Introduction to Color Science: Additive Color for Computer Graphics

CS 465, Prof. Steve Marschner
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Michael J. Murdoch
Eastman Kodak Company
Stare at the Red Dot.
Color

- Color is a perception resulting from a combination of physical stimulation and psychological interpretation.
- Color topics are myriad and interrelated.
Additive RGB Intensity

- Primaries: Red , Green , and Blue

- A monitor is an additive system, meaning colors may be synthesized using linear combinations of the primaries.

- Example: $0.8 \times \text{Red} = \text{Red}$
  
  $0.6 \times \text{Green} = \text{Green}$

  $0.2 \times \text{Blue} = \text{Blue}$

  $\text{Red} + \text{Green} + \text{Blue}$

- Perhaps you recall that Code Values are nonlinear? Using an sRGB monitor, linear intensity values (0.8, 0.6, 0.2) correspond to 8-bit nonlinear code values (231, 203, 124).
Additive Color Using Component Vectors

- In a 3-D (R,G,B) intensity space
  \( \mathbf{R} \) unit vector is \((1,0,0)\)
  \( \mathbf{G} \) unit vector is \((0,1,0)\)
  \( \mathbf{B} \) unit vector is \((0,0,1)\)

- \(0.8 \mathbf{R} + 0.6 \mathbf{G} + 0.2 \mathbf{B} = (0.8, 0.6, 0.2)\)

- Black is \((0, 0, 0)\)
- White is \((1, 1, 1)\)
Chromaticity Diagram

- Chromaticity: how much of each primary, relative to the others.
  \[ r = \frac{R}{R+G+B} \]
  \[ g = \frac{G}{R+G+B} \]
  \[ b = 1 - r - g \]

- Chromaticity is inherently 2-D, thus it has less information than RGB.

- Triangle connecting R, G, and B points is the Gamut Boundary, indicating the range of colors that may be synthesized.

- Additive colors found at the center of mass of primaries.

- White (R=G=B=1) means \((0.333, 0.333)\)
Additive Spectral Power

• Light may be described in terms of Spectral Power Distribution.
• Relative (or absolute) amount of power (i.e., photons, Watts/sr/m²/nm) in each of a number of wavelength bands
• Discrete spectra may be thought of has unit vectors.
• Example: CRT monitor R, G, and B emission

\[ 0.8 + 0.6 + 0.2 = \]
Human Eye Response

- Eyes have light-sensitive cells

- **Rods** (scotopic vision) work in low light levels, and see one band of the spectrum. *(plot is approximate)*

- Colors are not seen in low light.

- **Cones** (photopic vision) work in high light levels.

- There are three types of cones ($\rho$, $\gamma$, $\beta$); each sees a different band of the spectrum.

- Colors are discerned by their relative power in each band.
1931 Color Matching Experiment

- Observers looked at monochromatic colors.
- They were asked to dial in additive mixtures of primary colors that appeared to match each monochromatic color. **Metamers!**

Sometimes, the observer wanted a dial to go below zero! In this case, the primary was moved to the other side, like adding “negative light” to the monochrome color.
Color Matching Functions

- The plot shows the results of the 1931 color matching experiment: the relative amounts of R, G, and B to match each monochrome spectral color.

- The set of monochrome spectral colors (aka spectrum locus) is shown on an r,g chromaticity diagram.

- Note how many colors have $r < 0$
CIE (Commission Internationale de l’Eclairage) 1931 Color Matching Functions

- 1931 Standard Observer:
  The experimental color matching functions were transformed to this set of all-positive curves, called $\bar{x}$, $\bar{y}$, $\bar{z}$.

- $\bar{y}$ used to compute Luminance, $Y$.
- The “primaries,” called $X$, $Y$, $Z$, corresponding to these curves are imaginary! Meaning they can’t physically be made.

- The CIE 1931 xy Chromaticity Diagram

- Note that the spectrum locus fits inside the XYZ triangle.
CIE Colorimetry: $\bar{x} \ \bar{y} \ \bar{z}$, $XYZ$, $xyz$

- XYZ tristimulus values are computed from a spectral power distribution $\Phi(\lambda)$ and the three color matching functions, $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$.

\[
X = k \sum \Phi(\lambda) \bar{x}(\lambda) \Delta \lambda \\
Y = k \sum \Phi(\lambda) \bar{y}(\lambda) \Delta \lambda \\
Z = k \sum \Phi(\lambda) \bar{z}(\lambda) \Delta \lambda
\]

- Two colors with same XYZ are Metamers.
- $k$ often chosen so white $Y = 100$.
- $xyz$ chromaticity values are computed from XYZ tristimulus values.

\[
\begin{align*}
x &= \frac{X}{X+Y+Z} \\
y &= \frac{Y}{X+Y+Z} \\
z &= 1 - x - y
\end{align*}
\]

- $xyY$ often used, because $z$ is redundant.

Example: $Y$, using $\bar{y}$

Y is area under this curve

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Balancing RGB to Synthesize White

- White is a sum of 3 primaries.
- Example, sRGB, a standard set of RGB primaries

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>41</td>
<td>21</td>
</tr>
<tr>
<td>G</td>
<td>36</td>
<td>72</td>
</tr>
<tr>
<td>B</td>
<td>18</td>
<td>7</td>
</tr>
<tr>
<td>Sum</td>
<td>95</td>
<td>100</td>
</tr>
</tbody>
</table>

- White is a perceptual label.
  - Must be “achromatic”
  - Must be “bright”
  - Depends on surroundings

Luminance Contribution: 72%

RGB Power Spectra Scaled to Add to D65 White

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What is White?
Primary Matrix (RGB to XYZ)

- Need to mathematically predict XYZ tristimulus values resulting from a set of RGB values
- Enter the Primary Matrix, aka Pmat, aka Phosphor Matrix
- Example for sRGB primaries & D65 white

\[
\begin{bmatrix}
X \\
Y \\
Z \\
\end{bmatrix} = \begin{bmatrix}
41.2 & 35.8 & 18.0 \\
21.3 & 71.5 & 7.22 \\
1.93 & 11.9 & 95.0 \\
\end{bmatrix} \begin{bmatrix}
R \\
G \\
B \\
\end{bmatrix}
\]

Note the relationship

<table>
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<tr>
<th></th>
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<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>41</td>
<td>21</td>
<td>2</td>
</tr>
<tr>
<td>G</td>
<td>36</td>
<td>72</td>
<td>12</td>
</tr>
<tr>
<td>B</td>
<td>18</td>
<td>7</td>
<td>95</td>
</tr>
</tbody>
</table>

- Invert the 3×3 Pmat to compute RGBs required to attain desired XYZ.
- Think of RGB and XYZ as different Primary sets.
- An inverse and forward Pmat from different RGB Primary sets may be combined to transform from one set of RGB to another.

\[
\begin{bmatrix}
R_2 \\
G_2 \\
B_2 \\
\end{bmatrix} = \mathbf{P}_2 \mathbf{P}_1^{-1} \begin{bmatrix}
R_1 \\
G_1 \\
B_1 \\
\end{bmatrix}
\]
CIE 1931 xy Chromaticity Diagram

- sRGB primaries’ xy chromaticities

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>0.6400</td>
<td>0.3300</td>
</tr>
<tr>
<td>G</td>
<td>0.3000</td>
<td>0.6000</td>
</tr>
<tr>
<td>B</td>
<td>0.1500</td>
<td>0.0600</td>
</tr>
<tr>
<td>White</td>
<td>0.3127</td>
<td>0.3290</td>
</tr>
</tbody>
</table>

- Triangle is sRGB gamut boundary.
- sRGB standard white point has chromaticities equal to CIE Standard Illuminant D65.
- Addition via Center of Mass
Chromaticity Diagram: Conceptualizing a Spectrum
Primaries and Color Matching Functions

- Each set of primaries has a corresponding set of color matching functions

- 1931 Color-Matching Primaries

- XYZ

- sRGB
RGB Implies a System Model: Example sRGB

- Pmat is only half the story. A real system has a Characteristic Curve, recalling the nonlinearity between Code Value and Luminance.
- sRGB is an encoding standard based on an idealized monitor/TV.
  - sRGB (ITU-R Rec 709) Primaries (same as HDTV)
  - Gamma 2.2-like Characteristic Curve
- The model is simple.

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} =
\begin{bmatrix}
41.2 & 35.8 & 18.0 \\
21.3 & 71.5 & 7.22 \\
1.93 & 11.9 & 95.0
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]
Color Appearance

- Color is perceived, not measured.
- XYZ is useful for quantifying color stimuli and measuring how well they match one another.
- BUT, the appearance of a stimulus as a color depends on (among other things)
  - Absolute luminance of the stimulus
  - Surrounding color(s)
  - State of adaptation of the observer
- Predicting color appearance is difficult, especially in images.
Example: Surround Effects

- The appearance of colors can be influenced by their surroundings.
References & Further Reading

