CS 465 Homework 6

out: Friday 13 October 2006
due: Friday 20 October 2006

Problem 1: 3D parametric surfaces

Consider the Bézier Curve defined by the following control points:

$$\mathbf{p}_0 = (0, 0, 0)$$
$$\mathbf{p}_1 = (1, 0, 0)$$
$$\mathbf{p}_2 = (1, 1, 0)$$
$$\mathbf{p}_3 = (0, 2, 0)$$

A 3D surface of revolution can be formed by revolving this curve $360°$ about the y axis. This surface can be expressed as

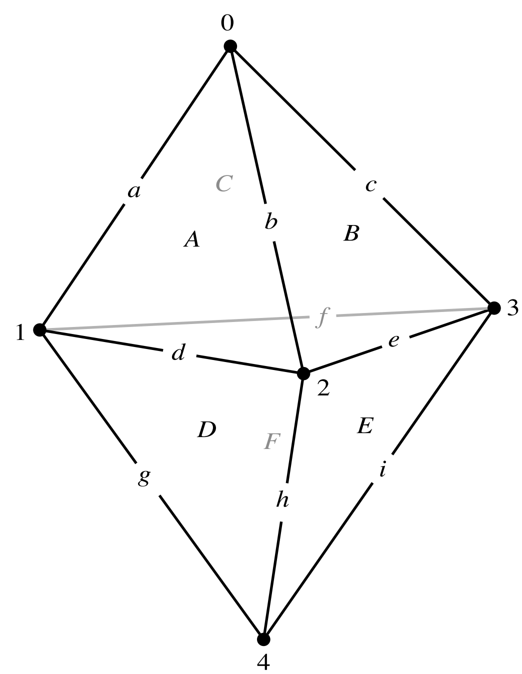
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{f}(u, v)$$

where $v \in [0, 1]$ parametrizes the position along the spline segment (just as t does), and $u \in [0, 1]$ parametrizes the counter-clockwise rotation around the positive y axis. With the conventions discussed in class, this parametrization defines the inside and outside of the surface in a way that is consistent with our intuition for closed surfaces.

1. Give equations for the three components of $\mathbf{f}(u, v)$.
2. Give the surface normal vector at $(u, v) = (0.5, 0.5)$, using the partial derivatives $\partial\mathbf{f}/\partial u$ and $\partial\mathbf{f}/\partial v$.
3. Is the normal to the surface well-defined when $(u, v) = (0, 0)$? Can it be computed using $\partial\mathbf{f}/\partial u$ and $\partial\mathbf{f}/\partial v$?
4. Is the normal to the surface well-defined when $(u, v) = (1, 1)$? Can it be computed using $\partial\mathbf{f}/\partial u$ and $\partial\mathbf{f}/\partial v$?

Problem 2: Mesh Data Structures

A trigonal dipyramidal mesh is illustrated below. It consists of five vertices (labeled 0–4), six triangular faces (labeled A–F), and nine edges (labeled a–i).



Assuming the vertex positions are already stored in a table by vertex number, give representations of this mesh in the following forms:

1. An indexed triangle mesh.
2. A set of triangle strips (use as few strips as possible).
3. A winged-edge structure (use a form similar to Figure 13.2 in Shirley).