Lecture 2: Edge detection

• Convert a 2D image into a set of curves
  – Extracts salient features of the scene
  – More compact than pixels
Origin of Edges

• Edges are caused by a variety of factors

Images as functions...

• Edges look like steep cliffs
Characterizing edges

- An edge is a place of *rapid change* in the image intensity function.

![Image of edge detection](image)

Source: L. Lazebnik

Image derivatives

- How can we differentiate a *digital* image $F[x,y]$?
  - Option 1: reconstruct a continuous image, $f$, then compute the derivative.
  - Option 2: take discrete derivative (finite difference)

$$\frac{\partial f}{\partial x}[x, y] \approx F[x + 1, y] - F[x, y]$$

How would you implement this as a linear filter?

![Linear filter for image derivatives](image)

Source: S. Seitz
Image gradient

- The gradient of an image: \( \nabla f = \left[ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right] \)

The gradient points in the direction of most rapid increase in intensity:

\[
\nabla f = \left[ \frac{\partial f}{\partial x}, 0 \right]
\]

\[
\nabla f = \left[ 0, \frac{\partial f}{\partial y} \right]
\]

The edge strength is given by the gradient magnitude:

\[
||\nabla f|| = \sqrt{\left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2}
\]

The gradient direction is given by:

\[
\theta = \tan^{-1} \left( \frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial x}} \right)
\]

• how does this relate to the direction of the edge?

Source: Steve Seitz

Source: L. Lazebnik
Effects of noise

Where is the edge?

Solution: smooth first

To find edges, look for peaks in $\frac{d}{dx}(f * h)$.
**Associative property of convolution**

- Differentiation is convolution, and convolution is associative: \[ \frac{d}{dx} (f * h) = f * \frac{d}{dx} h \]
- This saves us one operation:

**2D edge detection filters**

\[ h_\sigma(u, v) = \frac{1}{2\pi \sigma^2} e^{-\frac{u^2 + v^2}{2\sigma^2}} \]

\[ \frac{\partial}{\partial x} h_\sigma(u, v) \]
Derivative of Gaussian filter

The Sobel operator

- Common approximation of derivative of Gaussian

\[
\begin{array}{ccc}
& 1 & 0 & 1 \\
1 & -2 & 0 & 2 \\
& -1 & 0 & 1 \\
\end{array}
\]

\[
\begin{array}{ccc}
1 & 2 & 1 \\
0 & 0 & 0 \\
-1 & -2 & -1 \\
\end{array}
\]

- The standard defn. of the Sobel operator omits the 1/8 term
  - doesn’t make a difference for edge detection
  - the 1/8 term is needed to get the right gradient magnitude
Sobel operator: example

Example

- original image (Lena)
Finding edges

gradient magnitude

Finding edges

thresholding

where is the edge?
Questions?