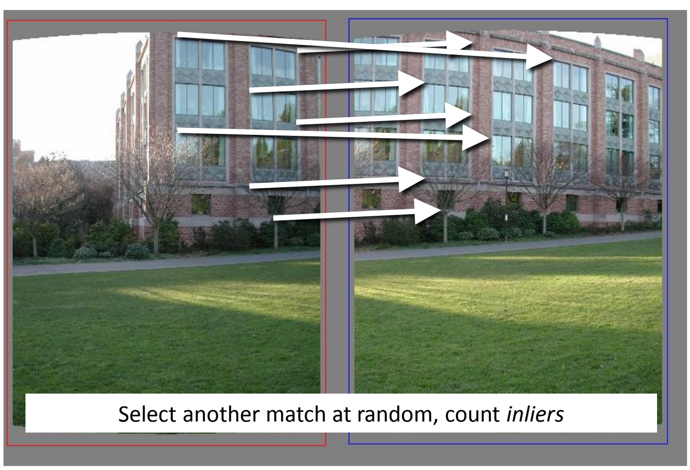


# RAndom SAmple Consensus



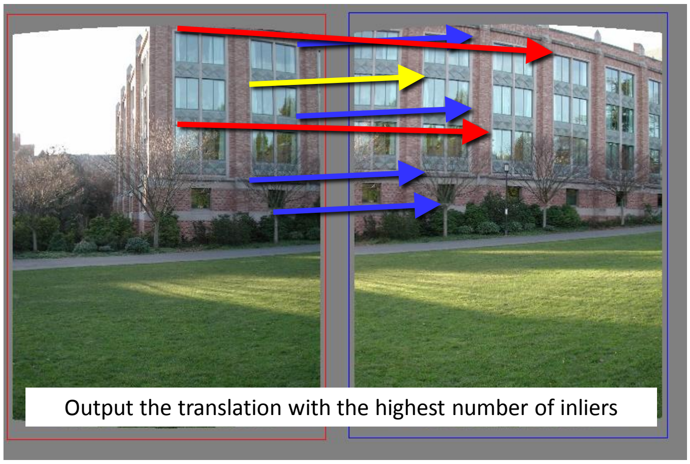
Select *one* match at random, count *inliers*

# RAndom SAmple Consensus



Select another match at random, count *inliers*

# RAndom SAmple Consensus



Output the translation with the highest number of inliers

# RANSAC

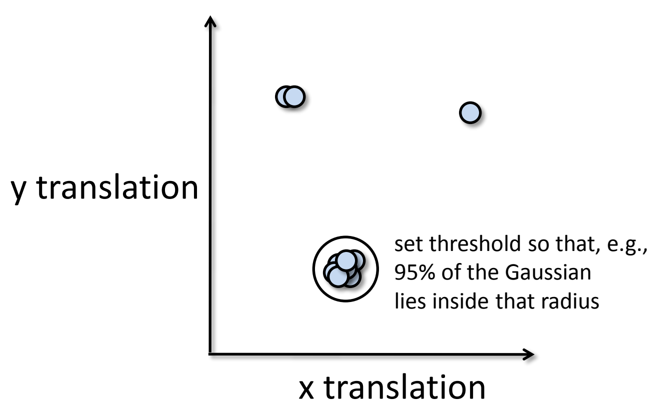
- Idea:
  - All the inliers will agree with each other on the translation vector; the (hopefully small) number of outliers will (hopefully) disagree with each other
    - RANSAC only has guarantees if there are < 50% outliers
  - "All good matches are alike; every bad match is bad in its own way."

– Tolstoy via Alyosha Efros

# RANSAC

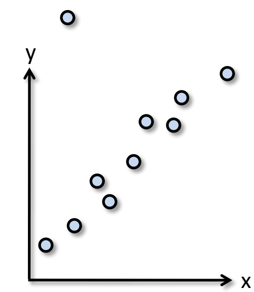
- **Inlier threshold** related to the amount of noise we expect in inliers
  - Often model noise as Gaussian with some standard deviation (e.g., 3 pixels)
- **Number of rounds** related to the percentage of outliers we expect, and the probability of success we'd like to guarantee
  - Suppose there are 20% outliers, and we want to find the correct answer with 99% probability
  - How many rounds do we need?

# RANSAC



# RANSAC

- Back to linear regression
- How do we generate a hypothesis?





# RANSAC

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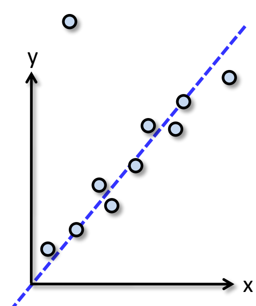


# RANSAC

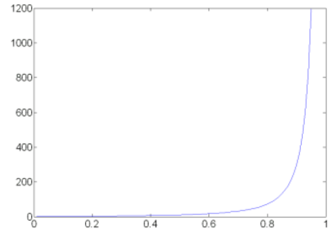
- General version:
  1. Randomly choose *s* samples
     - Typically *s* = minimum sample size that lets you fit a model
  2. Fit a model (e.g., line) to those samples
  3. Count the number of inliers that approximately fit the model
  4. Repeat *N* times
  5. Choose the model that has the largest set of inliers

# How many rounds?

- If we have to choose *s* samples each time
  – with an outlier ratio *e*
  – and we want the right answer with probability *p*

| | proportion of outliers *e* | | | | | | |
|---|---|---|---|---|---|---|---|
| s | 5% | 10% | 20% | 25% | 30% | 40% | 50% |
| 2 | 2 | 3 | 5 | 6 | 7 | 11 | 17 |
| 3 | 3 | 4 | 7 | 9 | 11 | 19 | 35 |
| 4 | 3 | 5 | 9 | 13 | 17 | 34 | 72 |
| 5 | 4 | 6 | 12 | 17 | 26 | 57 | 146 |
| 6 | 4 | 7 | 16 | 24 | 37 | 97 | 293 |
| 7 | 4 | 8 | 20 | 33 | 54 | 163 | 588 |
| 8 | 5 | 9 | 26 | 44 | 78 | 272 | 1177 |

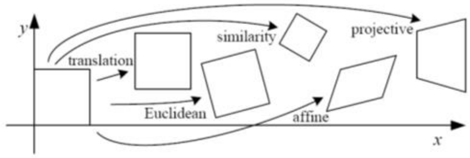
$p = 0.99$




Source: M. Pollefeys

# How big is *s*?

- For alignment, depends on the motion model
  - Here, each sample is a correspondence (pair of matching points)

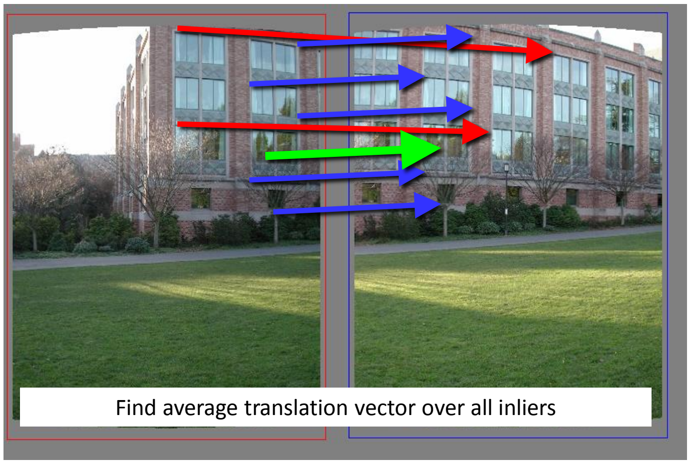



| Name | Matrix | # D.O.F. | Preserves: | Icon |
|---|---|---|---|---|
| translation | $\begin{bmatrix} I \mid t \end{bmatrix}_{2\times 3}$ | 2 | orientation + ··· | |
| rigid (Euclidean) | $\begin{bmatrix} R \mid t \end{bmatrix}_{2\times 3}$ | 3 | lengths + ··· | |
| similarity | $\begin{bmatrix} sR \mid t \end{bmatrix}_{2\times 3}$ | 4 | angles + ··· | |
| affine | $\begin{bmatrix} A \end{bmatrix}_{2\times 3}$ | 6 | parallelism + ··· | |
| projective | $\begin{bmatrix} \tilde{H} \end{bmatrix}_{3\times 3}$ | 8 | straight lines | |

# RANSAC pros and cons

- Pros
  - Simple and general
  - Applicable to many different problems
  - Often works well in practice
- Cons
  - Parameters to tune
  - Sometimes too many iterations are required
  - Can fail for extremely low inlier ratios
  - We can often do better than brute-force sampling

# Final step: least squares fit




# RANSAC

- An example of a "voting"-based fitting scheme
- Each hypothesis gets voted on by each data point, best hypothesis wins

- There are many other types of voting schemes
  - E.g., Hough transforms…

# Hough transform