What light is

- Light is electromagnetic radiation
  - exists as oscillations of different frequency (or, wavelength)

Measuring light

- Salient property is the spectral power distribution (SPD)
  - the amount of light present at each wavelength
  - units: Watts per nanometer (tells you how much power you’ll find in a narrow range of wavelengths)
  - for color, often use “relative units” when overall intensity is not important

\[ \text{amount of light} = 180 \, d\lambda \] (relative units)
What color is

- Colors are the sensations that arise from light energy of different wavelengths
  - we are sensitive from about 380 to 760 nm—one “octave”
- Color is a phenomenon of human perception; it is not a universal property of light
- Roughly speaking, things appear “colored” when they depend on wavelength and “gray” when they do not.

The problem of color science

- Build a model for human color perception
- That is, map a Physical light description to a Perceptual color sensation

The eye as a measurement device

- We can model the low-level behavior of the eye by thinking of it as a light-measuring machine
  - its optics are much like a camera
  - its detection mechanism is also much like a camera
- Light is measured by the photoreceptors in the retina
  - they respond to visible light
  - different types respond to different wavelengths

A simple light detector

- Produces a scalar value (a number) when photons land on it
  - this value depends strictly on the number of photons detected
  - each photon has a probability of being detected that depends on the wavelength
  - there is no way to tell the difference between signals caused by light of different wavelengths: there is just a number
- This model works for many detectors:
  - based on semiconductors (such as in a digital camera)
  - based on visual photopigments (such as in human eyes)
A simple light detector

Light detection math

- Same math carries over to power distributions
  - spectrum entering the detector has its spectral power distribution (SPD), $s(\lambda)$
  - detector has its spectral sensitivity or spectral response, $r(\lambda)$

\[ X = \int s(\lambda) r(\lambda) \, d\lambda \]

Light detection math

- If we think of $s$ and $r$ as vectors, this operation is a dot product (aka inner product)
  - in fact, the computation is done exactly this way, using sampled representations of the spectra.
  - let $\lambda_i$ be regularly spaced sample points $\Delta \lambda$ apart; then:
    \[ \tilde{s}[i] = s(\lambda_i); \tilde{r}[i] = r(\lambda_i) \]
    \[ \int s(\lambda) r(\lambda) \, d\lambda \approx \sum_i \tilde{s}[i] \tilde{r}[i] \Delta \lambda \]
  - this sum is very clearly a dot product

Cone Responses

- S,M,L cones have broadband spectral sensitivity
- S,M,L neural response is integrated w.r.t. $\lambda$
  - we'll call the response functions $r_S$, $r_M$, $r_L$
- Results in a trichromatic visual system
- S, M, and L are tristimulus values
Cone responses to a spectrum $s$

$$S = \int r_S(\lambda)s(\lambda) \, d\lambda = r_S \cdot s$$
$$M = \int r_M(\lambda)s(\lambda) \, d\lambda = r_M \cdot s$$
$$L = \int r_L(\lambda)s(\lambda) \, d\lambda = r_L \cdot s$$

Colorimetry: an answer to the problem

- Wanted to map a *Physical light description* to a *Perceptual color sensation*
- Basic solution was known and standardized by 1930
  - Though not quite in this form—more on that in a bit

![Physical Perceptual](image)

Basic fact of colorimetry

- Take a spectrum (which is a function)
- Eye produces three numbers
- This throws away a lot of information!
  - Quite possible to have two different spectra that have the same S, M, L tristimulus values
  - Two such spectra are **metamers**

Pseudo-geometric interpretation

- A dot product is a projection
- We are projecting a high dimensional vector (a spectrum) onto three vectors
  - differences that are perpendicular to all 3 vectors are not detectable
- For intuition, we can imagine a 3D analog
  - 3D stands in for high-D vectors
  - 2D stands in for 3D
  - Then vision is just projection onto a plane
Pseudo-geometric interpretation

- The information available to the visual system about a spectrum is three values
  - this amounts to a loss of information analogous to projection on a plane
- Two spectra that produce the same response are metamers

Luminance, mathematically

- Y just has another response curve (like S, M, and L)
  \[ Y = r_Y \cdot s \]
  - \( r_Y \) is really called “\( V_Y \)"
- \( V_Y \) is a linear combination of S, M, and L
  - Has to be, since it’s derived from cone outputs

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Basic colorimetric concepts

- Luminance
  - the overall magnitude of the visual response to a spectrum (independent of its color)
  - corresponds to the everyday concept “brightness”
  - determined by product of SPD with the luminous efficiency function \( V_\lambda \) that describes the eye’s overall ability to detect light at each wavelength
  - e.g. lamps are optimized to improve their luminous efficiency (tungsten vs. fluorescent vs. sodium vapor)

More basic colorimetric concepts

- Chromaticity
  - what’s left after luminance is factored out (the color without regard for overall brightness)
  - scaling a spectrum up or down leaves chromaticity alone
- Dominant wavelength
  - many colors can be matched by white plus a spectral color
  - correlates to everyday concept “hue”
- Purity
  - ratio of pure color to white in matching mixture
  - correlates to everyday concept “colorfulness” or “saturation”
Color reproduction

- Have a spectrum $s$; want to match on RGB monitor
  - “match” means it looks the same
  - any spectrum that projects to the same point in the visual color space is a good reproduction
- Must find a spectrum that the monitor can produce that is a metamer of $s$

Additive Color

CRT display primaries

- Curves determined by phosphor emission properties

LCD display primaries

- Curves determined by (fluorescent) backlight and filters
Combining Monitor Phosphors with Spatial Integration

Color reproduction

- Say we have a spectrum \( s \) we want to match on an RGB monitor
  - “match” means it looks the same
  - any spectrum that projects to the same point in the visual color space is a good reproduction
- So, we want to find a spectrum that the monitor can produce that matches \( s \)
  - that is, we want to display a metamer of \( s \) on the screen

Color reproduction as linear algebra

- The projection onto the three response functions can be written in matrix form:

\[
\begin{bmatrix}
S \\ M \\ L 
\end{bmatrix} =
\begin{bmatrix}
r_S \\ -r_M \\ -r_L
\end{bmatrix}
\begin{bmatrix}
1 \\ 1 \\ 1
\end{bmatrix}
\]

or,

\[V = M_{SML}s.\]
Color reproduction as linear algebra

• The spectrum that is produced by the monitor for the color signals R, G, and B is:
  \[ s_a(\lambda) = R s_r(\lambda) + G s_g(\lambda) + B s_b(\lambda). \]

• Again the discrete form can be written as a matrix:

\[
\begin{bmatrix}
  s_a \\
  s_R \\
  s_G \\
  s_B
\end{bmatrix} =
\begin{bmatrix}
  R \\
  G \\
  B
\end{bmatrix} \begin{bmatrix}
  s_r \\
  s_g \\
  s_b
\end{bmatrix}
\]

or,

\[ s_a = M_{RGB} C. \]

Color reproduction as linear algebra

• Goal of reproduction: visual response to \( s \) and \( s_a \) is the same:

\[ M_{SML} \tilde{s} = M_{SML} \tilde{s}_a. \]

• Substituting in the expression for \( s_a \),

\[ M_{SML} \tilde{s} = M_{SML} M_{RGB} C \]

\[ C = (M_{SML} M_{RGB})^{-1} M_{SML} \tilde{s} \]

*color matching matrix for RGB*
**Subtractive color**

- Produce desired spectrum by *subtracting* from white light (usually via absorption by pigments)
- Photographic media (slides, prints) work this way
- Leads to C, M, Y as primaries
- Approximately, 1–R, 1–G, 1–B

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**Color spaces**

- Need three numbers to specify a color
  - but what three numbers?
  - a *color space* is an answer to this question
- Common example: monitor RGB
  - define colors by what R, G, B signals will produce them on your monitor
    (in math, $s = RR + GG + BB$ for some spectra R, G, B)
  - device dependent (depends on gamma, phosphors, gains, …)
    - therefore if I choose RGB by looking at my monitor and send it to you, you may not see the same color
    - also leaves out some colors (limited *gamut*), e.g. vivid yellow

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**Standard color spaces**

- Standardized RGB (sRGB)
  - makes a particular monitor RGB standard
  - other color devices simulate that monitor by calibration
  - sRGB is usable as an interchange space; widely adopted today
  - gamut is still limited
A universal color space: XYZ

- Standardized by CIE (Commission Internationale de l’Eclairage, the standards organization for color science)
- Based on three “imaginary” primaries X, Y, and Z
  (in math, s = XX + YY + ZZ)
  - imaginary = only realizable by spectra that are negative at some wavelengths
  - key properties
    - any stimulus can be matched with positive X, Y, and Z
    - separates out luminance: X, Z have zero luminance, so Y tells you the luminance by itself

Separating luminance, chromaticity

- Luminance: Y
- Chromaticity: x, y, z, defined as
  \[
  x = \frac{X}{X + Y + Z} \\
  y = \frac{Y}{X + Y + Z} \\
  z = \frac{Z}{X + Y + Z}
  \]
  - since x + y + z = 1, we only need to record two of the three
    - usually choose x and y, leading to (x, y, Y) coords
Color Gamuts

Monitors/printers can’t produce all visible colors
Reproduction is limited to a particular domain
For additive color (e.g. monitor) gamut is the triangle defined by the chromaticities of the three primaries.

Perceptually organized color spaces

• Artists often refer to colors as tints, shades, and tones of pure pigments
  – tint: mixture with white
  – shade: mixture with black
  – tones: mixture with black and white
  – gray: no color at all (aka. neutral)
• This seems intuitive
  – tints and shades are inherently related to the pure color
    • “same” color but lighter, darker, paler, etc.

Perceptual dimensions of color

• Hue
  – the “kind” of color, regardless of attributes
  – colorimetric correlate: dominant wavelength
  – artist’s correlate: the chosen pigment color
• Saturation
  – the “colorfulness”
  – colorimetric correlate: purity
  – artist’s correlate: fraction of paint from the colored tube
• Lightness (or value)
  – the overall amount of light
  – colorimetric correlate: luminance
  – artist’s correlate: tints are lighter, shades are darker

Perceptual dimensions: chromaticity

• In x, y, Y (or another luminance/chromaticity space), Y corresponds to lightness
• hue and saturation are then like polar coordinates for chromaticity (starting at white, which way did you go and how far?)
Perceptual dimensions of color

- There's good evidence ("opponent color theory") for a neurological basis for these dimensions
  - the brain seems to encode color early on using three axes:
    white — black, red — green, yellow — blue
  - the white—black axis is lightness; the others determine hue and saturation
  - one piece of evidence: you can have a light green, a dark green, a yellow-green, or a blue-green, but you can't have a reddish green (just doesn't make sense)
    - thus red is the opponent to green
  - another piece of evidence: afterimages (next slide)
Perceptual organization for RGB: HSV

- Uses hue (an angle, 0 to 360), saturation (0 to 1), and value (0 to 1) as the three coordinates for a color
  - the brightest available
    RGB colors are those with one of R,G,B equal to 1 (top surface)
  - each horizontal slice is the surface of a sub-cube of the RGB cube

Perceptually uniform spaces

- Two major spaces standardized by CIE
  - designed so that equal differences in coordinates produce equally visible differences in color
  - LUV: earlier, simpler space; $L^*, u^*, v^*$
  - LAB: more complex but more uniform: $L^*, a^*, b^*$
  - both separate luminance from chromaticity
  - including a gamma-like nonlinear component is important