The graphics pipeline

- The standard approach to object-order graphics
- Many versions exist
  - software, e.g. Pixar's REYES architecture
    - many options for quality and flexibility
  - hardware, e.g. graphics cards in PCs
    - amazing performance: millions of triangles per frame
- We'll focus on an abstract version of hardware pipeline
- “Pipeline” because of the many stages
  - very parallelizable
  - leads to remarkable performance of graphics cards (many times the flops of the CPU at ~1/5 the clock speed)

Primitives

- Points
- Line segments
  - and chains of connected line segments
- Triangles
- And that's all!
  - Curves? Approximate them with chains of line segments
  - Polygons? Break them up into triangles
  - Curved regions? Approximate them with triangles
- Trend has been toward minimal primitives
  - simple, uniform, repetitive: good for parallelism
Rasterization

- First job: enumerate the pixels covered by a primitive
  - Simple, aliased definition: pixels whose centers fall inside
- Second job: interpolate values across the primitive
  - e.g., colors computed at vertices
  - e.g., normals at vertices
  - Will see applications later on

Rasterizing lines

- Define line as a rectangle
- Specify by two endpoints
- Ideal image: black inside, white outside

Point sampling

- Approximate rectangle by drawing all pixels whose centers fall within the line
- Problem: sometimes turns on adjacent pixels

Point sampling in action
Bresenham lines (midpoint alg.)

- Point sampling unit width rectangle leads to uneven line width
- Define line width parallel to pixel grid
- That is, turn on the single nearest pixel in each column
- Note that 45° lines are now thinner

Algorithms for drawing lines

- line equation: \( y = b + m \times x \)
- Simple algorithm: evaluate line equation per column
- W.l.o.g. assume:
  - \( x_0 < x_1 \)
  - \( 0 \leq m \leq 1 \)

\[
\begin{align*}
\text{for } x &= \text{ceil}(x_0) \text{ to floor}(x_1) \\
y &= b + m \times x \\
\text{output}(x, \text{round}(y))
\end{align*}
\]

Optimizing line drawing

- Multiplying and rounding is slow
- At each pixel the only options are E and NE
- \( d = m(x + 1) + b - y \)
- \( d > 0.5 \) decides between E and NE
Optimizing line drawing (DDA)

- \( d = m(x + 1) + b - y \)
- Only need to update \( d \) for integer steps in \( x \) and \( y \)
- Do that with addition
- Known as “DDA“ (digital differential analyzer)

Midpoint line algorithm

- \( x = \text{ceil}(x_0) \)
- \( y = \text{round}(m\times x + b) \)
- \( d = m(x + 1) + b - y \)
- while \( x < \text{floor}(x_1) \)
  - if \( d > 0.5 \)
    - \( y += 1 \)
    - \( d -= 1 \)
  - \( x += 1 \)
  - \( d += m \)
  - \( \text{output}(x, y) \)

Linear interpolation

- We often attach attributes to vertices
  - e.g. computed diffuse color of a hair being drawn using lines
  - want color to vary smoothly along a chain of line segments
- Recall basic definition
  - 1D: \( f(x) = (1 - \alpha) y_0 + \alpha y_1 \)
  - where \( \alpha = (x - x_0) / (x_1 - x_0) \)
- In the 2D case of a line segment, alpha is just the fraction of the distance from \((x_0, y_0)\) to \((x_1, y_1)\)

Linear interpolation

- Pixels are not exactly on the line
- Define 2D function by projection on line
  - this is linear in 2D
  - therefore can use DDA to interpolate
Alternate interpretation

- We are updating \(d\) and \(\alpha\) as we step from pixel to pixel
  - \(d\) tells us how far from the line we are
  - \(\alpha\) tells us how far along the line we are

- So \(d\) and \(\alpha\) are coordinates in a coordinate system oriented to the line

Alternate interpretation

- View loop as visiting all pixels the line passes through
  - Interpolate \(d\) and \(\alpha\) for each pixel
  - Only output frag. if pixel is in band

- This makes linear interpolation the primary operation

Pixel-walk line rasterization

\[
x = \text{ceil}(x_0) \\
y = \text{round}(m \times x + b) \\
d = m \times x + b - y \\
\text{while } x < \text{floor}(x_1) \\
\text{if } d > 0.5 \\
\quad y += 1; \ d -= 1; \\
\text{else} \\
\quad x += 1; \ d += m; \\
\quad \text{if } -0.5 < d \leq 0.5 \\
\quad \text{output}(x, y)
\]

Rasterizing triangles

- The most common case in most applications
  - with good antialiasing can be the only case
  - some systems render a line as two skinny triangles
- Triangle represented by three vertices
- Simple way to think of algorithm follows the pixel-walk interpretation of line rasterization
  - walk from pixel to pixel over (at least) the polygon’s area
  - evaluate linear functions as you go
  - use those functions to decide which pixels are inside
**Rasterizing triangles**

- **Input:**
  - three 2D points (the triangle’s vertices in pixel space)
    - \((x_0, y_0); (x_1, y_1); (x_2, y_2)\)
  - parameter values at each vertex
    - \(q_{00}, \ldots, q_{0n}; q_{10}, \ldots, q_{1n}; q_{20}, \ldots, q_{2n}\)
- **Output:** a list of fragments, each with
  - the integer pixel coordinates \((x, y)\)
  - interpolated parameter values \(q_0, \ldots, q_n\)

**Incremental linear evaluation**

- A linear (affine, really) function on the plane is:
  \[ q(x, y) = c_x x + c_y y + c_k \]
- Linear functions are efficient to evaluate on a grid:
  \[
  q(x+1, y) = c_x(x+1) + c_y y + c_k = q(x, y) + c_x \\
  q(x, y+1) = c_x x + c_y(y+1) + c_k = q(x, y) + c_y
  \]

```
linEval(xl, xh, yl, yh, cx, cy, ck) {
    // setup
    qRow = cx*xl + cy*yl + ck;
    // traversal
    for y = yl to yh {
        qPix = qRow;
        for x = xl to xh {
            output(x, y, qPix);
            qPix += cx;
        }
        qRow += cy;
    }
}
```
Rasterizing triangles

- Summary
  1. Evaluation of linear functions on pixel grid
  2. Functions defined by parameter values at vertices
  3. Using extra parameters to determine fragment set

Defining parameter functions

- To interpolate parameters across a triangle we need to find the \( c_x \), \( c_y \), and \( c_k \) that define the (unique) linear function that matches the given values at all 3 vertices
  - this is 3 constraints on 3 unknown coefficients:
    \[
    c_x x_0 + c_y y_0 + c_k = q_0 \\
    c_x x_1 + c_y y_1 + c_k = q_1 \\
    c_x x_2 + c_y y_2 + c_k = q_2
    \]
    (each states that the function agrees with the given value at one vertex)
  - leading to a 3x3 matrix equation for the coefficients:
    \[
    \begin{bmatrix}
    x_0 & y_0 & 1 \\
    x_1 & y_1 & 1 \\
    x_2 & y_2 & 1
    \end{bmatrix}
    \begin{bmatrix}
    c_x \\
    c_y \\
    c_k
    \end{bmatrix}
    =
    \begin{bmatrix}
    q_0 \\
    q_1 \\
    q_2
    \end{bmatrix}
    \]
    (singular iff triangle is degenerate)

Defining parameter functions

- More efficient version: shift origin to \((x_0, y_0)\)
  - now this is a 2x2 linear system (since \(q_0\) falls out):
  - solve using Cramer's rule (see Shirley):
    \[
    c_x = \frac{(q_1-y_0)(x_2-x_0) - (q_2-y_0)(x_1-x_0)}{det} \\
    c_y = \frac{(q_2-q_0)(x_1-x_0) - (q_1-q_0)(x_2-x_0)}{det}
    \]
    \[
    \begin{bmatrix}
    x_1 - x_0 & y_1 - y_0 \\
    x_2 - x_0 & y_2 - y_0
    \end{bmatrix}
    \begin{bmatrix}
    c_x \\
    c_y
    \end{bmatrix}
    =
    \begin{bmatrix}
    q_1 - q_0 \\
    q_2 - q_0
    \end{bmatrix}
    \]

```c
linInterp(xl, xh, yl, yh, x0, y0, q0, x1, y1, q1, x2, y2, q2) {
  // setup
  det = (x1-x0)*(y2-y0) - (x2-x0)*(y1-y0);
  cx = ((q1-q0)*(y2-y0) - (q2-q0)*(y1-y0)) / det;
  cy = ((q2-q0)*(x1-x0) - (q1-q0)*(x2-x0)) / det;
  qxyRow = cx*(xl-x0) + cy*(yl-y0) + q0;
  // traversal (same as before)
  for y = yl to yh {
    qPix += qxyRow;
    for x = xl to xh {
      output(x, y, qPix);
      qPix += cx;
    }
    qxyRow += cy;
  }
}
```

Defining parameter functions

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  - this is 3 constraints on 3 unknown coefficients:
    \[
    c_x x_0 + c_y y_0 + c_k = q_0 \\
    c_x x_1 + c_y y_1 + c_k = q_1 \\
    c_x x_2 + c_y y_2 + c_k = q_2
    \]
    (each states that the function agrees with the given value at one vertex)
  - leading to a 3x3 matrix equation for the coefficients:
    \[
    \begin{bmatrix}
    x_0 & y_0 & 1 \\
    x_1 & y_1 & 1 \\
    x_2 & y_2 & 1
    \end{bmatrix}
    \begin{bmatrix}
    c_x \\
    c_y \\
    c_k
    \end{bmatrix}
    =
    \begin{bmatrix}
    q_0 \\
    q_1 \\
    q_2
    \end{bmatrix}
    \]
    (singular iff triangle is degenerate)
Interpolating several parameters

```c
linInterp(xl, xh, yl, yh, n, x0, y0, q0[]);
x1, y1, q1[], x2, y2, q2[])

// setup
for k = 0 to n-1
// compute cx[k], cy[k], qRow[k]
// from q0[k], q1[k], q2[k]
// traversal
for y = yl to yh {
    for k = 1 to n, qPix[k] = qRow[k];
    for x = xl to xh {
        output(x, y, qPix);
        for k = 1 to n, qPix[k] += cx[k];
    }
    for k = 1 to n, qRow[k] += cy[k];
}
```

Rasterizing triangles

- Summary
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Clipping to the triangle

- Interpolate three **barycentric coordinates** across the plane
  - each barycentric coord is 1 at one vert. and 0 at the other two
- Output fragments only when all three are > 0.

Barycentric coordinates

- A coordinate system for triangles
  - algebraic viewpoint:
    \[ p = \alpha a + \beta b + \gamma c \]
    \[ \alpha + \beta + \gamma = 1 \]
  - geometric viewpoint (areas):
    - Triangle interior test:
      \[ \alpha > 0; \beta > 0; \gamma > 0 \]
Barycentric coordinates

- A coordinate system for triangles
  - geometric viewpoint: distances
  - linear viewpoint: basis of edges
    \[
    \alpha = 1 - \beta - \gamma \quad \text{and} \quad p = a + \beta(b - a) + \gamma(c - a)
    \]
    
- in this view, the triangle interior test is just
  \[
  \beta > 0; \quad \gamma > 0; \quad \beta + \gamma < 1
  \]

Edge equations

- In plane, triangle is the intersection of 3 half spaces
  \[
  (x - a) \cdot (b - a) > 0, \quad (x - b) \cdot (c - b) > 0, \quad (x - c) \cdot (a - c) > 0
  \]

Walking edge equations

- We need to update values of the three edge equations with single-pixel steps in x and y
- Edge equation already in form of dot product
- Components of vector are the increments
**Pixel-walk (Pineda) rasterization**

- Conservatively visit a superset of the pixels you want
- Interpolate linear functions
- Use those functions to determine when to emit a fragment

**Rasterizing triangles**

- Exercise caution with rounding and arbitrary decisions
  - need to visit these pixels once
  - but it’s important not to visit them twice!