Lecture 31: Photometric stereo
What happens when a light ray hits an object?

Some of the light gets absorbed
  • converted to other forms of energy (e.g., heat)

Some gets transmitted through the object
  • possibly bent, through “refraction”
  • a transmitted ray could possibly bounce back

Some gets reflected
  • as we saw before, it could be reflected in multiple directions (possibly all directions) at once
Classic reflection behavior

ideal specular

rough specular

Lambertian

from Steve Marschner
The BRDF

The Bidirectional Reflection Distribution Function

- Given an incoming ray \((\theta_i, \phi_i)\) and outgoing ray \((\theta_e, \phi_e)\) what proportion of the incoming light is reflected along outgoing ray?

Answer given by the BRDF: \(\rho(\theta_i, \phi_i, \theta_e, \phi_e)\)
Constraints on the BRDF

Energy conservation

- Quantity of outgoing light ≤ quantity of incident light
  - integral of BRDF ≤ 1

Helmholtz reciprocity

- reversing the path of light produces the same reflectance
Diffuse reflection governed by Lambert’s law

- Viewed brightness does not depend on viewing direction
- Brightness *does* depend on direction of illumination
- This is the model most often used in computer vision

Lambert’s Law: \( I_e = k_d N \cdot L I_i \)

\( k_d \) is called **albedo**

BRDF for Lambertian surface

\[ \rho(\theta_i, \phi_i, \theta_e, \phi_e) = k_d \cos \theta_i \]
Diffuse reflection

Demo

http://www.math.montana.edu/frankw/ccp/multiworld/twothree/lighting/applet1.htm
http://www.math.montana.edu/frankw/ccp/multiworld/twothree/lighting/learn2.htm
Specular reflection

For a perfect mirror, light is reflected about \( \mathbf{N} \)

\[
I_e = \begin{cases} 
I_i & \text{if } \mathbf{V} = \mathbf{R} \\
0 & \text{otherwise}
\end{cases}
\]

Near-perfect mirrors have a **highlight** around \( \mathbf{R} \)

- common model: 
  \[
  I_e = k_s (\mathbf{V} \cdot \mathbf{R})^{n_s} I_i
  \]
Specular reflection

Moving the light source

Changing $n_s$
Photometric Stereo

Readings

Diffuse reflection

\[ R_e = k_d N \cdot LR_i \]

Simplifying assumptions

- \( I = R_e \): camera response function is the identity function:

- \( R_i = 1 \): light source intensity is 1
  - can achieve this by dividing each pixel in the image by \( R_i \)
Shape from shading

Suppose $k_d = 1$

$$I = k_d N \cdot L$$

$$= N \cdot L$$

$$= \cos \theta_i$$

You can directly measure angle between normal and light source

- Not quite enough information to compute surface shape
- But can be if you add some additional info, for example
  - assume a few of the normals are known (e.g., along silhouette)
  - constraints on neighboring normals—“integrability”
  - smoothness
- Hard to get it to work well in practice
  - plus, how many real objects have constant albedo?
Photometric stereo

\[ I_1 = k_d N \cdot L_1 \]
\[ I_2 = k_d N \cdot L_2 \]
\[ I_3 = k_d N \cdot L_3 \]

Can write this as a matrix equation:

\[
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix} = k_d 
\begin{bmatrix}
L_1^T \\
L_2^T \\
L_3^T
\end{bmatrix} N
\]
Solving the equations

\[
\begin{bmatrix}
  I_1 \\
  I_2 \\
  I_3
\end{bmatrix}
= \begin{bmatrix}
  L_1^T \\
  L_2^T \\
  L_3^T
\end{bmatrix} k_d \mathbf{N}
\]

\[
\mathbf{G} = \mathbf{L}^{-1} \mathbf{I}
\]

\[
k_d = \| \mathbf{G} \|
\]

\[
\mathbf{N} = \frac{1}{k_d} \mathbf{G}
\]
More than three lights

Get better results by using more lights

\[
\begin{bmatrix}
I_1 \\
\vdots \\
I_n
\end{bmatrix} = \begin{bmatrix}
L_1 \\
\vdots \\
L_n
\end{bmatrix} k_d N
\]

Least squares solution:

\[
I = LG \\
L^T I = L^T LG \\
G = (L^T L)^{-1} (L^T I)
\]

Solve for \(N, k_d\) as before

What’s the size of \(L^T L\)?
Computing light source directions

Trick: place a chrome sphere in the scene

- the location of the highlight tells you where the light source is
Depth from normals

Get a similar equation for $V_2$

- Each normal gives us two linear constraints on $z$
- Compute $z$ values by solving a matrix equation

$$V_1 = (x + 1, y, z_{x+1,y}) - (x, y, z_{xy})$$
$$= (1, 0, z_{x+1,y} - z_{xy})$$
$$0 = N \cdot V_1$$
$$= (n_x, n_y, n_z) \cdot (1, 0, z_{x+1,y} - z_{xy})$$
$$= n_x + n_z (z_{x+1,y} - z_{xy})$$
Example
What if we don’t have mirror ball?

Limitations

Big problems

• doesn’t work for shiny things, semi-translucent things
• shadows, inter-reflections

Smaller problems

• camera and lights have to be distant
• calibration requirements
  – measure light source directions, intensities
  – camera response function

Newer work addresses some of these issues

Some pointers for further reading:

• Hertzmann & Seitz, “Example-Based Photometric Stereo: Shape Reconstruction with General, Varying BRDFs.” IEEE Trans. PAMI 2005
Application: Detecting composite photos

Which is the real photo?

Fake photo

Real photo