

CS465 Notes: Simple ray-triangle intersection

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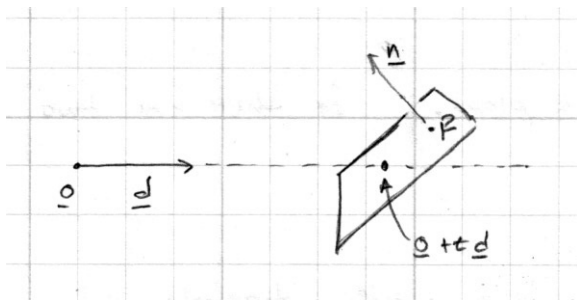
Intersecting a ray with a sphere is easy because the sphere can be defined by a single implicit equation. Intersecting a ray with a triangle is a little more complex because a triangle needs a slightly more complex definition. The method given by Shirley is efficient and elegant, and it's what you would probably use in a "real" ray tracer because it computes some values as a side effect that you need for various more advanced features. However, it's not conceptually the simplest way to do it. Here I present an alternate and more straightforward approach.

A triangle is a subset of a plane—the region of the plane defined by its three vertices that is bounded by its three edges. There are two steps to ray-triangle intersection:

1. Intersect the ray with the triangle's plane
2. Check whether the intersection is inside the triangle.

We'll assume we are given the 3D positions of three vertices of the triangle as three vectors \mathbf{p}_0 , \mathbf{p}_1 , and \mathbf{p}_2 .

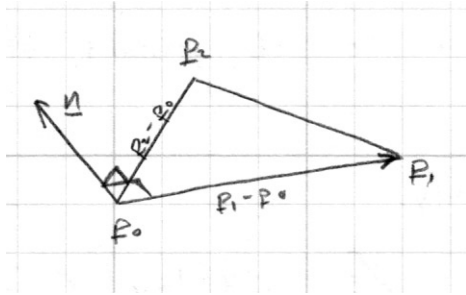
Ray-plane intersection is easy using the implicit equation for the plane; we discussed this before in the ray tracing lecture. For this ray and plane:



the solution is

$$t = -\frac{(\mathbf{o} - \mathbf{p}) \cdot \mathbf{n}}{\mathbf{d} \cdot \mathbf{n}}$$
$$\mathbf{x} = \mathbf{o} + t\mathbf{d}.$$

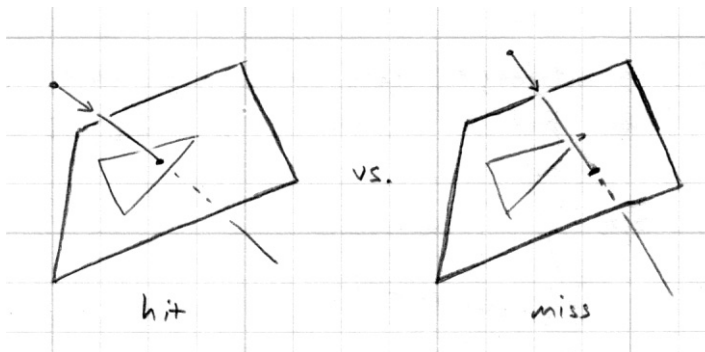
For a plane defined by the three vertices, we can just use any vertex, say \mathbf{p}_0 , as the point on the plane, and we can get the normal from the cross product of two edges:



$$\mathbf{n} = (\mathbf{p}_1 - \mathbf{p}_0) \times (\mathbf{p}_2 - \mathbf{p}_0).$$

Note that the order is important, to make sure the normal points towards the front of the triangle (the side from which the vertices are ordered counterclockwise).

Now that we can find the intersection of the point with the triangle's plane, we need to find out whether that point is inside the triangle or not:



We can answer this question by thinking of the triangle as the intersection of three half-spaces. Each edge of the triangle defines a line, and a point is inside the triangle if it's on the correct side of all three lines (Figure 1).

Given a point \mathbf{p} and a vector \mathbf{v} that define a line, let's arbitrarily define that the half of the plane on your left, as you travel along the line in the direction of the vector, is the "inside" and the half on your right is the "outside." We can determine whether a particular point \mathbf{x} is inside or outside by checking whether the vector $\mathbf{x} - \mathbf{p}$ is counterclockwise or clockwise from \mathbf{v} :

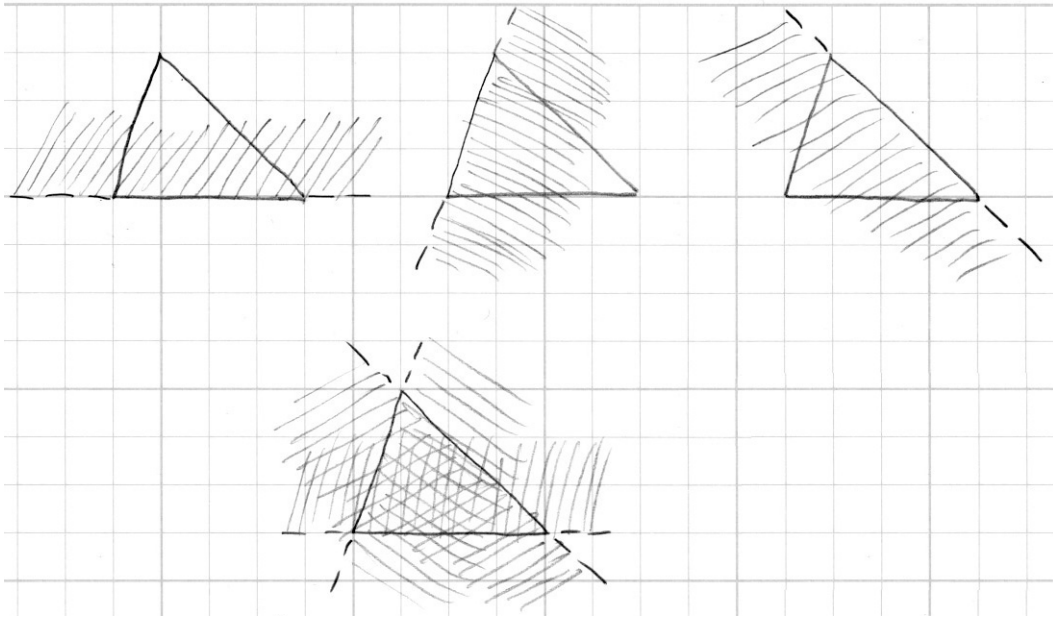
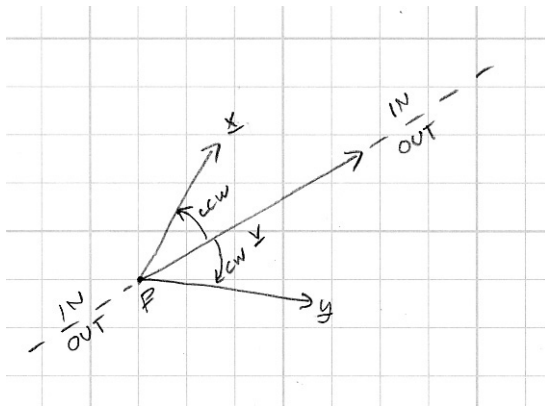
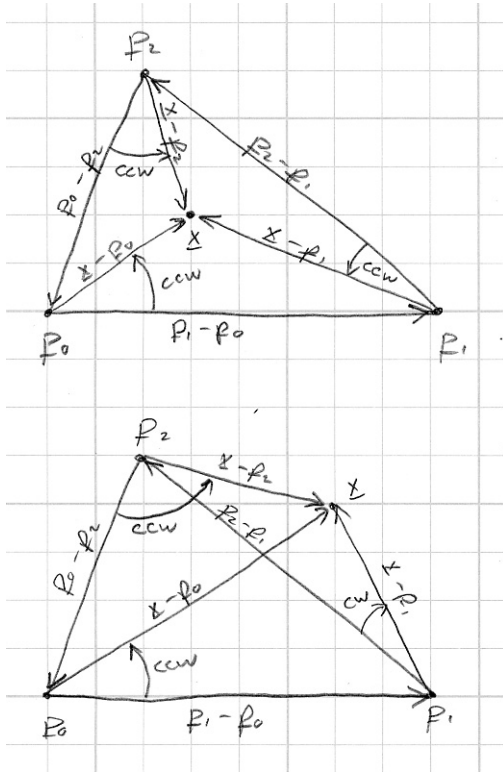


Figure 1: A triangle is the intersection of three half-spaces.

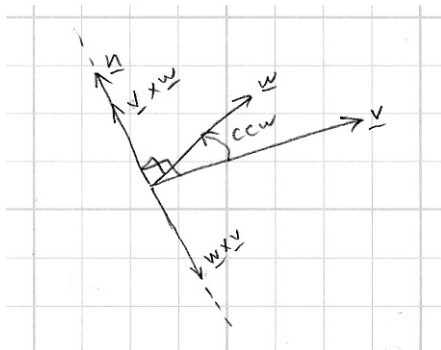


Using this definition, the triangle is the region that is on the left of the line defined by \mathbf{p}_0 and $\mathbf{p}_1 - \mathbf{p}_0$, on the left of the line defined by \mathbf{p}_1 and $\mathbf{p}_2 - \mathbf{p}_1$, and also on the left of the line defined by \mathbf{p}_2 and $\mathbf{p}_0 - \mathbf{p}_2$. (That is, it's to the left of the line defined by each vertex and the vector to the next vertex.) If it's on the outside of any of the edges it's outside the triangle:



Note that in this definition it's important that the three vertices are numbered in counterclockwise order; otherwise no point can ever be in the triangle.

Since the three vertices and the intersection point are all in 3D space, the easiest way to compute these clockwise/counterclockwise checks is using the cross product. We already defined a surface normal \mathbf{n} that points toward the “front”—that is, the side from which the vertices are ordered counterclockwise. Given two arbitrary vectors \mathbf{v} and \mathbf{w} in the triangle's plane, we know that $\mathbf{v} \times \mathbf{w}$ is perpendicular to that plane, and the right hand rule tells us that the cross product points toward the front if and only if \mathbf{w} is counterclockwise from \mathbf{v} (when viewed from the front). This means we can tell clockwise from counterclockwise by checking whether the cross product points in the same direction as the normal.



Two parallel vectors point in the same direction if and only if their dot product is positive, so the point-in-triangle test reduces to

$$(\mathbf{p}_1 - \mathbf{p}_0) \times (\mathbf{x} - \mathbf{p}_0) \cdot \mathbf{n} \geq 0$$

$$(\mathbf{p}_2 - \mathbf{p}_1) \times (\mathbf{x} - \mathbf{p}_1) \cdot \mathbf{n} \geq 0$$

$$(\mathbf{p}_0 - \mathbf{p}_2) \times (\mathbf{x} - \mathbf{p}_2) \cdot \mathbf{n} \geq 0$$