Reading

- Szeliski: Chapter 3.6
Announcements

• Project 1 voting will be released soon

• Project 2 out soon, to be done in groups of 2
  – Please form groups of 2 on CMS

Image alignment

Why don’t these image line up exactly?
What is the geometric relationship between these two images?

Answer: Similarity transformation (translation, rotation, uniform scale)

What is the geometric relationship between these two images?
What is the geometric relationship between these two images?

Very important for creating mosaics!

Image Warping

- image filtering: change range of image
  - \( g(x) = h(f(x)) \)

- image warping: change domain of image
  - \( g(x) = f(h(x)) \)
Image Warping

- image filtering: change *range* of image
  \[ g(x) = h(f(x)) \]

- image warping: change *domain* of image
  \[ g(x) = f(h(x)) \]

Parametric (global) warping

- Examples of parametric warps:
  - translation
  - rotation
  - aspect
Parametric (global) warping

- Transformation $T$ is a coordinate-changing machine:
  \[ p' = T(p) \]
- What does it mean that $T$ is global?
  - Is the same for any point $p$
  - Can be described by just a few numbers (parameters)
- Let’s consider linear xforms (can be represented by a 2D matrix):
  \[
  p' = Tp \quad \begin{bmatrix} x' \\ y' \end{bmatrix} = T \begin{bmatrix} x \\ y \end{bmatrix}
  \]

Common linear transformations

- Uniform scaling by $s$:
  \[
  S = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \quad \text{What is the inverse?}
  \]
Common linear transformations

• Rotation by angle $\theta$ (about the origin)

$$R = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

What is the inverse?

For rotations: $R^{-1} = R^T$

2x2 Matrices

• What types of transformations can be represented with a 2x2 matrix?

2D mirror about Y axis?

$$\begin{align*}
x' &= -x \\
y' &= y
\end{align*}$$

$$T = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

2D mirror across line $y = x$?

$$\begin{align*}
x' &= y \\
y' &= x
\end{align*}$$

$$T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$
2x2 Matrices

• What types of transformations can be represented with a 2x2 matrix?

2D Translation?
\[ x' = x + t_x \quad \text{NO!} \]
\[ y' = y + t_y \]

Translation is not a linear operation on 2D coordinates

All 2D Linear Transformations

• Linear transformations are combinations of ...
  – Scale,
  – Rotation,
  – Shear, and
  – Mirror

\[
\begin{bmatrix}
  x' \\
  y'
\end{bmatrix} =
\begin{bmatrix}
  a & b \\
  c & d
\end{bmatrix}
\begin{bmatrix}
  x \\
  y
\end{bmatrix}
\]

• Properties of linear transformations:
  – Origin maps to origin
  – Lines map to lines
  – Parallel lines remain parallel
  – Ratios are preserved
  – Closed under composition

\[
\begin{bmatrix}
  x' \\
  y'
\end{bmatrix} =
\begin{bmatrix}
  a & b & e & f \\
  c & d & g & h \\
  i & j & k & l
\end{bmatrix}
\begin{bmatrix}
  x \\
  y
\end{bmatrix}
\]
Homogeneous coordinates

Trick: add one more coordinate:

\[(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}\]

homogeneous image coordinates

Converting from homogeneous coordinates:

\[
\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)
\]

Translation

- Solution: homogeneous coordinates to the rescue

\[
\mathbf{T} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}
\]

\[
\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}
\]
Affine transformations

\[
T = \begin{bmatrix}
1 & 0 & t_x \\
0 & 1 & t_y \\
0 & 0 & 1
\end{bmatrix}
\]

any transformation with last row \([0 0 1]\) we call an affine transformation

Basic affine transformations

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Matrix</th>
</tr>
</thead>
</table>
| Translate     | \[
\begin{bmatrix}
x' \\
y' \\
1
\end{bmatrix} = \begin{bmatrix}
1 & 0 & t_x \\
0 & 1 & t_y \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
x \\
y \\
1
\end{bmatrix}
\] |
| Scale         | \[
\begin{bmatrix}
x' \\
y' \\
1
\end{bmatrix} = \begin{bmatrix}
s_x & 0 & 0 \\
0 & s_y & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
x \\
y \\
1
\end{bmatrix}
\] |
| 2D in-plane rotation | \[
\begin{bmatrix}
x' \\
y' \\
1
\end{bmatrix} = \begin{bmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
x \\
y \\
1
\end{bmatrix}
\] |
| Shear         | \[
\begin{bmatrix}
x' \\
y' \\
1
\end{bmatrix} = \begin{bmatrix}
1 & s_{x'} & 0 \\
0 & s_{y'} & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
x \\
y \\
1
\end{bmatrix}
\] |
Affine Transformations

• Affine transformations are combinations of ...
  – Linear transformations, and
  – Translations
  \[
  \begin{bmatrix}
  x' \\
y' \\
w
  \end{bmatrix} = \begin{bmatrix}
  a & b & c \\
d & e & f \\
0 & 0 & 1
  \end{bmatrix} \begin{bmatrix}
x \\
y \\
w
  \end{bmatrix}
  \]

• Properties of affine transformations:
  – Origin does not necessarily map to origin
  – Lines map to lines
  – Parallel lines remain parallel
  – Ratios are preserved
  – Closed under composition

Is this an affine transformation?
Where do we go from here?

\[
\begin{bmatrix}
  a & b & c \\
  d & e & f \\
  0 & 0 & 1
\end{bmatrix}
\]

affine transformation

what happens when we mess with this row?

Projective Transformations aka Homographies aka Planar Perspective Maps

\[
H = \begin{bmatrix}
  a & b & c \\
  d & e & f \\
  g & h & 1
\end{bmatrix}
\]

called a homography
(or planar perspective map)
Homographies

• Example on board

Image warping with homographies
Homographies

Projective Transformations

• Projective transformations ...
  – Affine transformations, and
  – Projective warps

\[
\begin{bmatrix}
  x' \\
  y' \\
  w'
\end{bmatrix} =
\begin{bmatrix}
  a & b & c \\
  d & e & f \\
  g & h & i
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  w
\end{bmatrix}
\]

• Properties of projective transformations:
  – Origin does not necessarily map to origin
  – Lines map to lines
  – Parallel lines do not necessarily remain parallel
  – Ratios are not preserved
  – Closed under composition
2D image transformations

These transformations are a nested set of groups
- Closed under composition and inverse is a member