# CS 465 Homework 6

out: Friday 21 October 2005
**due: Friday 28 October 2005**

**Problem 1:** Direct spline manipulation

Hermite splines are cubic splines defined by point and tangent (that is, value and derivative) constraints at the two endpoints of each segment. More specifically, the following constraints define a Hermite segment:

$$\begin{aligned} \mathbf{p}(0) &= \mathbf{p}_0 \\ \mathbf{p}'(0) &= \mathbf{v}_0 \\ \mathbf{p}(1) &= \mathbf{p}_1 \\ \mathbf{p}'(1) &= \mathbf{v}_1 \end{aligned}$$

where $\mathbf{p}_0$ and $\mathbf{p}_1$ are the endpoints of the segment and $\mathbf{v}_0$ and $\mathbf{v}_1$ are the tangents (derivatives) of the curve at the endpoints. The resulting spline is defined as follows:

$$\mathbf{p}(t) = \begin{bmatrix} t^3 \\ t^2 \\ t \\ 1 \end{bmatrix}^T \begin{bmatrix} 2 & 1 & -2 & 1 \\ -3 & -2 & 3 & -1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{p}_0 \\ \mathbf{v}_0 \\ \mathbf{p}_1 \\ \mathbf{v}_1 \end{bmatrix}$$

You are working on a graphical editor for Hermite splines. The editor is meant to allow *direct manipulation*: rather than changing the control points directly, the user can click directly on the curve and drag it to a new shape. In particular, the user clicks on a point on a curve segment, which is $\mathbf{p}(t_c)$ for some parameter value $t_c$, and drags it to a new location $\mathbf{x}_c$. After this manipulation, $\mathbf{p}_{\text{new}}(t_c) = \mathbf{x}_c$.

In this problem you'll work out three different ways of making the spline pass through the user's desired point:

1. *By adjusting the tangents at the ends.* Make the spline pass through the new point while keeping the endpoints fixed by adjusting the tangents $\mathbf{v}_0$ and $\mathbf{v}_1$.

   (a) How many variables have been introduced and how many constraints? Is the solution expected to be unique?

(b) Give an equation that constrains $\mathbf{v}_0$ and $\mathbf{v}_1$ so that the curve will pass through the point.

(c) Keep $\mathbf{v}_1$ fixed, and give an expression for $\mathbf{v}_0$ in terms of $t_c$ and $\mathbf{x}_c$.

2. *By promoting the degree of the spline.* Make the spline pass through the new point while still maintaining the endpoint and tangent constraints by using a quartic spline that satisfies the Hermite constraints and the new constraint.

   (a) How many variables have been introduced and how many constraints? Is the solution expected to be unique?

   (b) Give the matrix equation for the new spline, but it is OK to write the spline matrix as the inverse of another matrix. Please put the controls in the order $\mathbf{p}_0$, $\mathbf{v}_0$, $\mathbf{x}_c$, $\mathbf{p}_1$, $\mathbf{v}_1$.

   (c) Show that your spline reduces to a cubic again if the user drags the point back exactly to where it started.

   (d) Plot the basis functions for your new spline when $t = \frac{1}{3}$ and $t = \frac{2}{3}$.

   (e) What is the order of continuity of the curve at the manipulation point?

3. *By splitting the spline into two segments.* Make the spline pass through the new point by splitting it into two segments at the parameter $t_c$ and then adjusting the endpoints of the new segments.

   (a) How many variables have been introduced and how many constraints? Is the solution expected to be unique?

   (b) Write expressions for the endpoints and tangents of the two curves that result from the split. Resolve ambiguity by keeping the curve continuous and keeping the tangent at the manipulation point unchanged.

   (c) Spot-check that the curve coincides with the original if the user drags the point back exactly to where it started, by verifying that the points at $t = \frac{1}{2}$ on each of the two pieces are also on the original curve.

   (d) What is the order of continuity of the curve at the manipulation point?