

# CS 465 Homework 5 Solution

## Problem 1: Rasterization

1. Write down the projection matrix and the viewport matrix.

For this problem, we were asking for the perspective transform matrices as specified by Shirley; that is, for the projection matrix, the one just above the OpenGL one listed. The appropriate equations for those are:

$$M_{\text{view}} = \begin{bmatrix} \frac{n_x}{2} & 0 & 0 & \frac{n_x-1}{2} \\ 0 & \frac{n_y}{2} & 0 & \frac{n_y-1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad M_{\text{projection}} = \begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{l+r}{l-r} & 0 \\ 0 & \frac{2n}{t-b} & \frac{b+t}{b-t} & 0 \\ 0 & 0 & \frac{f+n}{f-n} & \frac{2fn}{n-f} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

This gives you the matrices:

$$M_{\text{view}} = \begin{bmatrix} 4 & 0 & 0 & 3.5 \\ 0 & 3 & 0 & 2.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad M_{\text{projection}} = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -\frac{8}{3} & 0 & 0 \\ 0 & 0 & \frac{13}{11} & \frac{96}{11} \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -2.67 & 0 & 0 \\ 0 & 0 & 1.18 & 8.73 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

2. Transform the vertices into clip space and write them down as 4-vectors. Which of these points are inside the view volume and which are outside?

To transform into clip space, multiply the projection matrix and eye space vector and homogenize.

$$M_{\text{projection}} v = v'$$

$$v'_0 = \begin{bmatrix} 0 \\ -\frac{128}{3} \\ -\frac{320}{11} \\ -32 \end{bmatrix} \sim \begin{bmatrix} 0 \\ \frac{4}{3} \\ \frac{10}{11} \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.33 \\ 0.91 \\ 1 \end{bmatrix} \quad v'_1 = \begin{bmatrix} 8 \\ 0 \\ -\frac{112}{11} \\ -16 \end{bmatrix} \sim \begin{bmatrix} -0.5 \\ 0 \\ \frac{7}{11} \\ 1 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 0 \\ 0.63 \\ 1 \end{bmatrix}$$

$$v'_2 = \begin{bmatrix} -48 \\ \frac{40}{3} \\ -\frac{736}{11} \\ -64 \end{bmatrix} \sim \begin{bmatrix} 0.75 \\ -\frac{5}{24} \\ \frac{23}{22} \\ 1 \end{bmatrix} = \begin{bmatrix} 0.75 \\ -0.2083 \\ 1.045 \\ 1 \end{bmatrix}$$

3. Transform the vertices into screen space and give their  $(x', y', z')$  coordinates.

To then go to screen space from clip space, multiply it with the viewport matrix. Note that the viewport matrix doesn't change the  $z$  and  $w$  values.

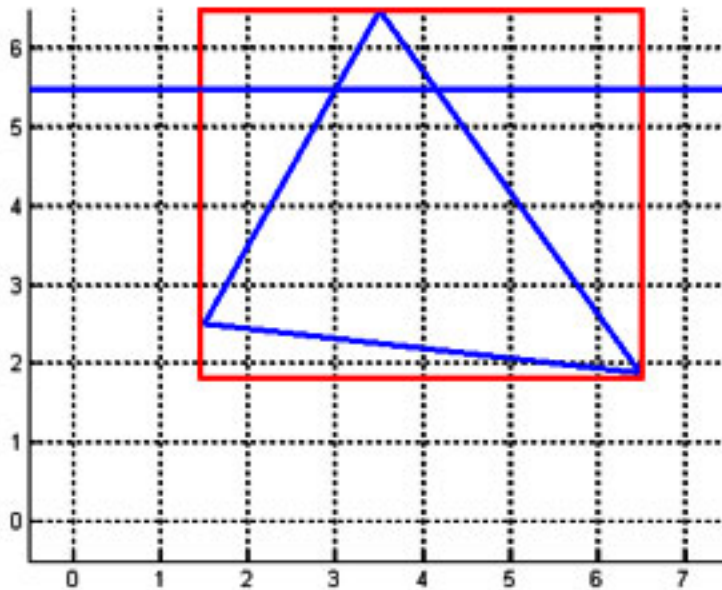
$$M_{\text{view}}v' = v''$$

$$v''_0 = \begin{bmatrix} 3.5 \\ 6.5 \\ 0.91 \\ 1 \end{bmatrix} \quad v''_1 = \begin{bmatrix} 1.5 \\ 2.5 \\ 0.63 \\ 1 \end{bmatrix} \quad v''_2 = \begin{bmatrix} 6.5 \\ \frac{15}{8} \\ 1.045 \\ 1 \end{bmatrix} = \begin{bmatrix} 6.5 \\ 1.875 \\ 1.045 \\ 1 \end{bmatrix}$$

4. What is the rectangle of pixels enclosed by intersection of the triangles screen-space bounding box with the image rectangle? *Hint:* There are 20 pixels in this rectangle.

Using Shirley's notation, the screen space coordinate run in the range  $[-0.5, n_x - 0.5 = 7.5]$  for the  $x$ -axis and  $[-0.5, n_y - 0.5 = 5.5]$  for the  $y$ -axis, with pixels on the integer coordinates. The pixel range of the bounding box is found by taking the ceiling of the least value and the floor of the highest value for the  $x$  and  $y$  coordinates. Given the corresponding ranges of  $[1.5, 6.5]$  and  $[1.875, 6.5]$  for  $x$  and  $y$ , you get the enclosed pixel ranges of  $[2, 6]$  for both. However, clamping it to the screen reduces the pixel's  $y$  maximum to 5.

5. Draw a picture (graph paper or drawing software will help) that shows the boundaries of the image, the screen-space position of the triangle, and the bounding rectangle.



6. Assume the rasterizer interpolates screen-space barycentric coordinates to determine which fragments to generate, and its also doing  $z$ -buffering. This means it needs to interpolate four attributes (3 barycentric coordinates plus  $z'$ ). What are the values of the attributes at pixel (2, 2) and what are the  $x$  and  $y$  increments? *Hint*: This pixel is inside the bounding box you computed in part 4. Computing the  $x$  and  $y$  increments for the various quantities can be done by simply plugging numbers into the equation

$$c_x^q = (\Delta q_1 \Delta y_2 - \Delta q_2 \Delta y_1) / d$$

$$c_y^q = (\Delta q_2 \Delta x_1 - \Delta q_1 \Delta x_2) / d$$

where  $\Delta a_n = a_n - a_0$  for  $a$  being any of the quantities  $x$ ,  $y$ , or  $q$  (an attribute value), and

$$d = \Delta x_1 \Delta y_2 - \Delta x_2 \Delta y_1$$

$$\Delta x_1 = 1.5 - 3.5 = -2, \quad \Delta x_2 = 6.5 - 3.5 = 3, \quad \Delta y_1 = 2.5 - 6.5 = -4, \quad \Delta y_2 = \frac{15}{8} - 6.5 = \frac{-37}{8}$$

$$d = -2 \times \frac{-37}{8} - 3 \times -4 = 21.25$$

- $\alpha : \Delta q_1 = \Delta q_2 = 0 - 1 = -1$

$$c_x^\alpha = (-1 \times \frac{-37}{8} - -1 \times -4) / 21.25 = \frac{1}{34} = 0.0294$$

$$c_y^\alpha = (-1 \times -2 - -1 \times 3) / 21.25 = \frac{4}{17} = 0.2353$$

- $\beta : \Delta q_1 = 1 - 0 = 1, \quad \Delta q_2 = 0 - 0 = 0$

$$c_x^\beta = (1 \times \frac{-37}{8} - 0 \times -4) / 21.25 = \frac{-37}{170} = -0.2176$$

$$c_y^\beta = (0 \times -2 - 1 \times 3) / 21.25 = \frac{-12}{85} = -0.1412$$

- $\gamma : \Delta q_1 = 0 - 0 = 0, \quad \Delta q_2 = 1 - 0 = 1$

$$c_x^\gamma = (0 \times \frac{-37}{8} - 1 \times -4) / 21.25 = \frac{16}{85} = 0.1882$$

$$c_y^\gamma = (1 \times -2 - 0 \times 3) / 21.25 = \frac{-8}{85} = -0.0941$$

- $z' : \Delta q_1 = \frac{7}{11} - \frac{10}{11} = \frac{-3}{11}, \quad \Delta q_2 = \frac{23}{22} - \frac{10}{11} = \frac{3}{22}$

$$c_x^{z'} = (\frac{-3}{11} \times \frac{-37}{8} - \frac{3}{22} \times -4) / 21.25 = \frac{159}{1870} = 0.0850$$

$$c_y^{z'} = (\frac{3}{22} \times -2 - \frac{-3}{11} \times 3) / 21.25 = \frac{24}{935} = 0.0257$$

This gives  $(\alpha, \beta, \gamma, z')$  values of  $(-0.1029, 0.9618, 0.1412, 0.6660)$  for the point (2, 2).

7. Compute the barycentric coordinates at the pixels (5, 2) and (5, 4). Which is inside and which is outside the triangle?

Using the same increments from (6), the  $(\alpha, \beta, \gamma)$  values for (5, 2) and (5, 4) are respectively

$$(-0.0147, 0.3088, 0.7059)$$

$$(0.4559, 0.0265, 0.5176)$$

This puts the point (5, 2) outside the triangle since the  $\alpha$  value for it is negative and (5, 4) inside as the barycentric coordinates are all in the range  $[0, 1]$ .

8. List all the fragments that will be generated for this triangle. Each one consists of  $(x, y)$  pixel coordinates and a value for the attribute  $z'$ . Assume the rasterizer will never generate fragments that are outside the view volume. *Hint:* There are nine fragments.

(Refer to (5)) Of the 10 pixels contained within the triangle, the lower rightmost pixel (6,2) will have a  $z' = 1.006$  and be culled by z-buffering and not be rasterized. The other pixels have interpolated  $0 < z' < 1$  and will be rasterized: (2, 3), (3, 3-5), (4, 3-5), (5, 3-4).