CS 465 Homework 4 Solutions

Problem 1: 2D Transformations

Here are four letter shapes that, in this particular font, are simple transformations of one another:

q b p d

Each letter is positioned with its baseline at $y = 0$ and its left edge aligned with $x = 0$. Express the transformation required to turn p into each of q, b, and d in the three following ways:

(Note these solutions may not be the only way to perform these transformations. Other equivalent solutions will be accepted as well.)

1. as a sequence of affine transformations, using only translation, rotation about the origin, and reflections across coordinate axes. Describe the transformations in words.

   q: Translation to the left by 10, followed by a reflection across y.
   d: Rotation by $180^\circ$ around the origin, followed by a translation by 10 to the right and up.
   b: Translation by 10 down, followed by a reflection across x.

2. as a single 3 x 3 homogeneous transformation matrix.

   $q:\begin{bmatrix}1 & 0 & -10 \\
                  0 & -1 & 0 \\
                  0 & 0 & 1 \\
   \end{bmatrix}$
   $d:\begin{bmatrix}-1 & 0 & 10 \\
                  0 & -1 & 0 \\
                  0 & 0 & 1 \\
   \end{bmatrix}$
   $b:\begin{bmatrix}-1 & 0 & 0 \\
                  0 & 1 & -10 \\
                  0 & 0 & 1 \\
   \end{bmatrix}$

3. as a single rotation about a point or a single reflection across a line.

   q: Reflection across the line $x=5$.
   d: Rotation around the point $[5,5]$ by $180^\circ$.
   b: Reflection across the line $y=5$.

Problem 2: 3D Transformations

Suppose I apply a rotation that maps the x axis to the y axis, the y axis to the z axis, and the z axis to the x axis.

1. What axis and angle can be used to describe this rotation?
   You can rotate around the axis $[1,1,1]$ by 60 degrees.

2. What is the 3-by-3 matrix of the rotation?
   The change of frame matrix will transform each of the original $e_1$, $e_2$, and $e_3$ vectors to $u$, $v$, $w$. 

Using the change of frame matrix:

\[
\begin{bmatrix}
  u & v & w & p \\
  0 & 0 & 0 & 1
\end{bmatrix}
\rightarrow
\begin{bmatrix}
  0 & 0 & 1 & 0 \\
  1 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 \\
  0 & 0 & 0 & 1
\end{bmatrix}
\]

We have a matrix that will take the x axis to the y axis, y axis to the z axis, and the z axis to the x axis.