

# CS4670 / 5670: Computer Vision

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## Lecture 5: Feature detection and matching



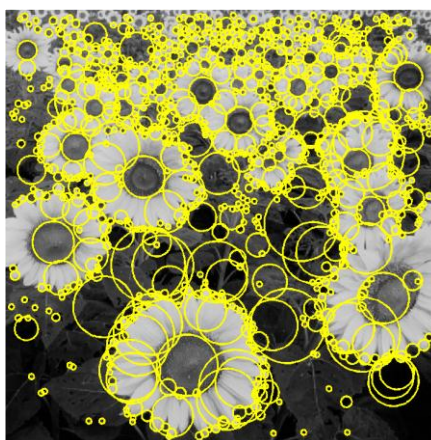
## Announcements

- Assignment 1 due **Sunday**
- Turn-in by 11:59pm Sunday evening
- Demo sessions on Monday (to be scheduled)

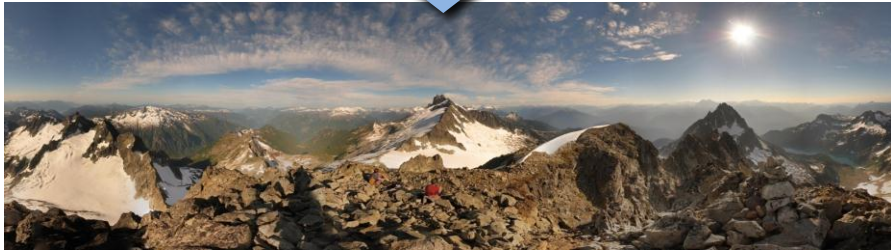
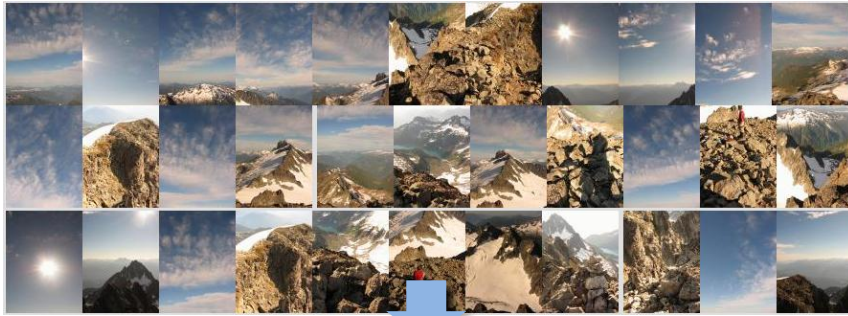
## Reading

- Szeliski: 4.1

### Feature extraction: Corners and blobs



## Motivation: Automatic panoramas



Credit: Matt Brown

## Motivation: Automatic panoramas



HD View

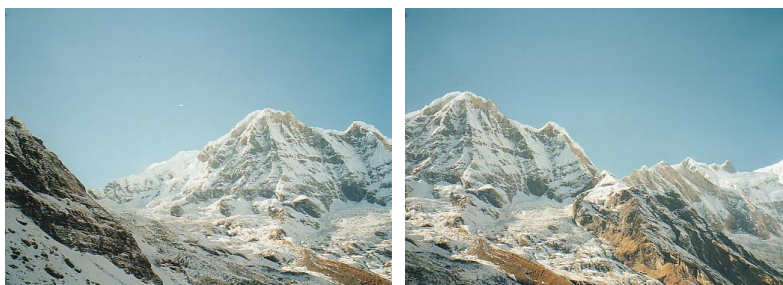
<http://research.microsoft.com/en-us/um/redmond/groups/ivm/HDView/HDGigapixel.htm>

Also see GigaPan:

<http://gigapan.org/>

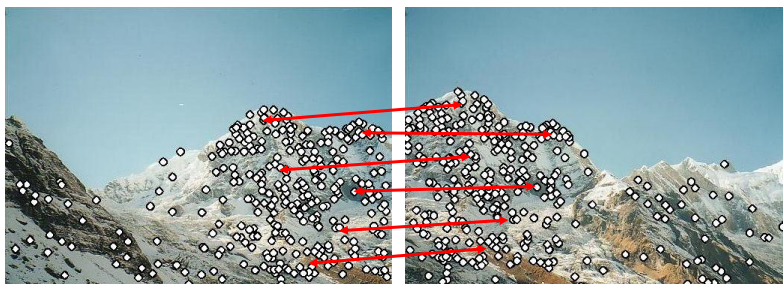
## Why extract features?

- Motivation: panorama stitching
  - We have two images – how do we combine them?



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Step 1: extract features

Step 2: match features

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- Motivation: panorama stitching
  - We have two images – how do we combine them?



Step 1: extract features  
Step 2: match features  
Step 3: align images

## Image matching



by [Diva Sian](#)



by [swashford](#)

## Harder case



by [Diva Sian](#)

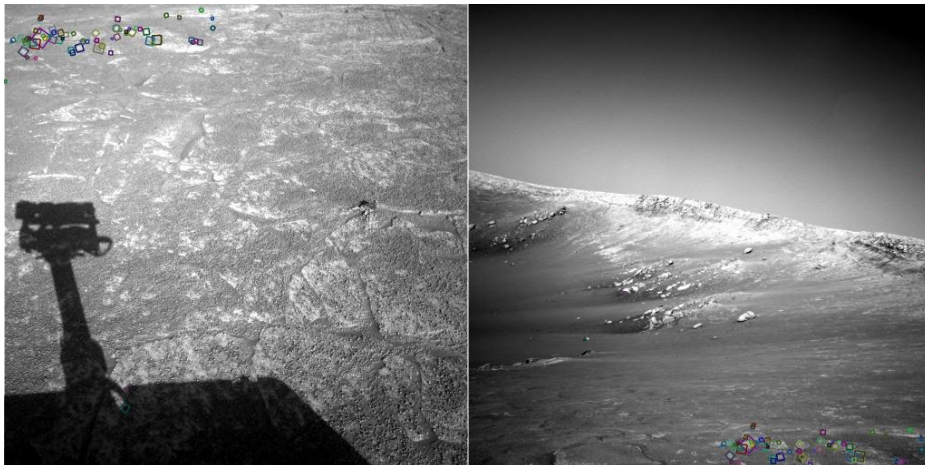


by [scgbt](#)

## Harder still?

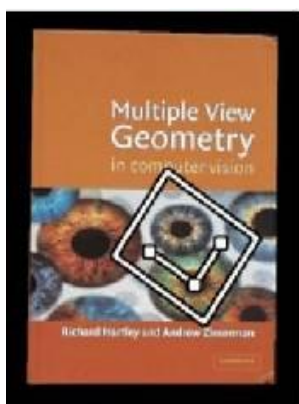


Answer below (look for tiny colored squares...)

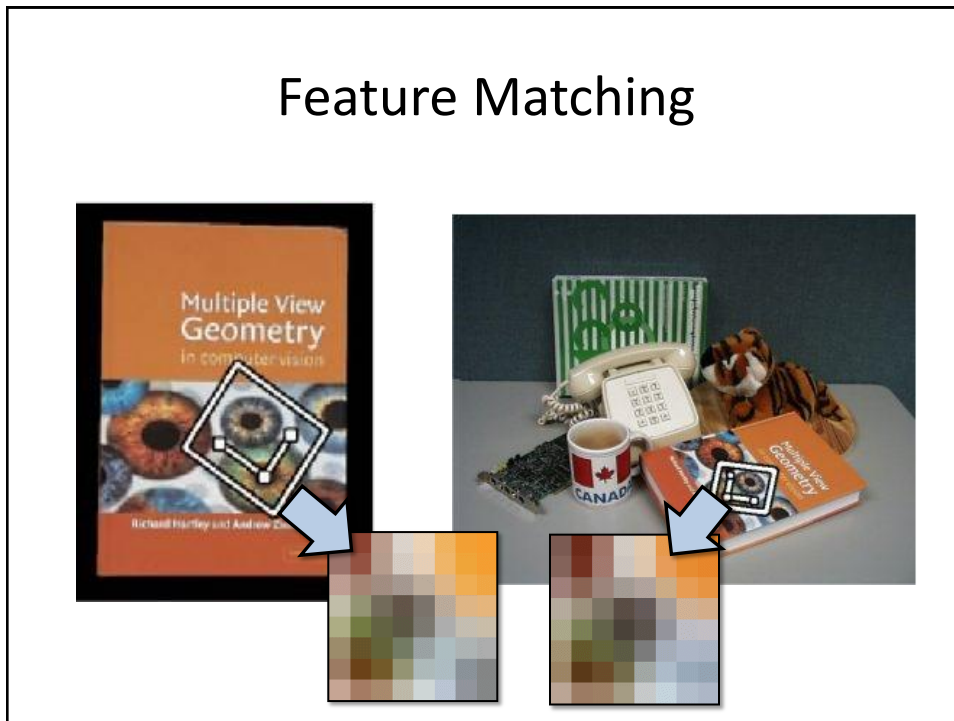


NASA Mars Rover images  
with SIFT feature matches

## Feature Matching



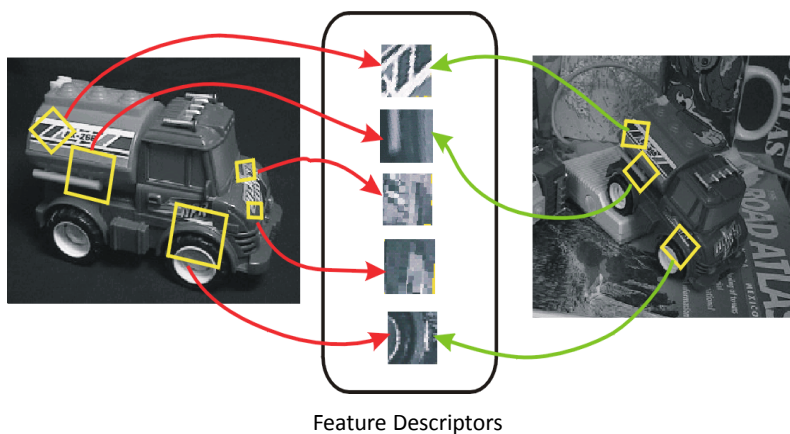
## Feature Matching



## Invariant local features

Find features that are invariant to transformations

- geometric invariance: translation, rotation, scale
- photometric invariance: brightness, exposure, ...





## Advantages of local features

### Locality

- features are local, so robust to occlusion and clutter

### Quantity

- hundreds or thousands in a single image

### Distinctiveness:

- can differentiate a large database of objects

### Efficiency

- real-time performance achievable

## More motivation...

### Feature points are used for:

- Image alignment (e.g., mosaics)
- 3D reconstruction
- Motion tracking
- Object recognition
- Indexing and database retrieval
- Robot navigation
- ... other



## Want uniqueness

Look for image regions that are unusual

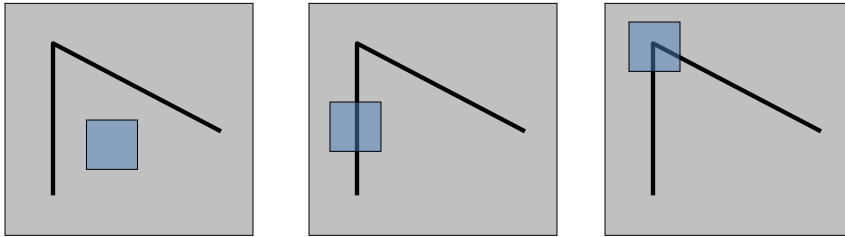
- Lead to unambiguous matches in other images

How to define “unusual”?

## Local measures of uniqueness

Suppose we only consider a small window of pixels

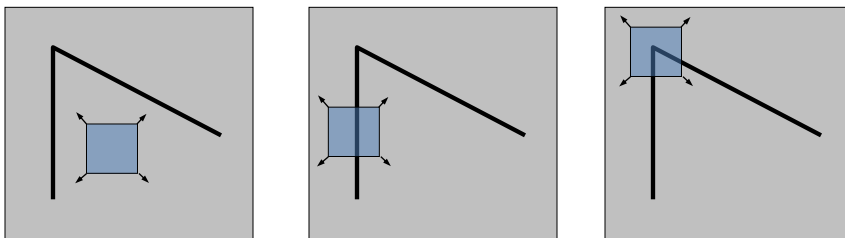
- What defines whether a feature is a good or bad candidate?



Credit: S. Seitz, D. Frolova, D. Simakov

## Local measure of feature uniqueness

- How does the window change when you shift it?
- Shifting the window in any direction causes a big change



“flat” region:  
no change in all  
directions

“edge”:  
no change along the  
edge direction

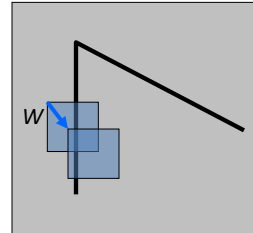
“corner”:  
significant change in  
all directions

Credit: S. Seitz, D. Frolova, D. Simakov

## Harris corner detection: the math

Consider shifting the window  $W$  by  $(u, v)$

- how do the pixels in  $W$  change?
- compare each pixel before and after by summing up the squared differences (SSD)
- this defines an SSD “error”  $E(u, v)$ :



$$E(u, v) = \sum_{(x, y) \in W} [I(x + u, y + v) - I(x, y)]^2$$

## Small motion assumption

Taylor Series expansion of  $I$ :

$$I(x + u, y + v) = I(x, y) + \frac{\partial I}{\partial x} u + \frac{\partial I}{\partial y} v + \text{higher order terms}$$

If the motion  $(u, v)$  is small, then first order approximation is good

$$\begin{aligned} I(x + u, y + v) &\approx I(x, y) + \frac{\partial I}{\partial x} u + \frac{\partial I}{\partial y} v \\ &\approx I(x, y) + [I_x \ I_y] \begin{bmatrix} u \\ v \end{bmatrix} \end{aligned}$$

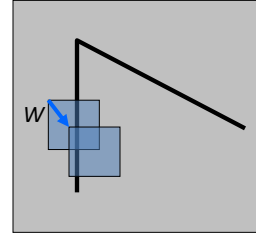
$$\text{shorthand: } I_x = \frac{\partial I}{\partial x}$$

Plugging this into the formula on the previous slide...

## Corner detection: the math

Consider shifting the window  $W$  by  $(u, v)$

- define an SSD "error"  $E(u, v)$ :

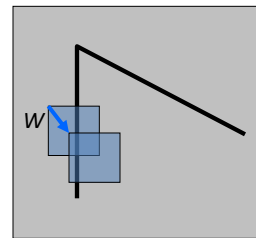


$$\begin{aligned}
 E(u, v) &= \sum_{(x, y) \in W} [I(x + u, y + v) - I(x, y)]^2 \\
 &\approx \sum_{(x, y) \in W} [I(x, y) + I_x u + I_y v - I(x, y)]^2 \\
 &\approx \sum_{(x, y) \in W} [I_x u + I_y v]^2
 \end{aligned}$$

## Corner detection: the math

Consider shifting the window  $W$  by  $(u, v)$

- define an SSD "error"  $E(u, v)$ :



$$\begin{aligned}
 E(u, v) &\approx \sum_{(x, y) \in W} [I_x u + I_y v]^2 \\
 &\approx Au^2 + 2Buv + Cv^2
 \end{aligned}$$

$$A = \sum_{(x, y) \in W} I_x^2 \quad B = \sum_{(x, y) \in W} I_x I_y \quad C = \sum_{(x, y) \in W} I_y^2$$

- Thus,  $E(u, v)$  is locally approximated as a quadratic error function

## The second moment matrix

The surface  $E(u,v)$  is locally approximated by a quadratic form.

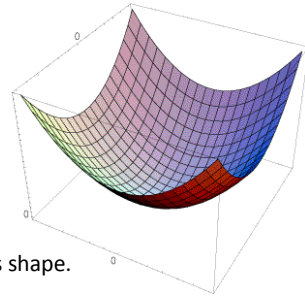
$$E(u, v) \approx Au^2 + 2Buv + Cv^2$$

$$\approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_H \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{(x,y) \in W} I_x^2$$

$$B = \sum_{(x,y) \in W} I_x I_y$$

$$C = \sum_{(x,y) \in W} I_y^2$$



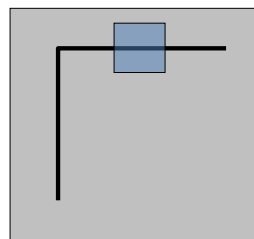
Let's try to understand its shape.

$$E(u, v) \approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_H \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{(x,y) \in W} I_x^2$$

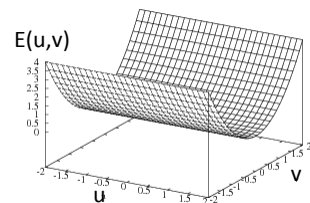
$$B = \sum_{(x,y) \in W} I_x I_y$$

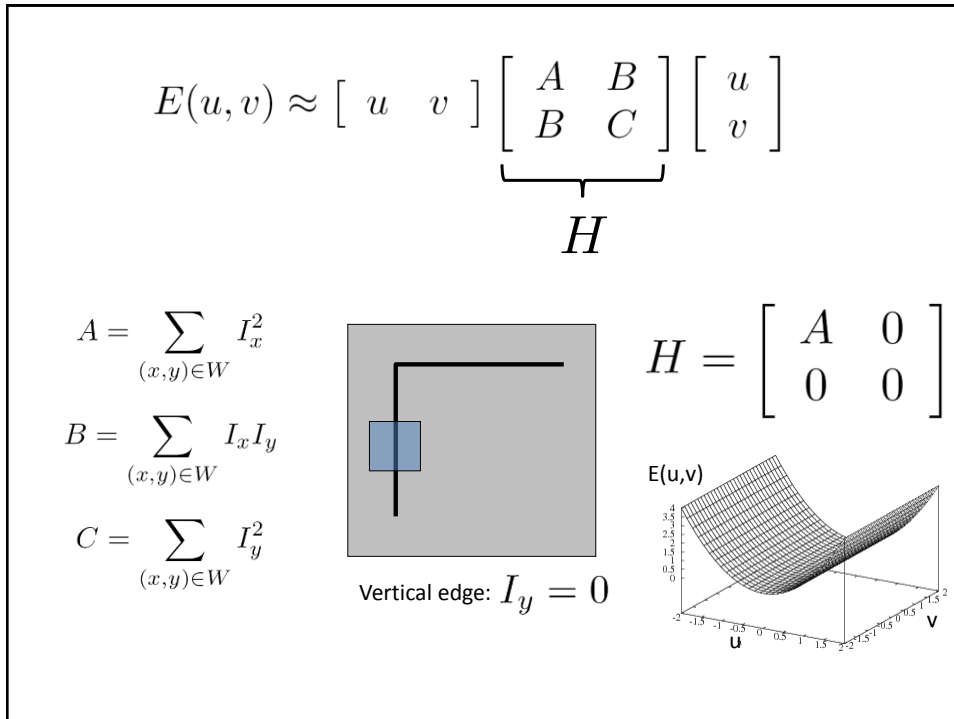
$$C = \sum_{(x,y) \in W} I_y^2$$



Horizontal edge:  $I_x = 0$

$$H = \begin{bmatrix} 0 & 0 \\ 0 & C \end{bmatrix}$$



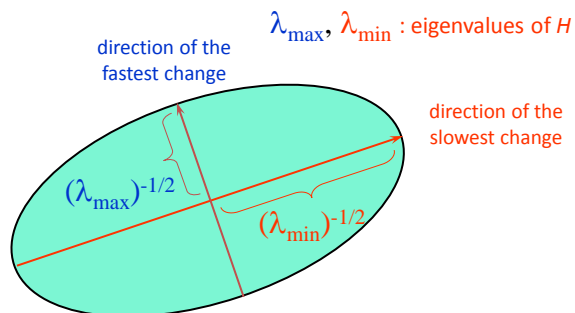


## General case

We can visualize  $H$  as an ellipse with axis lengths determined by the *eigenvalues* of  $H$  and orientation determined by the *eigenvectors* of  $H$

Ellipse equation:

$$[u \ v] H \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$



Questions?