Problem 1: [Convolution]

In this question you will apply a discrete 2D blur filter \( a[i, j] \) to the following 6-by-6 sample image.

![Original Values Blurred](image)

Figure 1: Image before convolution (with pixel values), and after blurring.

The blur filter mask, \( a[i, j] \), is defined as the outer product of the vector \( v = [1/4, 1/2, 1/4] \) with its transpose,

\[
a[i, j] = vv^T = \begin{bmatrix}
1/16 & 1/8 & 1/16 \\
1/8 & 1/4 & 1/8 \\
1/16 & 1/8 & 1/16
\end{bmatrix}.
\]

Note that the values of the filter \( a \) (and \( v \)) sum to one.

In the following two questions, you will evaluate the blurred image manually in two different ways:

(a) **Brute force approach**: Apply the 3-by-3 blur filter to the 6-by-6 image manually. *State the resulting matrix of pixel values using integers*, rounding as needed.

Boundary cases should be handled by ignoring missing/exterior filter locations, and renormalizing the remaining/interior filter values, i.e., divide the interior filter values by their sum. In this way, your filter will always produce a convex combination of pixel values, and preserve image intensity. *State the filters for these boundary cases.*

(b) **Multi-pass algorithm**: Since \( a[i, j] \) is a 2D separable filter, you can accelerate your blur calculation using a two-pass algorithm (as described in class): (1) generate an intermediate image array that results from blurring in the vertical direction, then (2) appropriately blur the intermediate image in the horizontal direction to produce the final result. Handle boundary cases with the 1D filter mask by using renormalization as before. Verify that your resulting image is the same as in part (a). State all final and intermediate results clearly.