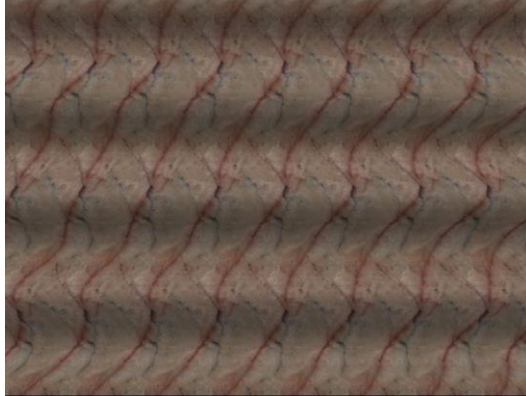


# CS4670 / 5670: Computer Vision

Noah Snavely

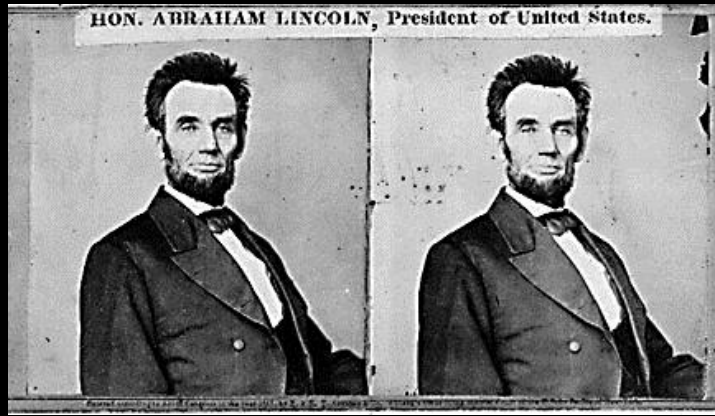
## Lecture 16: Stereo



Single image stereogram, by [Niklas Een](#)

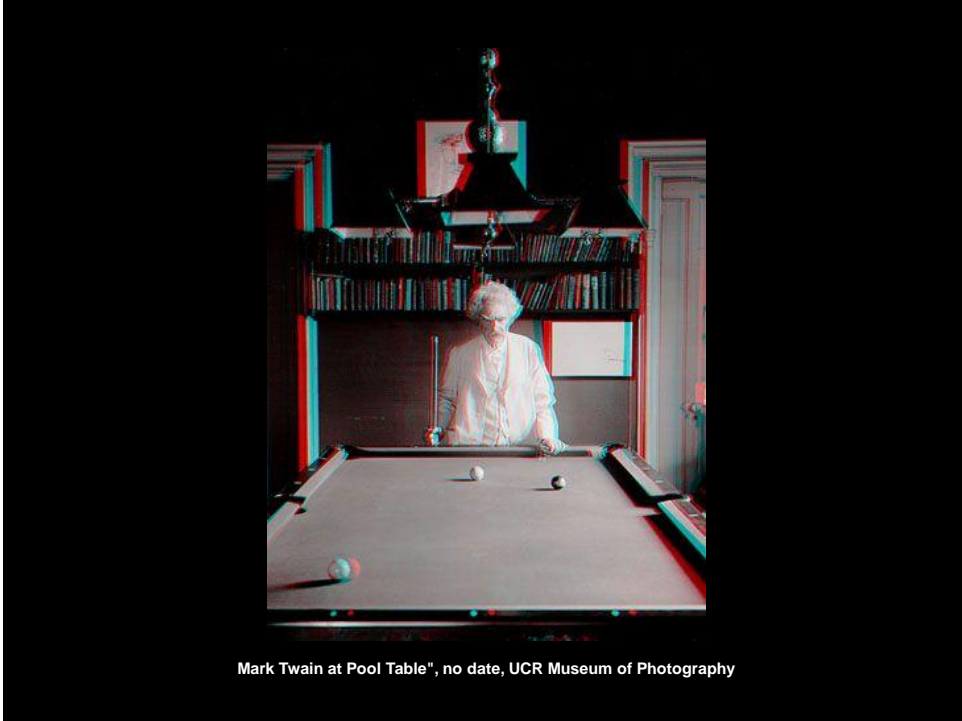
## Readings

- Szeliski, Chapter 10 (through 10.5)

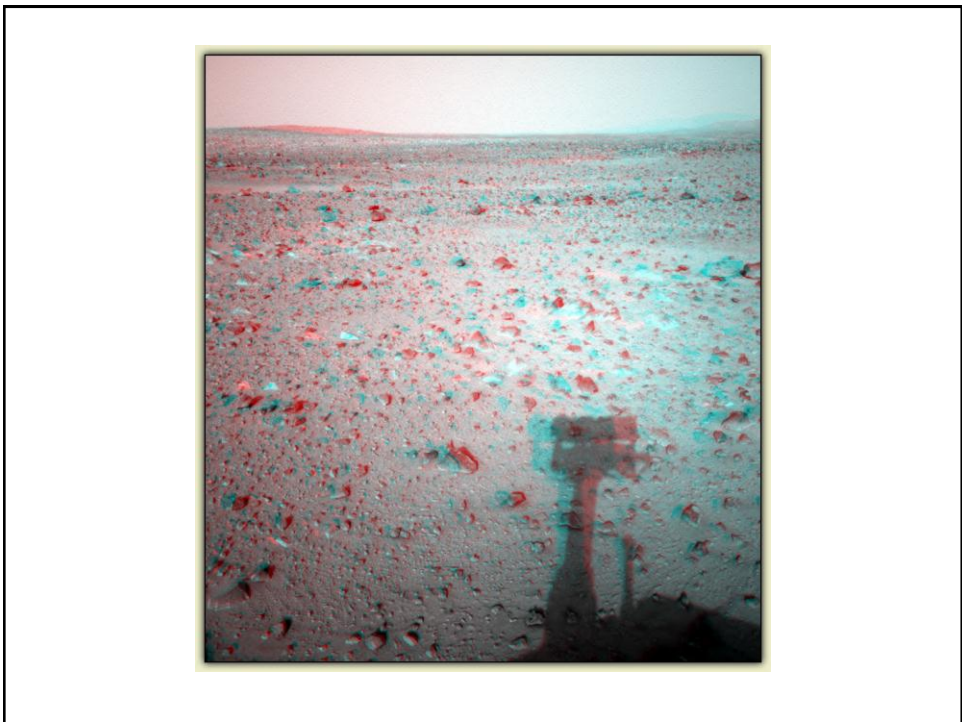


Public Library, Stereoscopic Looking Room, Chicago, by Phillips, 1923

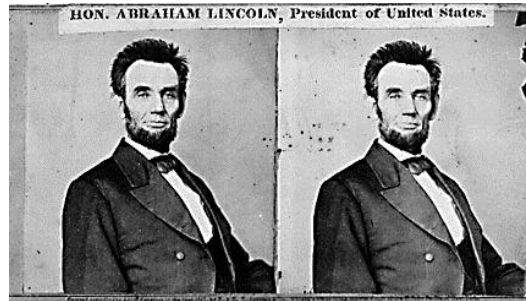




Mark Twain at Pool Table", no date, UCR Museum of Photography

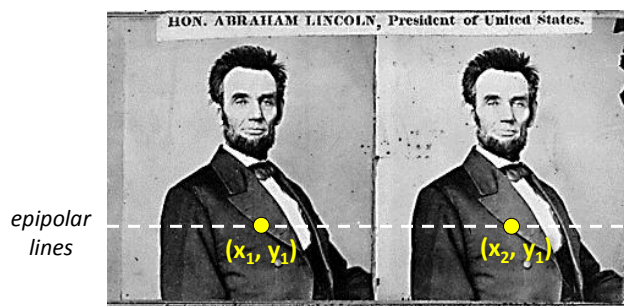


## Stereo



- Given two images from different viewpoints
  - How can we compute the depth of each point in the image?
  - Based on *how much each pixel moves* between the two images

## Epipolar geometry



Two images captured by a purely horizontal translating camera  
(*rectified* stereo pair)

$$x_2 - x_1 = \text{the } \textit{disparity} \text{ of pixel } (x_1, y_1)$$

## Stereo matching algorithms

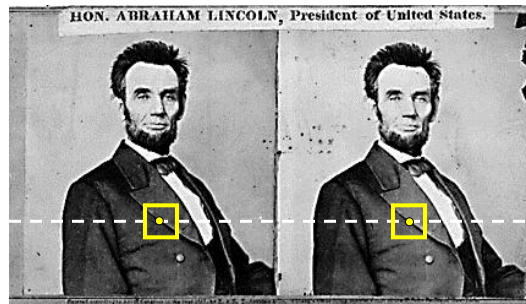
---

### Match Pixels in Conjugate Epipolar Lines

- Assume brightness constancy
- This is a tough problem
- Numerous approaches
  - A good survey and evaluation: <http://www.middlebury.edu/stereo/>

## Your basic stereo algorithm

---



For each epipolar line

For each pixel in the left image

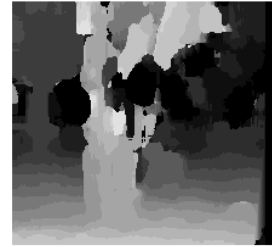
- compare with every pixel on same epipolar line in right image
- pick pixel with minimum match cost

Improvement: match **windows**

## Window size



$W = 3$



$W = 20$

### Effect of window size

- Smaller window
  - +
  -
- Larger window
  - +
  -

### Better results with *adaptive window*

- T. Kanade and M. Okutomi, [A Stereo Matching Algorithm with an Adaptive Window: Theory and Experiment](#), Proc. International Conference on Robotics and Automation, 1991.
- D. Scharstein and R. Szeliski, [Stereo matching with nonlinear diffusion](#), International Journal of Computer Vision, 28(2):155-174, July 1998

## Stereo results

- Data from University of Tsukuba
- Similar results on other images without ground truth



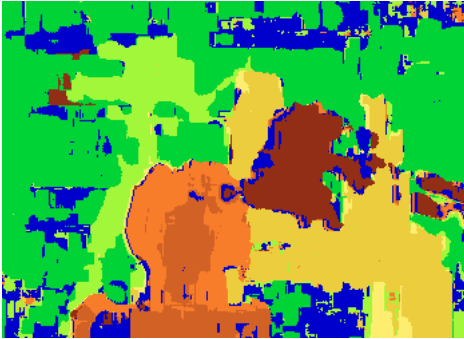
Scene



Ground truth

## Results with window search

---



Window-based matching  
(best window size)



Ground truth

## Better methods exist...

---



State of the art method



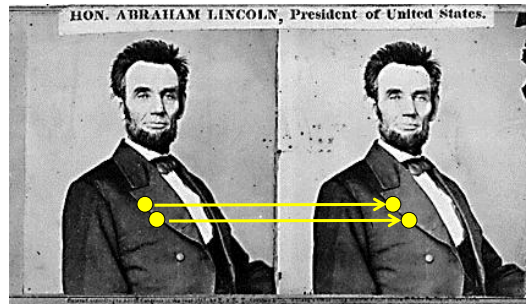
Ground truth

Boykov et al., [Fast Approximate Energy Minimization via Graph Cuts](#),  
International Conference on Computer Vision, September 1999.

For the latest and greatest: <http://www.middlebury.edu/stereo/>

## Stereo as energy minimization

---



What defines a good stereo correspondence?

1. Match quality
  - Want each pixel to find a good match in the other image
2. Smoothness
  - If two pixels are adjacent, they should (usually) move about the same amount

## Stereo as energy minimization

---

- Find disparity map  $d$  that minimizes an energy function

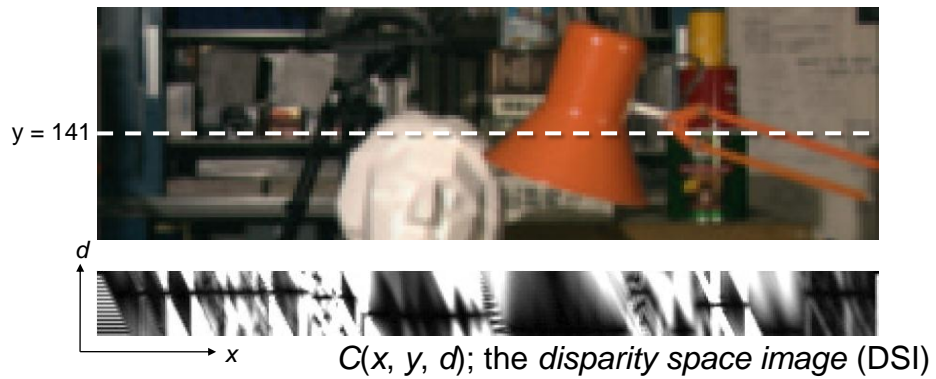
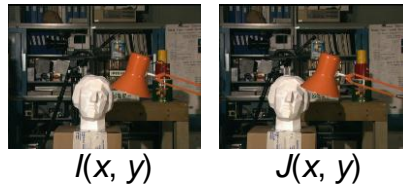
- Simple pixel / window matching

$$E(d) = \sum_{(x,y) \in I} C(x, y, d(x, y))$$

$$C(x, y, d(x, y)) = \text{SSD distance between windows } I(x, y) \text{ and } J(x + d(x, y), y)$$



## Stereo as energy minimization



## Stereo as energy minimization



Simple pixel / window matching: choose the minimum of each column in the DSI independently:

$$d(x, y) = \arg \min_{d'} C(x, y, d')$$

## Stereo as energy minimization

Better objective function

$$E(d) = \underbrace{E_d(d)}_{\text{match cost}} + \lambda \underbrace{E_s(d)}_{\text{smoothness cost}}$$

Want each pixel to find a good match in the other image

Adjacent pixels should (usually) move about the same amount

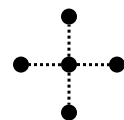
## Stereo as energy minimization

$$E(d) = E_d(d) + \lambda E_s(d)$$

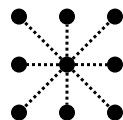
match cost:  $E_d(d) = \sum_{(x,y) \in I} C(x, y, d(x, y))$

smoothness cost:  $E_s(d) = \sum_{(p,q) \in \mathcal{E}} V(d_p, d_q)$

$\mathcal{E}$  : set of neighboring pixels



4-connected neighborhood



8-connected neighborhood

## Smoothness cost

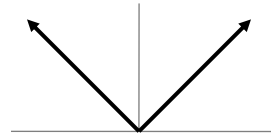
---

$$E_s(d) = \sum_{(p,q) \in \mathcal{E}} V(d_p, d_q)$$

How do we choose  $V$ ?

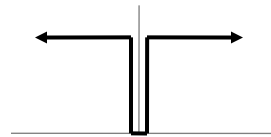
$$V(d_p, d_q) = |d_p - d_q|$$

$L_1$  distance



$$V(d_p, d_q) = \begin{cases} 0 & \text{if } d_p = d_q \\ 1 & \text{if } d_p \neq d_q \end{cases}$$

"Potts model"



## Dynamic programming

---

$$E(d) = E_d(d) + \lambda E_s(d)$$

Can minimize this independently per scanline using  
dynamic programming (DP)



$D(x, y, d)$  : minimum cost of solution such that  $d(x, y) = d$

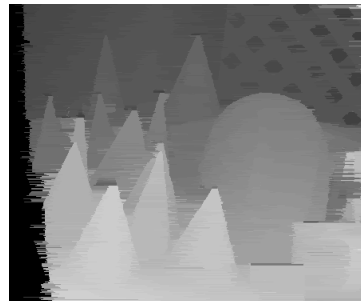
$$D(x, y, d) = C(x, y, d) + \min_{d'} \{D(x-1, y, d') + \lambda |d - d'|\}$$

## Dynamic programming



Finds “smooth” path through DPI from left to right

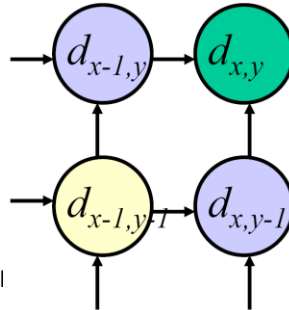
## Dynamic Programming



## Dynamic programming

---

Can we apply this trick in 2D as well?



No:  $d_{x,y-1}$  and  $d_{x-1,y}$

ent values of  $d_{x-1,y-1}$

Slide credit: D. Huttenlocher

## Stereo as a minimization problem

---

$$E(d) = E_d(d) + \lambda E_s(d)$$

The 2D problem has many local minima

- Gradient descent doesn't work well

And a large search space

- $n \times m$  image w/  $k$  disparities has  $k^{nm}$  possible solutions
- Finding the global minimum is NP-hard in general

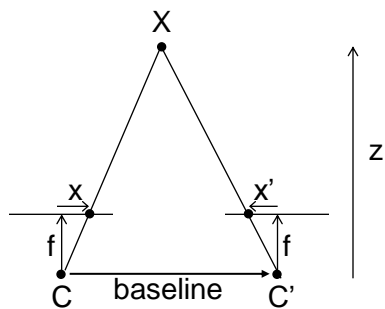
Good approximations exist... we'll see this soon

Questions?

---

Depth from disparity

---



$$disparity = x - x' = \frac{baseline * f}{z}$$

## Real-time stereo

---



[Nomad robot](http://www.frc.ri.cmu.edu/projects/meteorobot/index.html) searches for meteorites in Antarctica  
<http://www.frc.ri.cmu.edu/projects/meteorobot/index.html>

Used for robot navigation (and other tasks)

- Several software-based real-time stereo techniques have been developed (most based on simple discrete search)

## Stereo reconstruction pipeline

---

### Steps

- Calibrate cameras
- Rectify images
- Compute disparity
- Estimate depth

What will cause errors?

- Camera calibration errors
- Poor image resolution
- Occlusions
- Violations of brightness constancy (specular reflections)
- Large motions
- Low-contrast image regions

## Active stereo with structured light



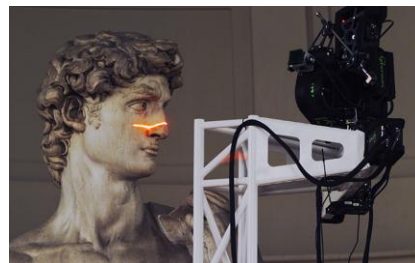
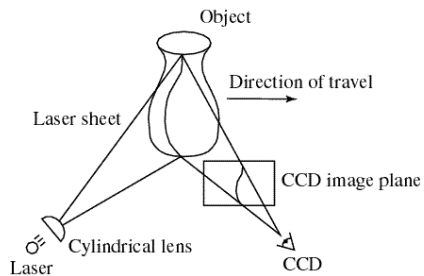
Li Zhang's one-shot stereo



Project “structured” light patterns onto the object

- simplifies the correspondence problem

## Laser scanning



Digital Michelangelo Project

<http://graphics.stanford.edu/projects/mich/>

Optical triangulation

- Project a single stripe of laser light
- Scan it across the surface of the object
- This is a very precise version of structured light scanning



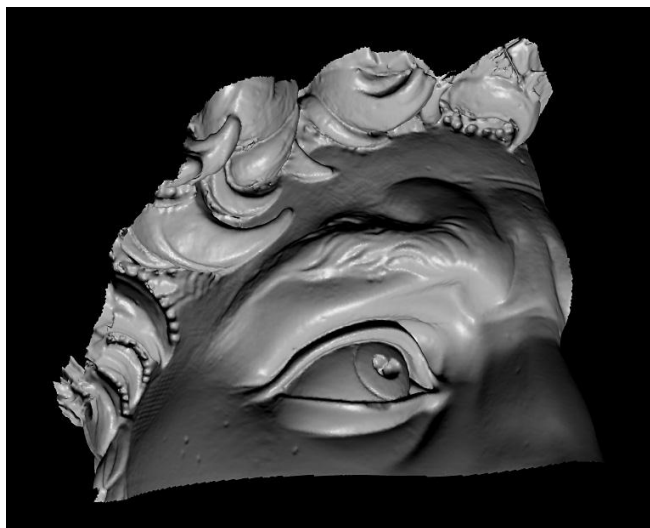
## Laser scanned models



*The Digital Michelangelo Project, Levoy et al.*

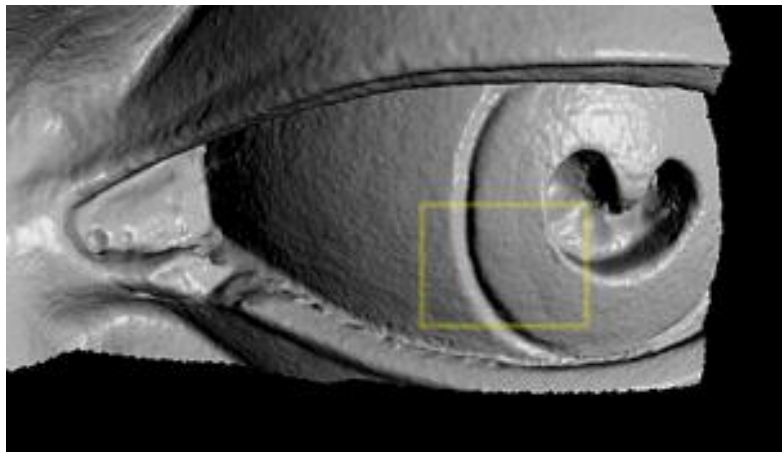
## Laser scanned models

---



*The Digital Michelangelo Project, Levoy et al.*

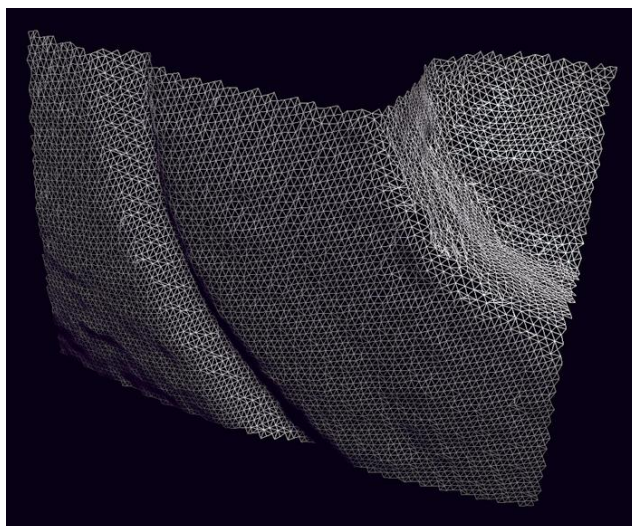
## Laser scanned models



*The Digital Michelangelo Project, Levoy et al.*

## Laser scanned models

---



*The Digital Michelangelo Project, Levoy et al.*