CS 465 Homework 5

out: Friday 6 October 2006
due: Friday 13 October 2006

Problem 1: 3D Transformations

Any rotation in 3D can be expressed as a rotation around an axis that passes through the origin. This *axis-angle* form can be specified by a unit vector $\hat{\mathbf{v}}$ around which a counter-clockwise rotation by θ is performed.

Let $\hat{\mathbf{v}} = [4\ 2\ 1]^T/\sqrt{21}$ and $\theta = 45°$.

Your goal is to derive the transformation matrix that corresponds to the axis-angle rotation defined by $\hat{\mathbf{v}}$ and θ.

1. Express the axis-angle rotation matrix as the product of 5 coordinate-axis rotations. *Hint: The first two rotations should align* $\hat{\mathbf{v}}$ *with a coordinate axis.*

2. Construct a rotation matrix that maps the vector $\mathbf{e}_3 = [0\ \ 0\ \ 1]^T$ to the vector $\hat{\mathbf{v}}$ by constructing its columns as an orthonormal basis. Use this matrix to derive the matrix for the axis-angle rotation specified by the $\hat{\mathbf{v}}$ and θ above.

You might want to check your answer using a computer by ensuring that it is orthonormal and does not move the vector $\hat{\mathbf{v}}$.

Problem 2: Cubic Curves

Consider the following set of 4 control points

$$\mathbf{p}_0 = (-1, 0)$$
$$\mathbf{p}_1 = (1, 0)$$
$$\mathbf{p}_2 = (0, 1)$$
$$\mathbf{p}_3 = (0, -1)$$

You will use these control points to construct two different cubic curves:

1. B-Spline
2. Bézier Curve

For each type of curve, please do the following:

(a) Give the polynomials $x(t)$ and $y(t)$ that define the curve on the interval $t \in [0, 1]$. For the B-spline, we are talking about the one segment that is controlled by these four control points; you don't need to worry about what happens outside that segment.

(b) Give the (x, y) positions of the endpoints of the curve.

(c) Give all values of $(t, x(t))$ where $\delta x/\delta t = 0$, on the interval $t \in [0, 1]$. Do the same for $y(t)$.

(d) Carefully sketch each polynomial, including all of the positions and local extrema found above, on the interval $t \in [0, 1]$. Labels will be helpful.

(e) Sketch the resulting curve $f(t) = [x(t)\ y(t)]^T$. Again, be sure to be consistent with the polynomials in (a) and accurately include all of the features found in (b) and (c).