CS 465 Homework 4 Solutions

Problem 1: 2D Transformations

Here are four letter shapes that, in this particular font, are simple transformations of one another:

q b p d

Each letter is positioned with its baseline at y = 0 and its left edge aligned with x = 0. Express the transformation required to turn p into each of q, b, and d in the three following ways:
(Note these solutions may not be the only way to perform these transformations. Other equivalent solutions will be accepted as well.)

1. as a sequence of affine transformations, using only translation, rotation about the origin, and reflections across coordinate axes. Describe the transformations in words.

q: Translation to the left by 10, followed by a reflection across y.
d: Rotation by 180^o around the origin, followed by a translation by 10 to the right and up.
b: Translation by 10 down, followed by a reflection across x.

2. as a single 3 x 3 homogeneous transformation matrix.

q: $\begin{bmatrix} 1 & 0 & -10 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ d: $\begin{bmatrix} -1 & 0 & 10 \\ 0 & -1 & 10 \\ 0 & 0 & 1 \end{bmatrix}$ b: $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{bmatrix}$

3. as a single rotation about a point or a single reflection across a line.
q: Reflection across the line x=5.
d: Rotation around the point [5,5] by 180^0.
b: Reflection across the line y=5.

Problem 2: 3D Transformations

Suppose I apply a rotation that maps the x axis to the y axis, the y axis to the z axis, and the z axis to the x axis.

1. What axis and angle can be used to describe this rotation?
You can rotate around the axis [1,1,1] by 60 degrees.

2. What is the 3-by-3 matrix of the rotation?
The change of frame matrix will transform each of the original e_1, e_2, and e_3 vectors to u, v, w.

Using the change of frame matrix:

$$\begin{bmatrix} u & v & w & p \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We have a matrix that will take the x axis to the y axis, y axis to the z axis, and the z axis to the x axis.