CS4670: Intro to Computer Vision

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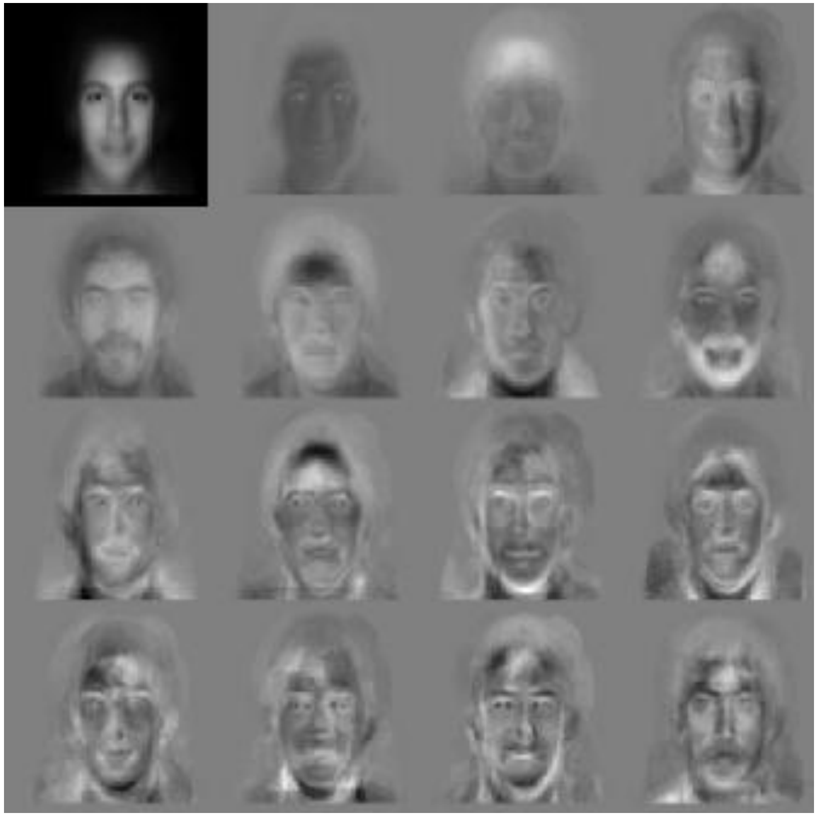
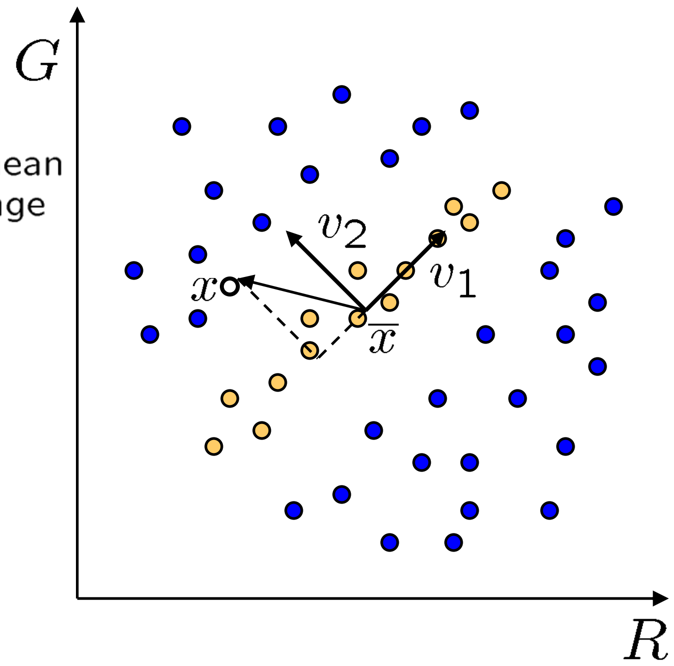
Lecture 27: Eigenfaces



Linear subspaces



$\overline{x}$ is the mean of the orange points

convert **x** into $\mathbf{v_1}$, $\mathbf{v_2}$ coordinates

$$\mathbf{x} \rightarrow ((\mathbf{x} - \overline{x}) \cdot \mathbf{v_1}, (\mathbf{x} - \overline{x}) \cdot \mathbf{v_2})$$

What does the $\mathbf{v_2}$ coordinate measure?

- distance to line
- use it for classification—near 0 for orange pts

What does the $\mathbf{v_1}$ coordinate measure?

- position along line
- use it to specify which orange point it is

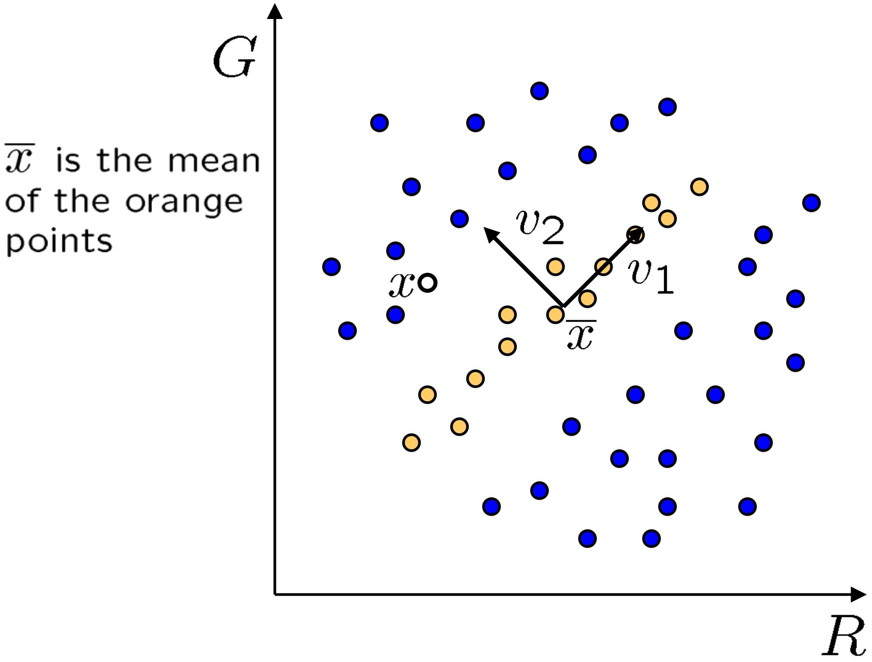
Classification can be expensive

- Must either search (e.g., nearest neighbors) or store large PDF's

Suppose the data points are arranged as above

- Idea—fit a line, classifier measures distance to line

Dimensionality reduction



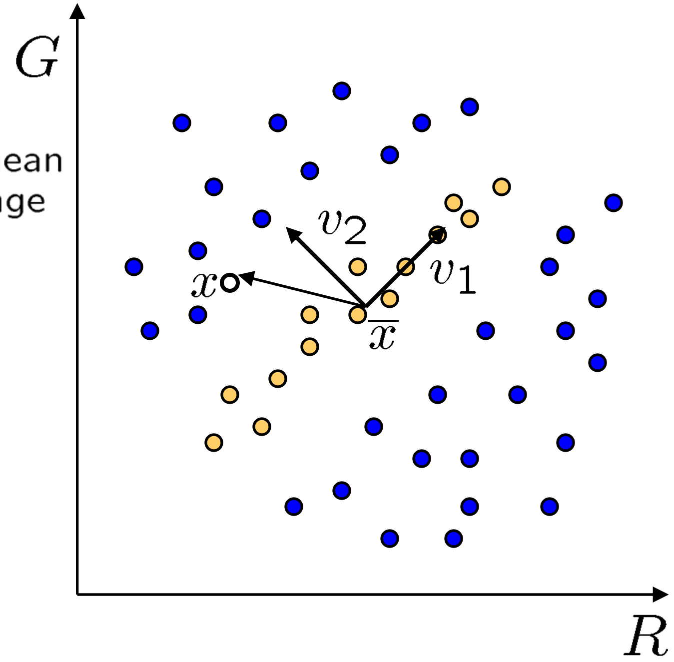
How to find $\mathbf{v_1}$ and $\mathbf{v_2}$?

Dimensionality reduction

- We can represent the orange points with *only* their $\mathbf{v_1}$ coordinates
  - since $\mathbf{v_2}$ coordinates are all essentially 0
- This makes it much cheaper to store and compare points
- A bigger deal for higher dimensional problems

Linear subspaces

$\overline{x}$ is the mean of the orange points



Consider the variation along direction **v** among all of the orange points:

$$var(\mathbf{v}) = \sum_{\text{orange point } \mathbf{x}} \|(\mathbf{x} - \overline{\mathbf{x}})^{\mathbf{T}} \cdot \mathbf{v}\|^2$$

What unit vector **v** minimizes *var*?

$$\mathbf{v}_2 = min_{\mathbf{v}} \ \{var(\mathbf{v})\}$$

What unit vector **v** maximizes *var*?

$$\mathbf{v}_1 = max_{\mathbf{v}} \ \{var(\mathbf{v})\}$$

$$\begin{aligned} var(\mathbf{v}) &= \sum_{\mathbf{x}} \|(\mathbf{x} - \overline{\mathbf{x}})^{\mathbf{T}} \cdot \mathbf{v}\|^2 \\ &= \sum_{\mathbf{x}} \mathbf{v}^{\mathbf{T}}(\mathbf{x} - \overline{\mathbf{x}})(\mathbf{x} - \overline{\mathbf{x}})^{\mathbf{T}}\mathbf{v} \\ &= \mathbf{v}^{\mathbf{T}}\left[\sum_{\mathbf{x}}(\mathbf{x} - \overline{\mathbf{x}})(\mathbf{x} - \overline{\mathbf{x}})^{\mathbf{T}}\right]\mathbf{v} \\ &= \mathbf{v}^{\mathbf{T}}\mathbf{A}\mathbf{v} \quad \text{where } \mathbf{A} = \sum_{\mathbf{x}}(\mathbf{x} - \overline{\mathbf{x}})(\mathbf{x} - \overline{\mathbf{x}})^{\mathbf{T}} \end{aligned}$$

Solution: $\mathbf{v_1}$ is eigenvector of **A** with *largest* eigenvalue
$\mathbf{v_2}$ is eigenvector of **A** with *smallest* eigenvalue

Principal component analysis

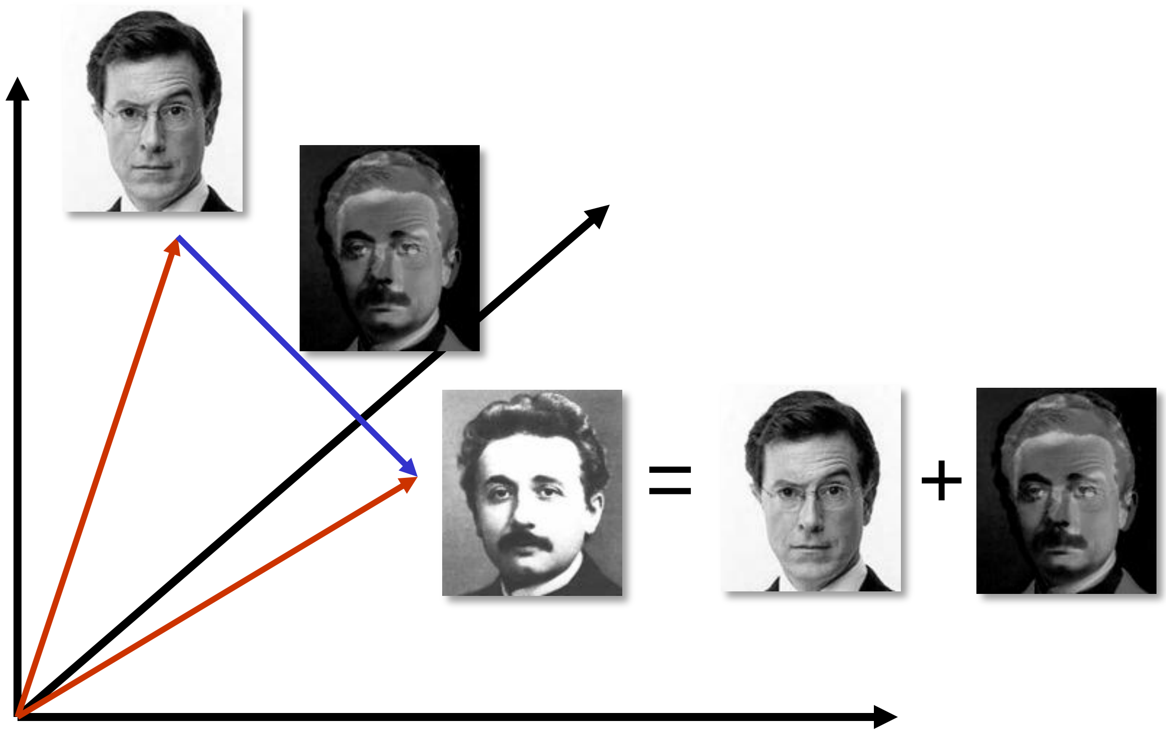
Suppose each data point is N-dimensional

- Same procedure applies:

$$\begin{aligned} var(\mathbf{v}) &= \sum_{\mathbf{x}} \|(\mathbf{x} - \overline{\mathbf{x}})^{\mathbf{T}} \cdot \mathbf{v}\| \\ &= \mathbf{v}^{\mathbf{T}} \mathbf{A} \mathbf{v} \quad \text{where } \mathbf{A} = \sum_{\mathbf{x}} (\mathbf{x} - \overline{\mathbf{x}})(\mathbf{x} - \overline{\mathbf{x}})^{\mathbf{T}} \end{aligned}$$

- The eigenvectors of **A** define a new coordinate system
  - eigenvector with largest eigenvalue captures the most variation among training vectors **x**
  - eigenvector with smallest eigenvalue has least variation
- We can compress the data by only using the top few eigenvectors
  - corresponds to choosing a “linear subspace”
    - » represent points on a line, plane, or “hyper-plane”
  - these eigenvectors are known as the ***principal components***

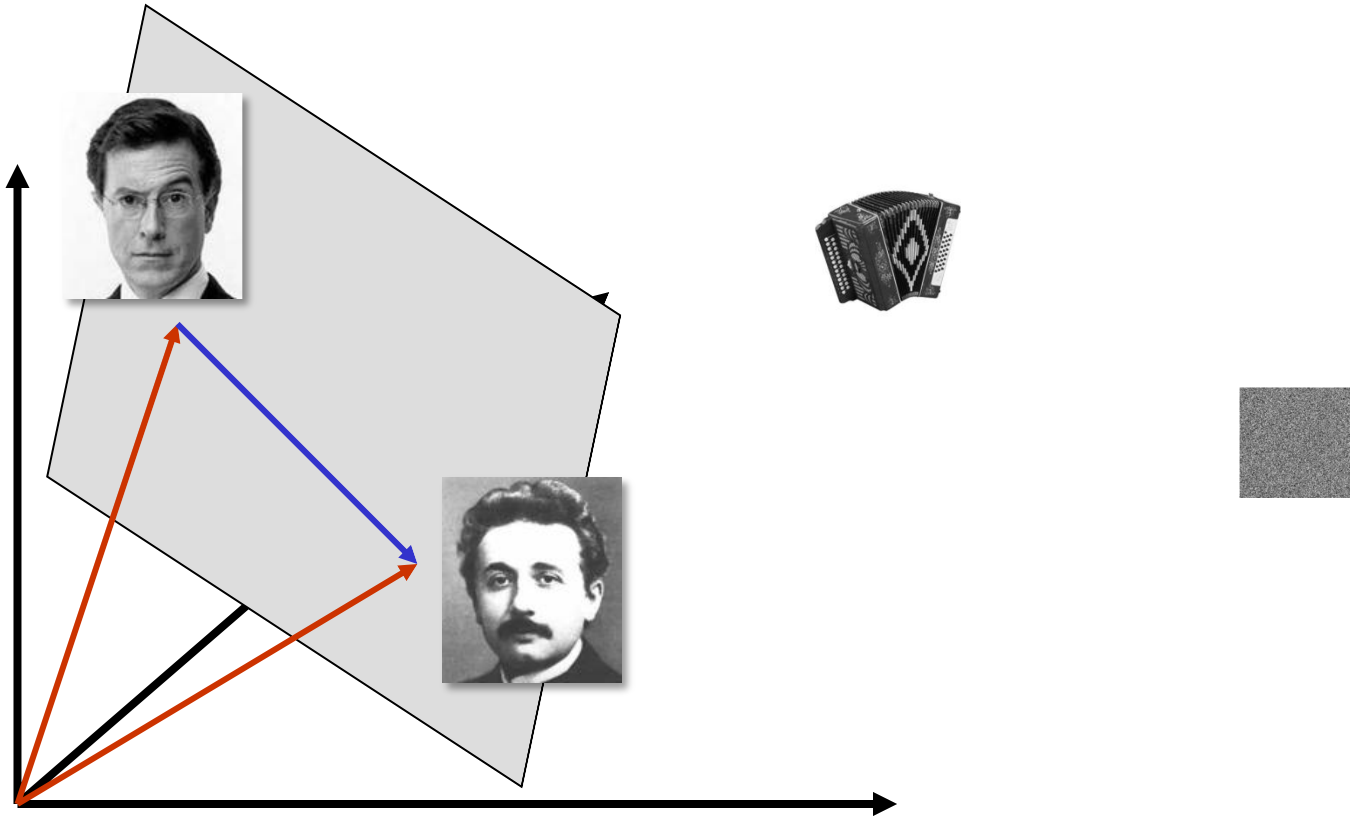
The space of faces



An image is a point in a high dimensional space

- An N x M intensity image is a point in R^{NM}
- We can define vectors in this space as we did in the 2D case

Dimensionality reduction



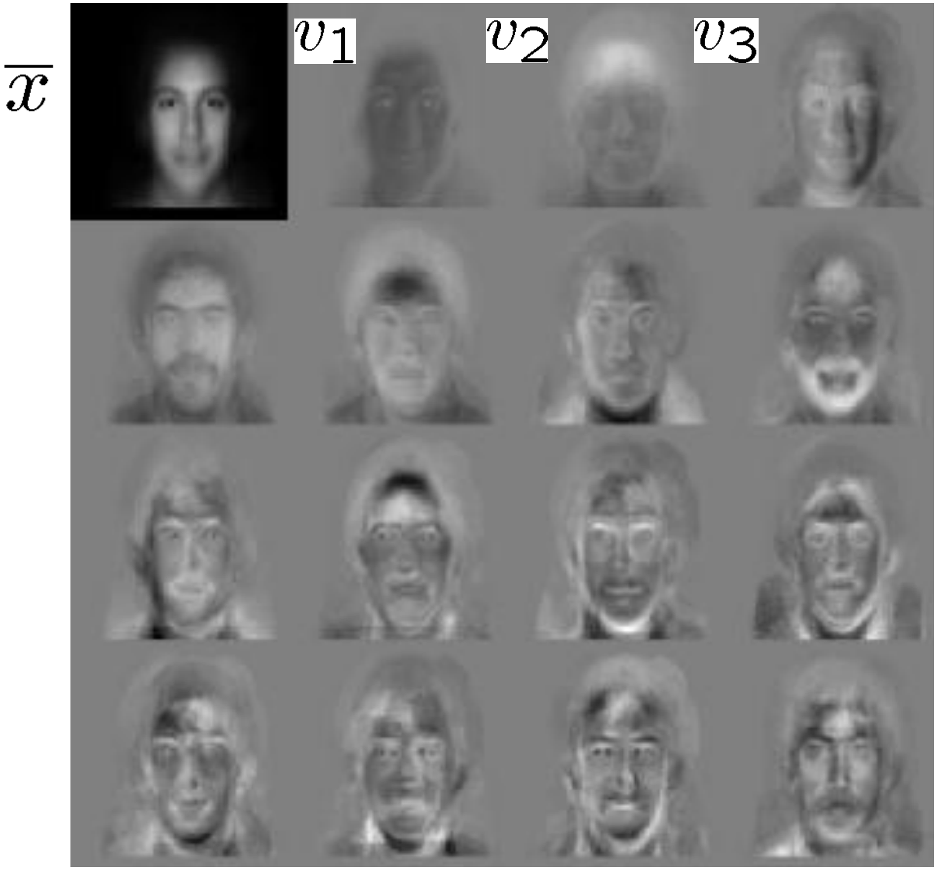
The set of faces is a "subspace" of the set of images

- Suppose it is K dimensional
- We can find the best subspace using PCA
- This is like fitting a "hyper-plane" to the set of faces
  - spanned by vectors $\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_K}$
  - any face $\mathbf{x} \approx \overline{\mathbf{x}} + a_1\mathbf{v_1} + a_2\mathbf{v_2} + \ldots + a_k\mathbf{v_k}$

Eigenfaces

PCA extracts the eigenvectors of **A**

- Gives a set of vectors $\mathbf{v_1}$, $\mathbf{v_2}$, $\mathbf{v_3}$, ...
- Each one of these vectors is a direction in face space
  - what do these look like?



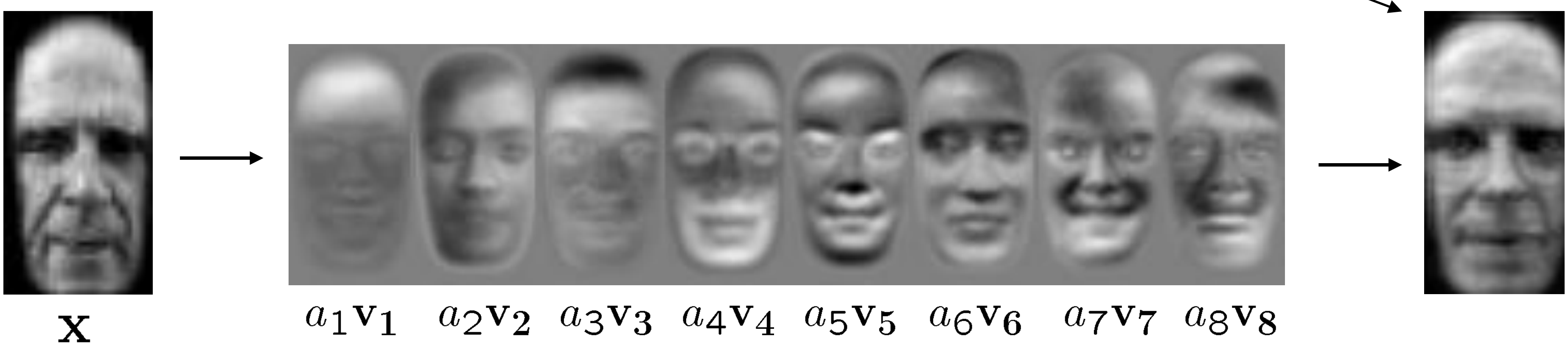
Projecting onto the eigenfaces

The eigenfaces $\mathbf{v_1}, ..., \mathbf{v_K}$ span the space of faces

- A face is converted to eigenface coordinates by

$$\mathbf{x} \rightarrow (\underbrace{(\mathbf{x} - \overline{\mathbf{x}}) \cdot \mathbf{v_1}}_{a_1}, \underbrace{(\mathbf{x} - \overline{\mathbf{x}}) \cdot \mathbf{v_2}}_{a_2}, \ldots, \underbrace{(\mathbf{x} - \overline{\mathbf{x}}) \cdot \mathbf{v_K}}_{a_K})$$

$$\mathbf{x} \approx \overline{\mathbf{x}} + a_1\mathbf{v_1} + a_2\mathbf{v_2} + \ldots + a_K\mathbf{v_K}$$



Detection and recognition with eigenfaces

Algorithm

1. Process the image database (set of images with labels)
   - Run PCA—compute eigenfaces
   - Calculate the K coefficients for each image
2. Given a new image (to be recognized) **x**, calculate K coefficients

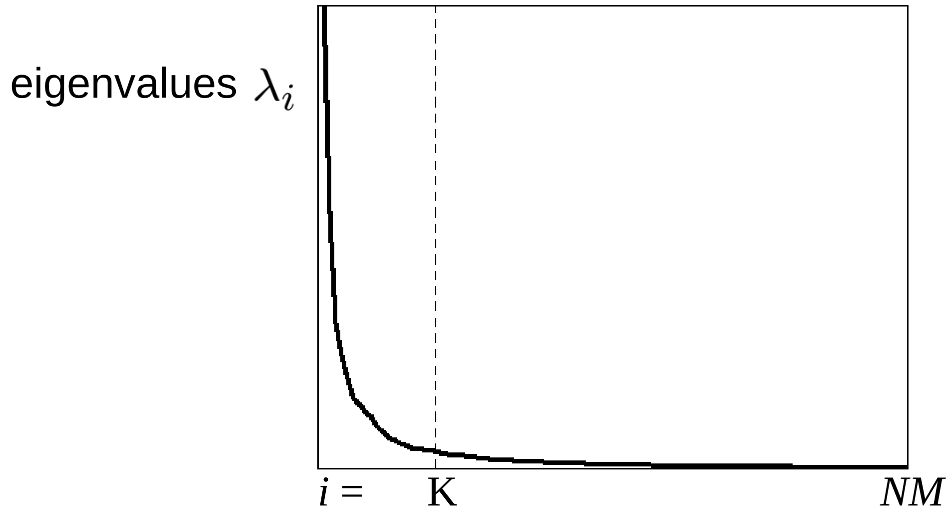
$$\mathbf{x} \rightarrow (a_1, a_2, \ldots, a_K)$$

3. Detect if x is a face

$$\|\mathbf{x} - (\overline{\mathbf{x}} + a_1\mathbf{v_1} + a_2\mathbf{v_2} + \ldots + a_K\mathbf{v_K})\| < \text{threshold}$$

4. If it is a face, who is it?
   - Find closest labeled face in database
     - nearest-neighbor in K-dimensional space

Choosing the dimension K



How many eigenfaces to use?

Look at the decay of the eigenvalues

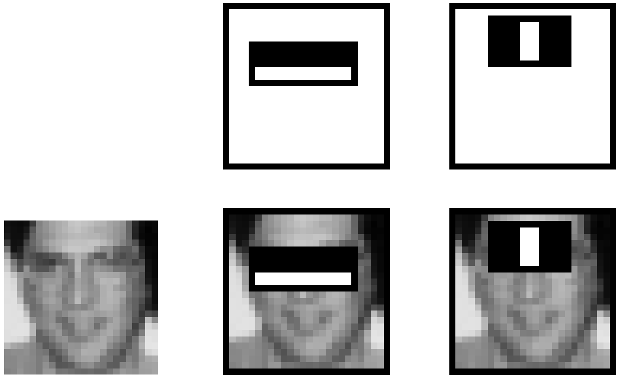
- the eigenvalue tells you the amount of variance “in the direction” of that eigenface
- ignore eigenfaces with low variance

Issues: metrics

What's the best way to compare images?

- need to define appropriate features
- depends on goal of recognition task

exact matching
complex features work well
(SIFT, MOPS, etc.)



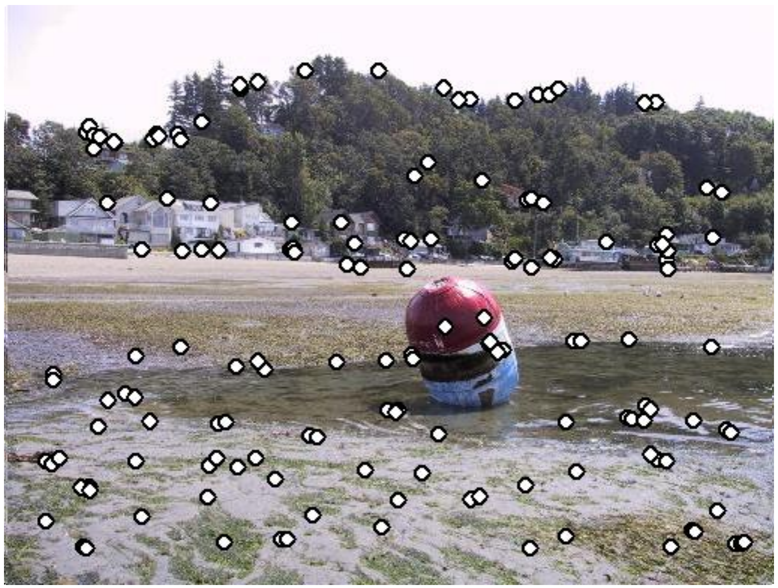
classification/detection
simple features work well
(Viola/Jones, etc.)

Metrics

Lots more feature types that we haven't mentioned

- moments, statistics
  - metrics: Earth mover's distance, ...
- edges, curves
  - metrics: Hausdorff, shape context, ...
- 3D: surfaces, spin images
  - metrics: chamfer (ICP)
- ...

Issues: feature selection



If all you have is one image:
non-maximum suppression, etc.

If you have a training set of images:
AdaBoost, etc.

Issues: data modeling

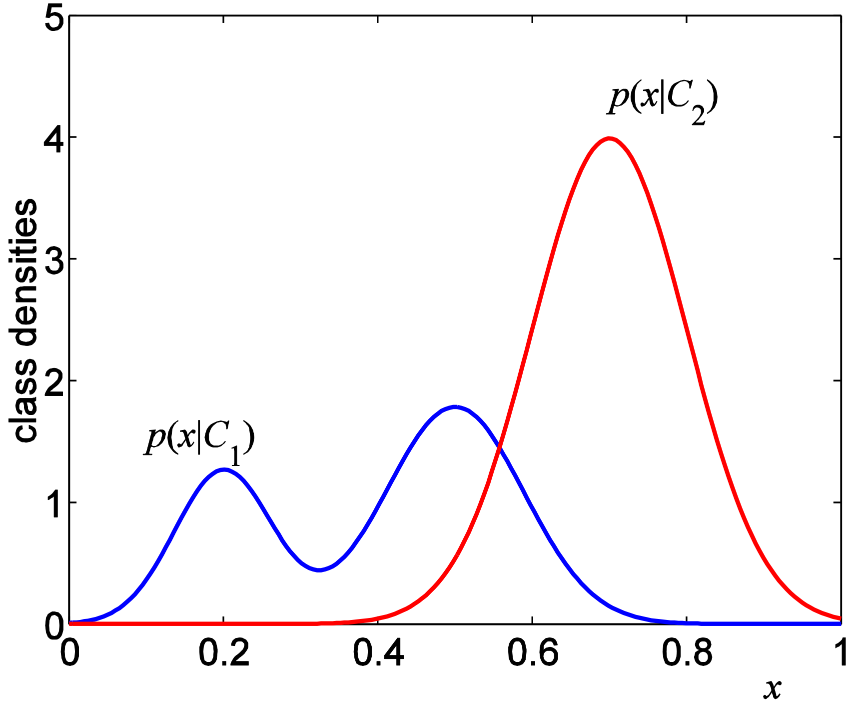
Generative methods

- model the “shape” of each class
  - histograms, PCA, mixtures of Gaussians
  - graphical models (HMM’s, belief networks, etc.)
  - ...

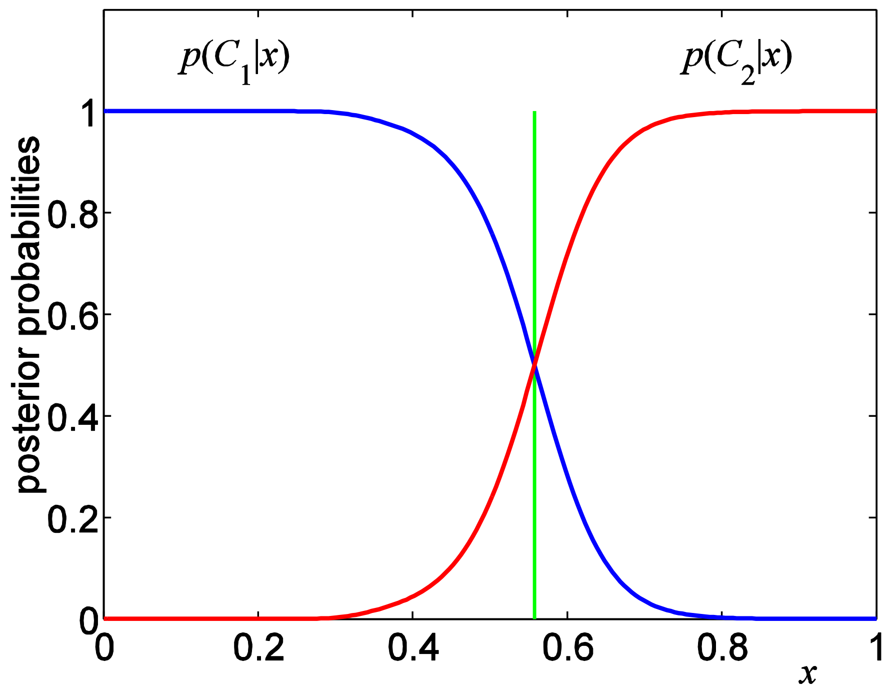
Discriminative methods

- model boundaries between classes
  - perceptrons, neural networks
  - support vector machines (SVM’s)

Generative vs. Discriminative



Generative Approach
model individual classes, priors



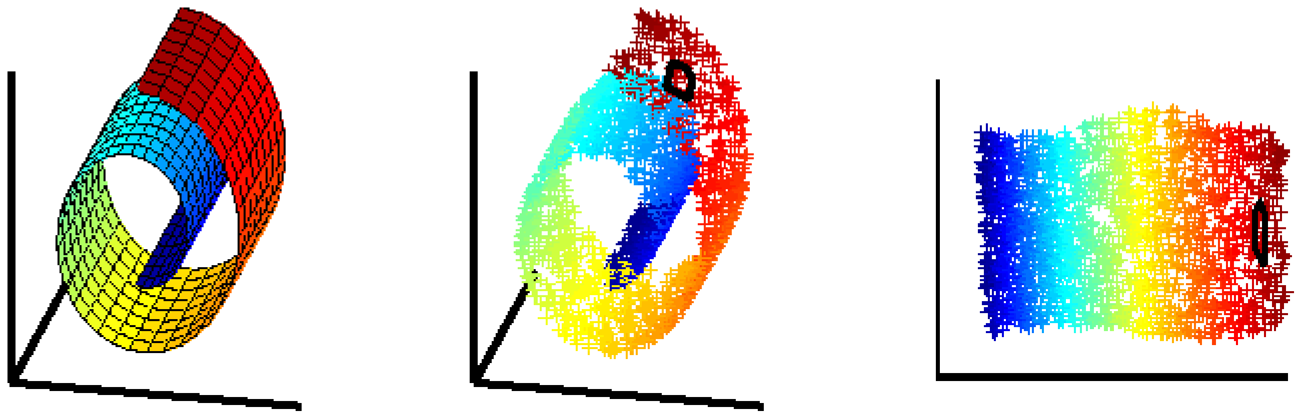
Discriminative Approach
model posterior directly

from Chris Bishop

Issues: dimensionality

What if your space isn't *flat*?

- PCA may not help



Nonlinear methods
LLE, MDS, etc.

Moving forward

- Faces are pretty well-behaved
  - Mostly the same basic shape
  - Lie close to a low-dimensional subspace

- Not all objects are as nice

Different appearance, similar parts