## CS4670: Computer Vision Noah Snavely

## Lecture 32: Photometric stereo, Part 2



## BRDF's can be incredibly complicated...



## Shape from shading



Suppose $k_{d}=1$

$$
\begin{aligned}
I & =k_{d} \mathbf{N} \cdot \mathbf{L} \\
& =\mathbf{N} \cdot \mathbf{L} \\
& =\cos \theta_{i}
\end{aligned}
$$

You can directly measure angle between normal and light source

- Not quite enough information to compute surface shape
- But can be if you add some additional info, for example
- assume a few of the normals are known (e.g., along silhouette)
- constraints on neighboring normals-"integrability"
- smoothness
- Hard to get it to work well in practice
- plus, how many real objects have constant albedo?


## Photometric stereo



Can write this as a matrix equation:

$$
\left[\begin{array}{c}
I_{1} \\
I_{2} \\
I_{3}
\end{array}\right]=k_{d}\left[\begin{array}{l}
\mathbf{L}_{1}^{T} \\
\mathbf{L}_{2}^{T} \\
\mathbf{L}_{3}^{T}
\end{array}\right] \mathbf{N}
$$

## Solving the equations

$$
\begin{aligned}
& {\left[\begin{array}{l}
I_{1} \\
I_{2} \\
I_{3}
\end{array}\right] }=\underbrace{\left[\begin{array}{l}
\mathbf{L}_{1}{ }^{T} \\
\mathbf{L}_{2}{ }^{T} \\
\mathbf{L}_{3}^{T}
\end{array}\right]}_{\mathbf{I}_{\times 1}} \underbrace{}_{\mathbf{J}^{T}} k_{d} \mathbf{N} \\
& \mathbf{G} \\
& \mathbf{G}=\mathbf{L}^{-1} \mathbf{I} \\
& k_{d}=\|\mathbf{G}\| \\
& \mathbf{N}=\frac{1}{k_{d}} \mathbf{G}
\end{aligned}
$$

## More than three lights

Get better results by using more lights

$$
\left[\begin{array}{c}
I_{1} \\
\vdots \\
I_{n}
\end{array}\right]=\left[\begin{array}{c}
\mathbf{L}_{1} \\
\vdots \\
\mathbf{L}_{\mathbf{n}}
\end{array}\right] k_{d} \mathbf{N}
$$

Least squares solution:

$$
\begin{aligned}
\mathbf{I} & =\mathbf{L G} \\
\mathbf{L}^{\mathbf{T}} \mathbf{I} & =\mathbf{L}^{\mathrm{T}} \mathbf{L G} \\
\mathbf{G} & =\left(\mathbf{L}^{\mathrm{T}} \mathbf{L}\right)^{-1}\left(\mathbf{L}^{\mathrm{T}} \mathbf{I}\right)
\end{aligned}
$$

Solve for $\mathrm{N}, \mathrm{k}_{\mathrm{d}}$ as before
What's the size of $L^{\top} L$ ?

## Example



Recovered normal field


## Computing light source directions

Trick: place a chrome sphere in the scene


- the location of the highlight tells you where the light source is


## Depth from normals



Get a similar equation for $\mathbf{V}_{\mathbf{2}}$

- Each normal gives us two linear constraints on z
- compute $z$ values by solving a matrix equation


## Example



## Limitations

## Big problems

- doesn't work for shiny things, semi-translucent things
- shadows, inter-reflections

Smaller problems

- camera and lights have to be distant
- calibration requirements
- measure light source directions, intensities
- camera response function

Newer work addresses some of these issues
Some pointers for further reading:

- Zickler, Belhumeur, and Kriegman, "Helmholtz Stereopsis: Exploiting Reciprocity for Surface Reconstruction." IJCV, Vol. 49 No. 2/3, pp 215-227.
- Hertzmann \& Seitz, "Example-Based Photometric Stereo: Shape Reconstruction with General, Varying BRDFs." IEEE Trans. PAMI 2005


## Finding the direction of the light source


P. Nillius and J.-O. Eklundh, "Automatic estimation of the projected light source direction," CVPR 2001

## Application: Detecting composite photos

Which is the real photo?


Fake photo


## Questions?

