Problem 1: [2D Transformations] Consider a 2D transformation that transforms point \( p_1 \rightarrow p_1', p_2 \rightarrow p_2', p_3 \rightarrow p_3' \).

1. Express the affine transformation solution to this general case using a matrix representation.

2. If point \((0,1)\) is transformed to \((3,4)\), \((1,1)\) to \((7,1)\), and \((1,0)\) to \((4,-3)\), where will point \((2,1)\) be transformed to?

Problem 2: [3D Transformations] Rodrigues’ rotation formula gives an efficient method for computing the rotation matrix corresponding to a rotation by an angle \( \theta \in \mathbb{R} \) about a fixed axis specified by the unit vector \( \tilde{\omega} = (\omega_x, \omega_y, \omega_z) \in \mathbb{R}^3 \). The rotation matrix is given by

\[
e^{\tilde{\omega} \theta} = I + \tilde{\omega} \sin \theta + \tilde{\omega}^2 (1 - \cos \theta)
\]

where \( I \) is the \( 3 \times 3 \) identity matrix and \( \tilde{\omega} \) denotes the antisymmetric matrix with entries

\[
\tilde{\omega} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}.
\]

1. State the three familiar canonical rotations, \( R_x(\theta) \), \( R_y(\theta) \) and \( R_z(\theta) \), and verify that Rodrigues’ formula reproduces them.

2. Compute the matrix for a rotation around the axis vector \([1, 2, 3]^T\) for \( \theta = 60^\circ \). Verify that \( \det(R) = 1 \).

Problem 3: [Splines] Clearly state the formula for a cubic Bezier curve, and show that the curve is affine invariant for a general affine transformation.

Problem 4: [Spline Conversion] Show explicitly how to convert from a Catmull-Rom representation for a cubic spline segment with control points \( p_1, p_2, p_3, p_4 \) to a Bezier spline with control points \( q_1, q_2, q_3 \) and \( q_4 \). Provide a geometric illustration of the quantities involved.