The He model [58] is, to-date, the most comprehensive and expensive shading model available; however, it is beyond the scope of this book to present this model.

Cook-Torrance model. The Cook-Torrance model includes a micro-facet model that assumes that a surface is made of a random collection of small smooth planar facets. The assumption in this model is that an incoming ray randomly hits one of these smooth facets. Given a specification of the distribution of micro-facets for a material, this model captures the shadowing effects of these micro-facets. In addition to the facet distribution, the Cook-Torrance model also includes the Fresnel reflection and refraction terms,

$$f_r(x, \Psi \leftrightarrow \Theta) = \frac{F(\beta)}{\pi} \frac{D(\theta_h)G}{(N \cdot \Psi)(N \cdot \Theta)} + k_d,$$

where the three terms in the nondiffuse component of the BRDF are the Fresnel reflectance F, the microfacet distribution D, and a geometric shadowing term G. We now present each of these terms.

The Fresnel terms, as given in Equations 2.28 and 2.29, are used in the Cook-Torrance model. This model assumes that the light is unpolarized; therefore, $F = \frac{|r_p|^2+|r_s|^2}{2}$. The Fresnel reflectance term is computed with respect to the angle β, which is the angle between the incident direction and the half vector: $\cos\beta = \Psi \cdot H = \Theta \cdot H$. By the definition of the half-vector, this angle is the same as the angle between the outgoing direction and the half-vector.

The distribution function D specifies the distribution of the micro-facets for the material. Various functions can be used to specify this distribution. One of the most common distributions is the distribution by Beckmann,

$$D(\theta_h) = \frac{1}{m^2 \cos^4 \theta_h} e^{-(\frac{\tan \theta_h}{m})^2},$$

where θ_h is the angle between the normal and the half-vector and $\cos\theta_h = N \cdot H$.

The geometry term G captures masking and self-shadowing by the micro-facets:

$$G = \min\{1, \frac{2(N \cdot H)(N \cdot \Theta)}{\Theta \cdot H}, \frac{2(N \cdot H)(N \cdot \Psi)}{\Theta \cdot H}\}.$$

Empirical Models

Models, such as Ward [181] and LaFortune [84], are based on empirical data. These models aim at ease of use and an intuitive parameterization of he BRDF. For isotropic surfaces, the Ward model has the following RDF,

$$f_r(x, \Psi \leftrightarrow \Theta) = \frac{\rho_d}{\pi} + \rho_s \frac{e^{\frac{-tan^2\theta_h}{\alpha^2}}}{4\pi\alpha^2\sqrt{(N \cdot \Psi)(N \cdot \Theta)}},$$

ere θ_h is the angle between the half-vector and the normal.

The Ward model includes three parameters to describe the BRDF: ρ_d, diffuse reflectance; ρ_s, the specular reflectance; and α, a measure of surface roughness. This model is energy-conserving and relatively in-tive to use because of the small set of parameters; with the appropriate rameter settings, it can be used to represent a wide range of materials.

Lafortune et al. [84] introduced an empirically based model to represent asurements of real materials. This model fits modified Phong lobes to asured BRDF data. The strength of this technique is that it exploits simplicity of the Phong model while capturing realistic BRDFs from asured data. More detailed descriptions of several models can be found Glassner's books [46].

2.6 Rendering Equation

w we are ready to mathematically formulate the equilibrium distribution light energy in a scene as the rendering equation. The goal of a global mination algorithm is to compute the steady-state distribution of light ergy. As mentioned earlier, we assume the absence of participating me-a. We also assume that light propagates instantaneously; therefore, the ady-state distribution is achieved instantaneously. At each surface point and in each direction Θ, the rendering equation formulates the exitant diance $L(x \to \Theta)$ at that surface point in that direction.

2.6.1 Hemispherical Formulation

he hemispherical formulation of the rendering equation is one of the most mmonly used formulations in rendering. In this section, we derive this rmulation using energy conservation at the point x. Let us assume that $(x \to \Theta)$ represents the radiance emitted by the surface at x and in e outgoing direction Θ, and $L_r(x \to \Theta)$ represents the radiance that is flected by the surface at x in that direction Θ.

By conservation of energy, the total outgoing radiance at a point and a particular outgoing direction is the sum of the emitted radiance and e radiance reflected at that surface point in that direction. The outgoing