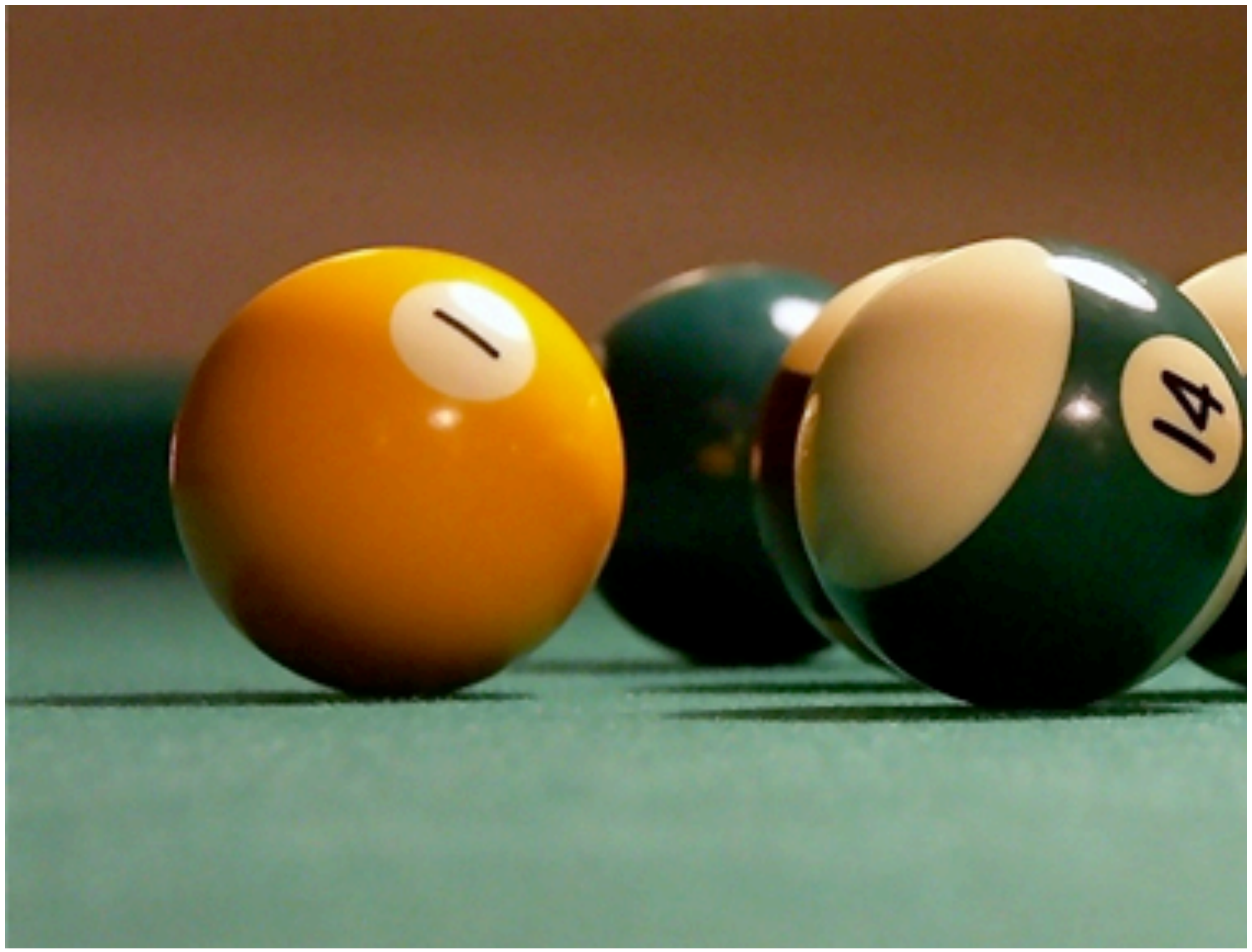
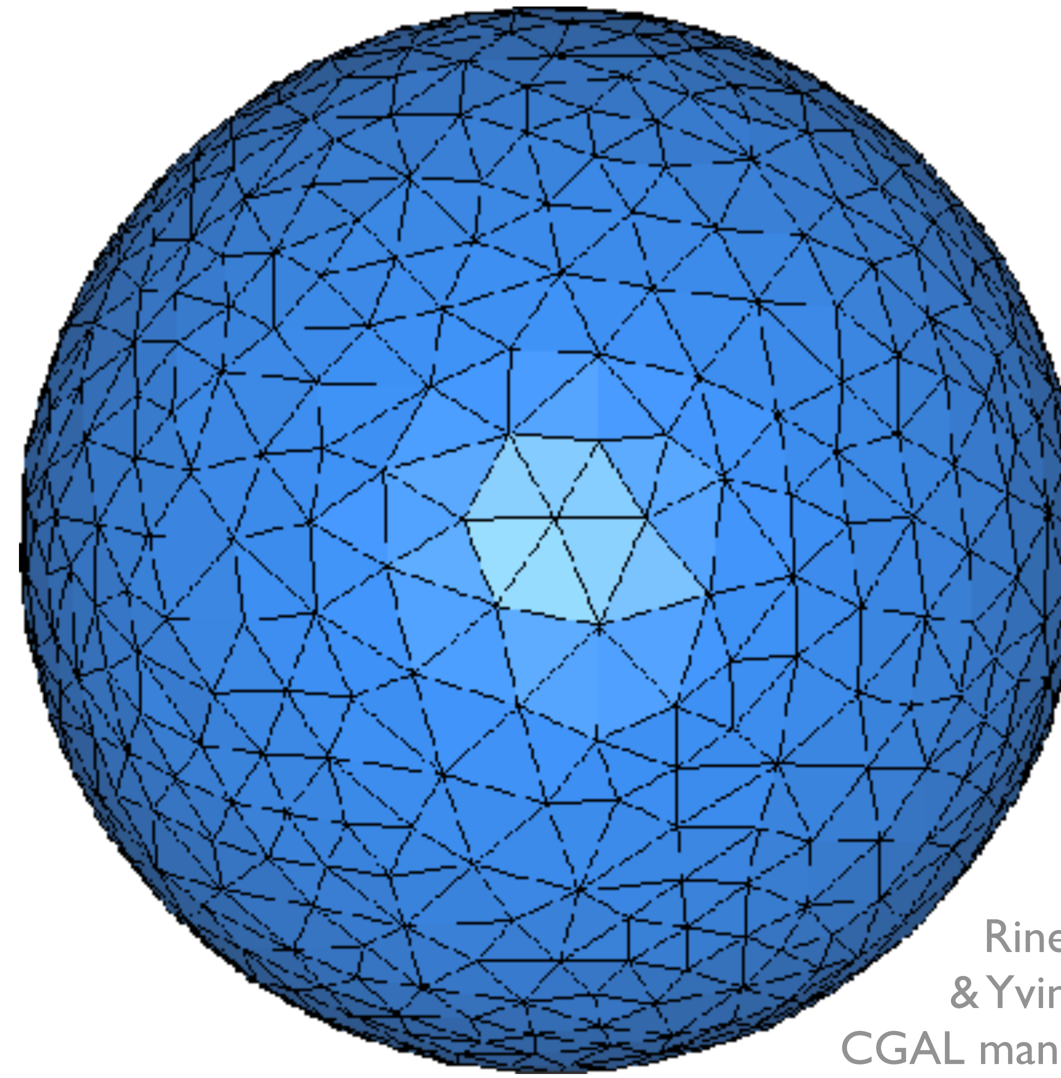
Triangle meshes I

CS 4620 Lecture 2



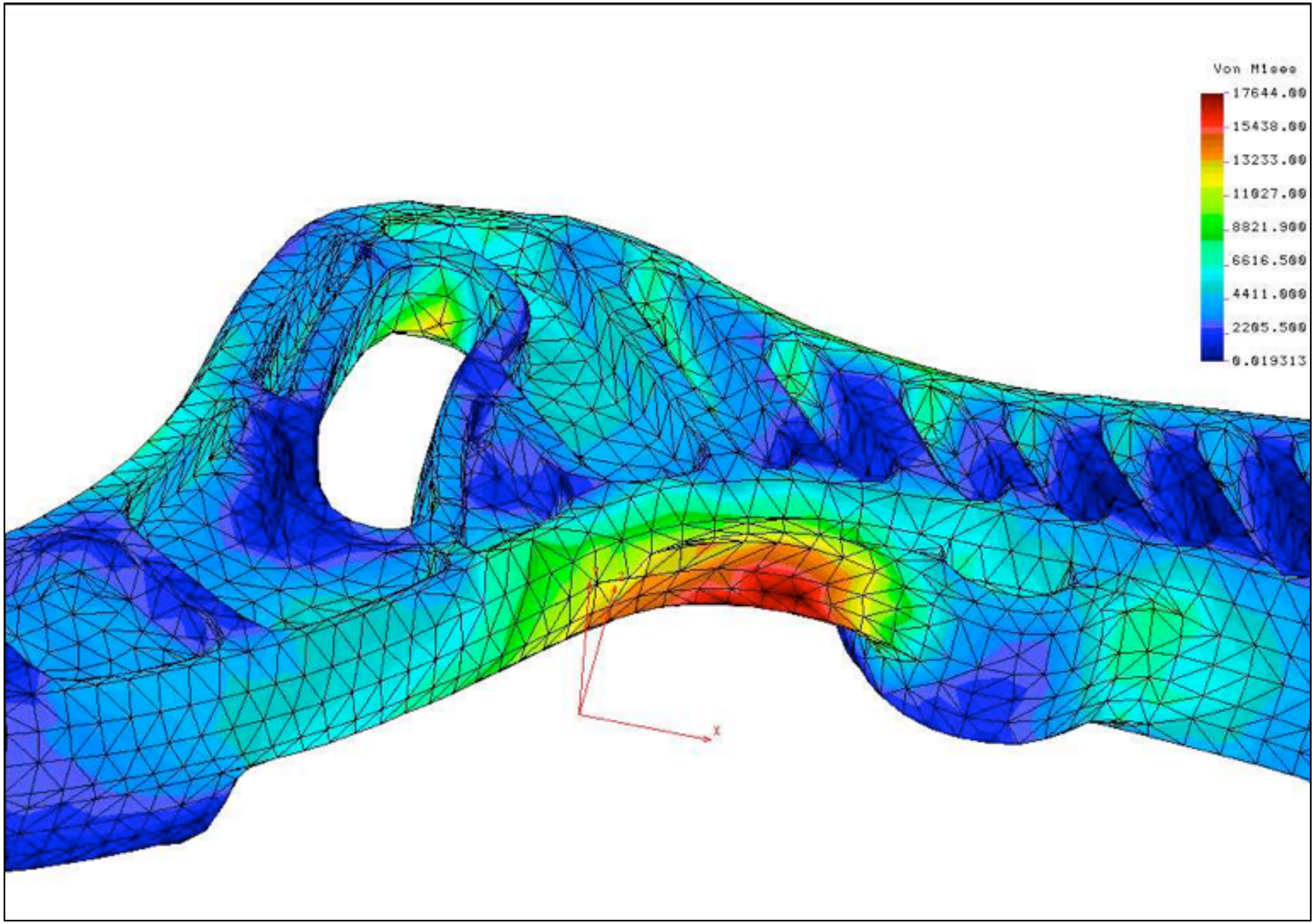
Andrzej Barabasz

spheres



Rineau & Yvinec CGAL manual

approximate sphere

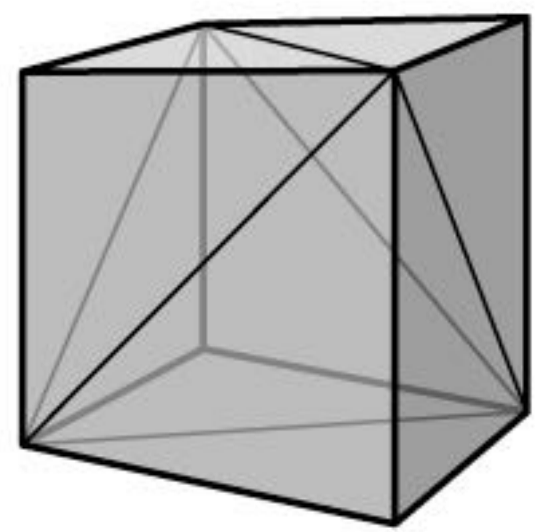


PATRIOT Engineering

finite element analysis

Ottawa Convention Center

A small triangle mesh

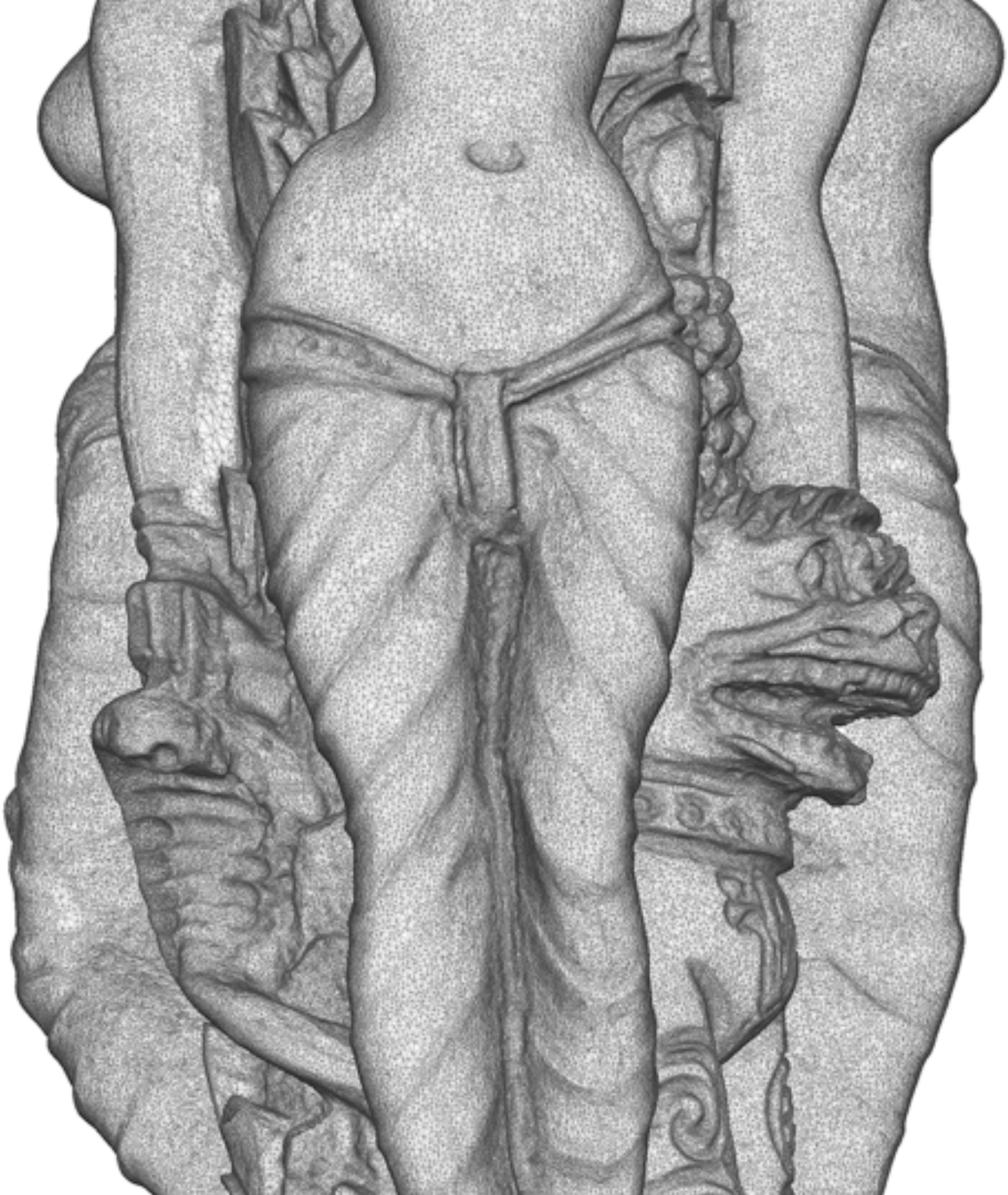


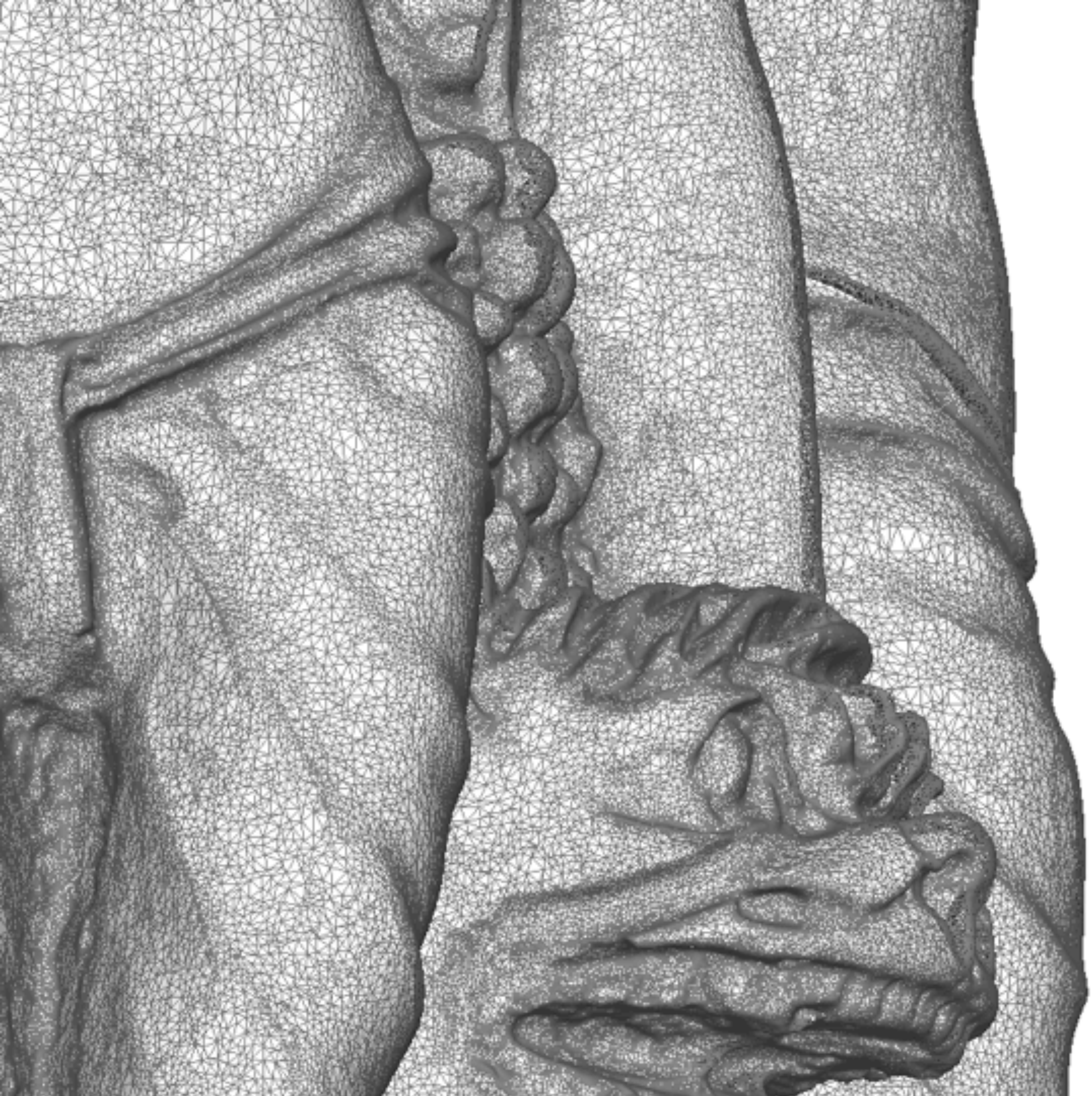
12 triangles, 8 vertices

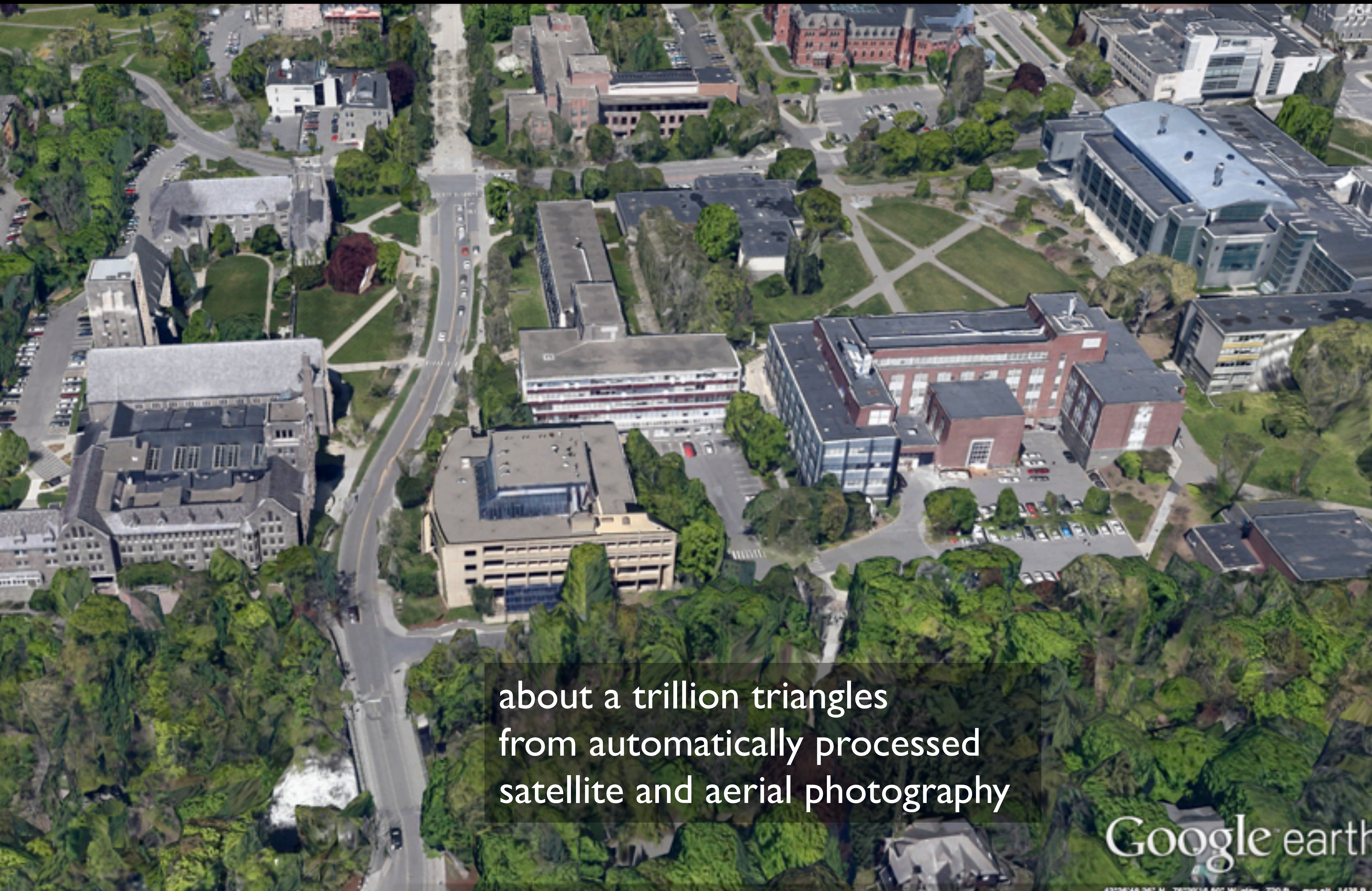
A large mesh

10 million triangles
from a high-resolution
3D scan

Traditional Thai sculpture—scan by XYZRGB, inc., image by MeshLab project






about a trillion triangles
from automatically processed
satellite and aerial photography
Google earth

Triangles

- **Defined by three *vertices***
- **Lives in the plane containing those vertices**
- **Vector normal to plane is the triangle's normal**
- **Conventions (for this class, not everyone agrees):**
  - vertices are counter-clockwise as seen from the "outside" or "front"
  - surface normal points towards the outside ("outward facing normals")

Triangle meshes

- **A bunch of triangles in 3D space that are connected together to form a surface**
- **Geometrically, a mesh is a *piecewise planar* surface**
  - almost everywhere, it is planar
  - exceptions are at the edges where triangles join
- **Often, it's a piecewise planar approximation of a smooth surface**
  - in this case the creases between triangles are artifacts—we don't want to see them

Representation of triangle meshes

- **Compactness**
- **Efficiency for rendering**
  - enumerate all triangles as triples of 3D points
- **Efficiency of queries**
  - all vertices of a triangle
  - all triangles around a vertex
  - neighboring triangles of a triangle
  - (need depends on application)
    - finding triangle strips
    - computing subdivision surfaces
    - mesh editing

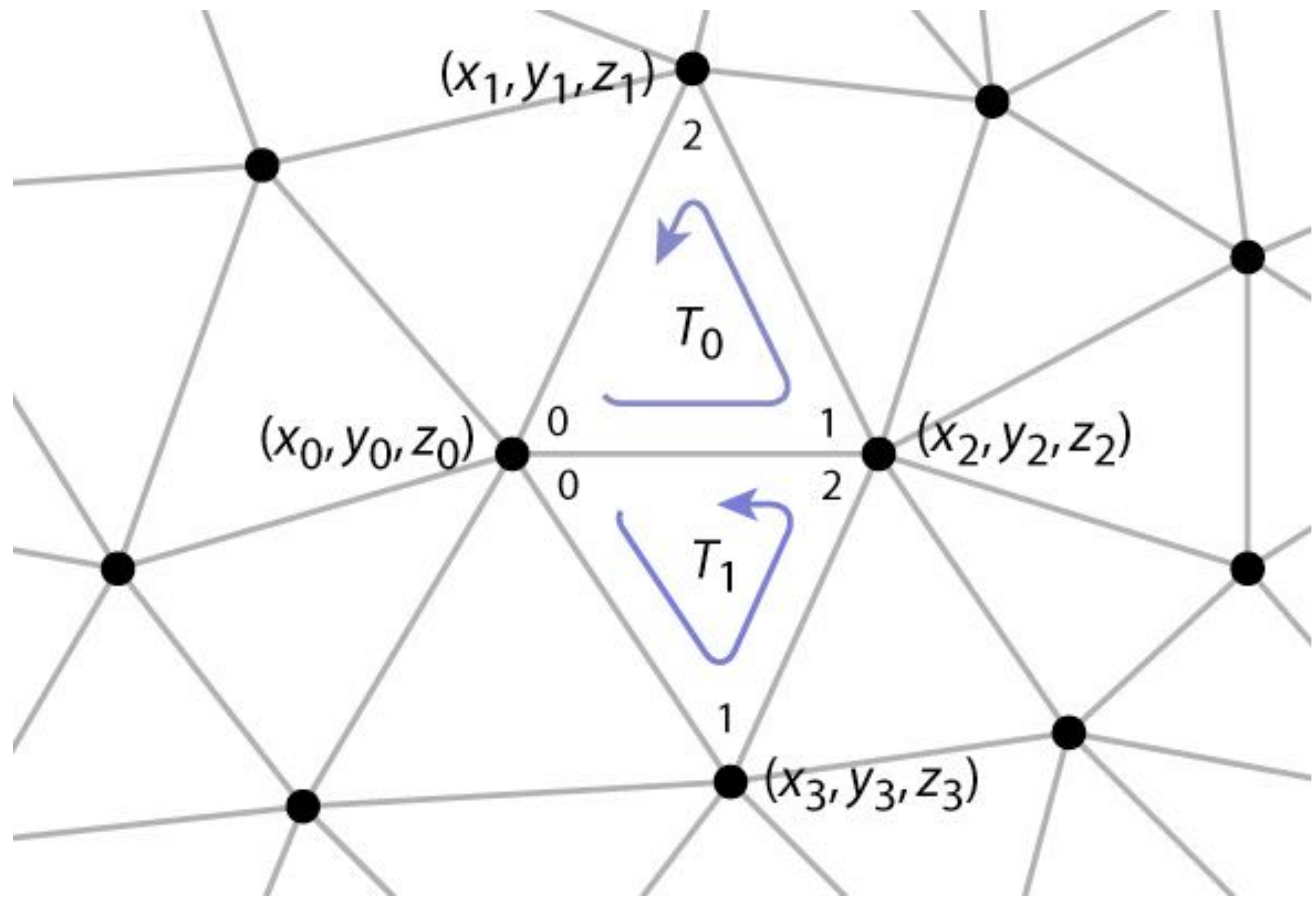
Representations for triangle meshes

- **Separate triangles**
- **Indexed triangle set** ← crucial for first assignment
  - shared vertices
- **Triangle strips and triangle fans**
  - compression schemes for fast transmission
- **Triangle-neighbor data structure**
  - supports adjacency queries
- **Winged-edge data structure**
  - supports general polygon meshes

} Interesting and useful but not used in Mesh assignment

Separate triangles

	[0]	[1]	[2]
tris[0]	x_0, y_0, z_0	x_2, y_2, z_2	x_1, y_1, z_1
tris[1]	x_0, y_0, z_0	x_3, y_3, z_3	x_2, y_2, z_2
	⋮	⋮	⋮



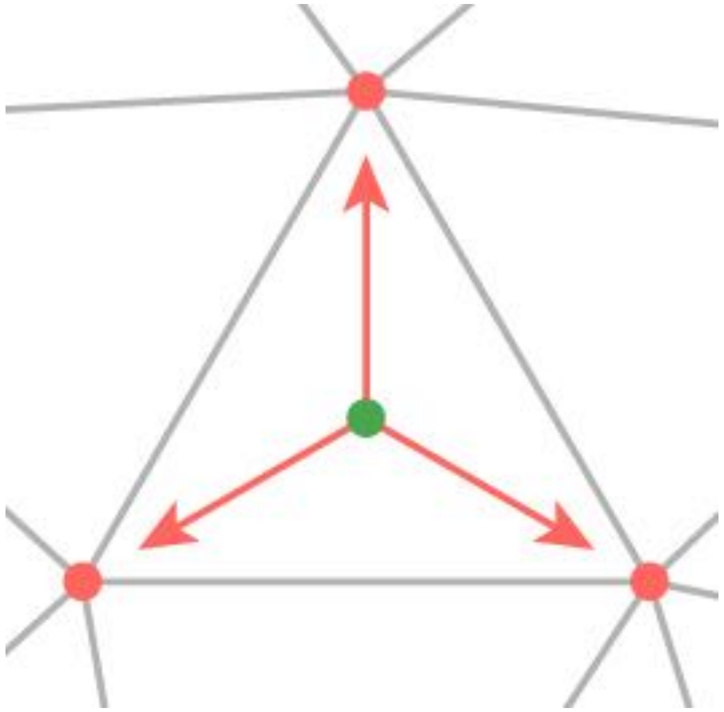
Separate triangles

- **array of triples of points**
  - float[n_T][3][3]: about 72 bytes per vertex
    - 2 triangles per vertex (on average)
    - 3 vertices per triangle
    - 3 coordinates per vertex
    - 4 bytes per coordinate (float)
- **various problems**
  - wastes space (each vertex stored 6 times)
  - cracks due to roundoff
  - difficulty of finding neighbors at all

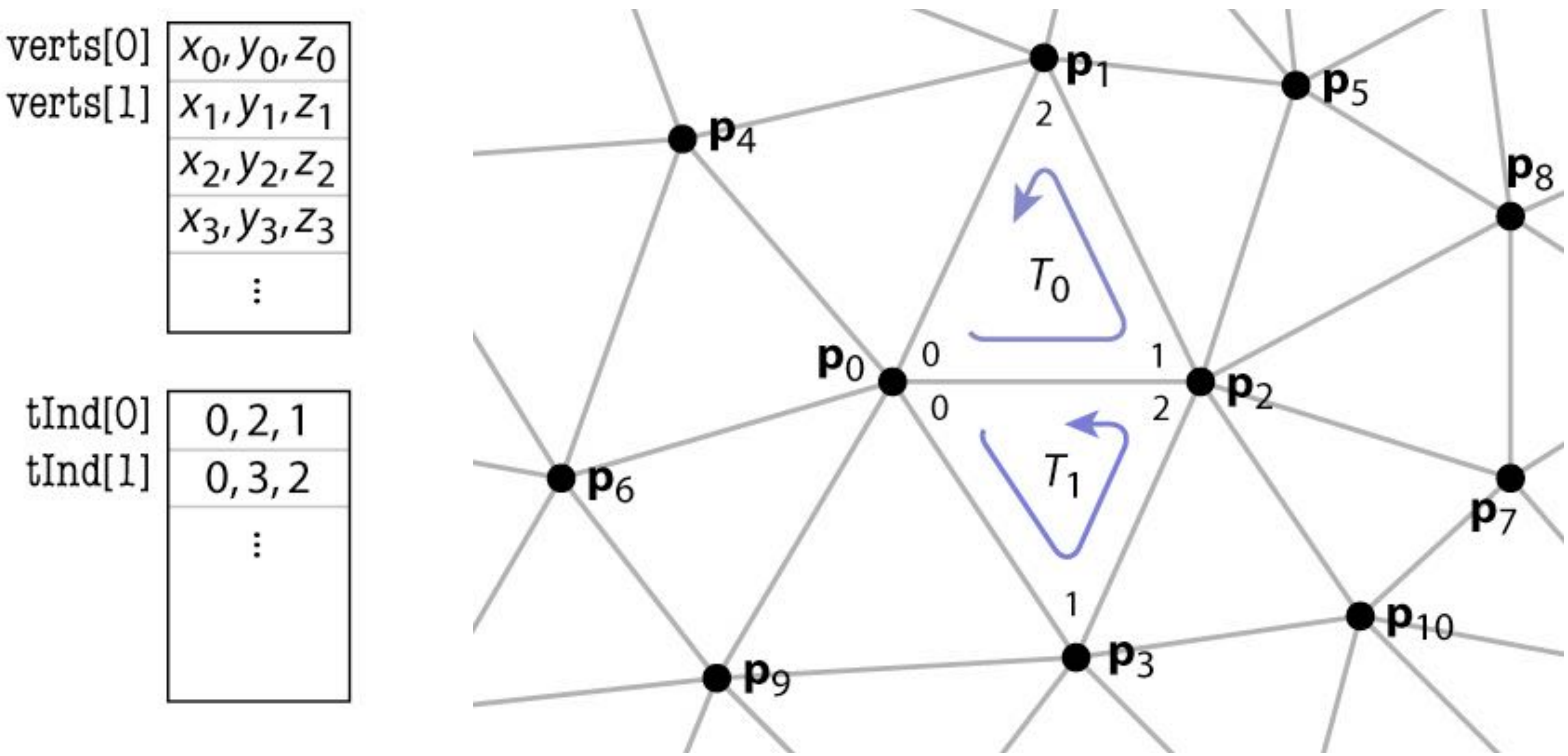
Indexed triangle set

- **Store each vertex once**
- **Each triangle points to its three vertices**

```
Triangle {
  Vertex vertex[3];
  }

Vertex {
  float position[3];  // or other data
  }

// ... or ...

Mesh {
  float verts[nv][3];  // vertex positions (or other data)
  int tInd[nt][3];  // vertex indices
  }
```

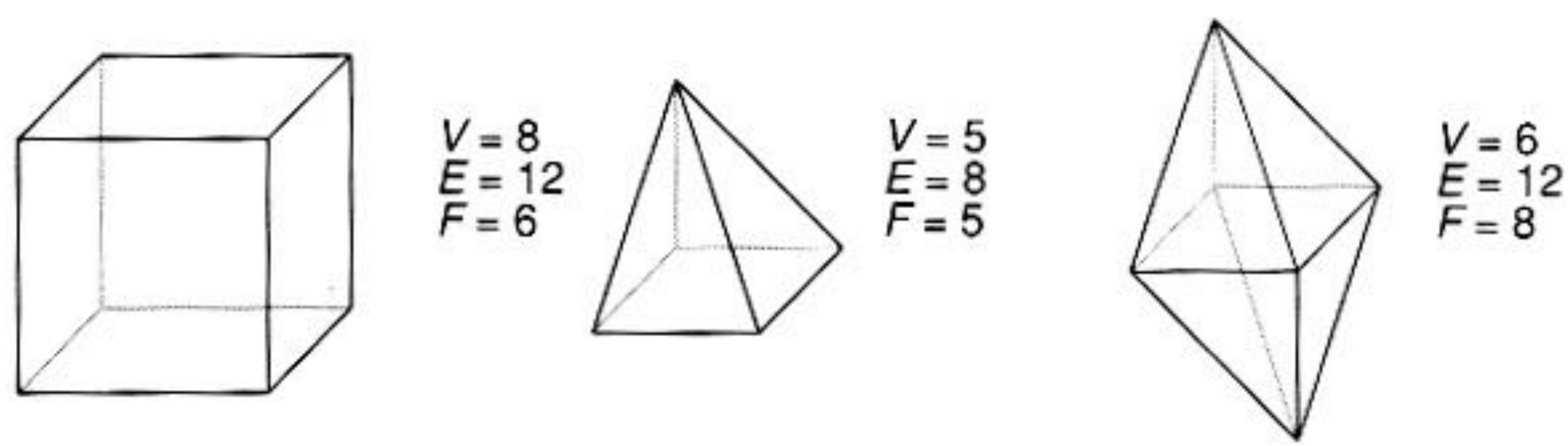


Indexed triangle set



Estimating storage space

- **n_T = #tris; n_V = #verts; n_E = #edges**
- **Euler: $n_V - n_E + n_T = 2$ for a simple closed surface**
  - and in general sums to small integer
  - argument for implication that $n_T{:}n_E{:}n_V$ is about 2:3:1



[Foley et al.]

Indexed triangle set

- **array of vertex positions**
  - float[n_V][3]: 12 bytes per vertex
    - (3 coordinates x 4 bytes) per vertex
- **array of triples of indices (per triangle)**
  - int[n_T][3]: about 24 bytes per vertex
    - 2 triangles per vertex (on average)
    - (3 indices x 4 bytes) per triangle
- **total storage: 36 bytes per vertex (factor of 2 savings)**
- **represents topology and geometry separately**
- **finding neighbors is at least well defined**

Data on meshes

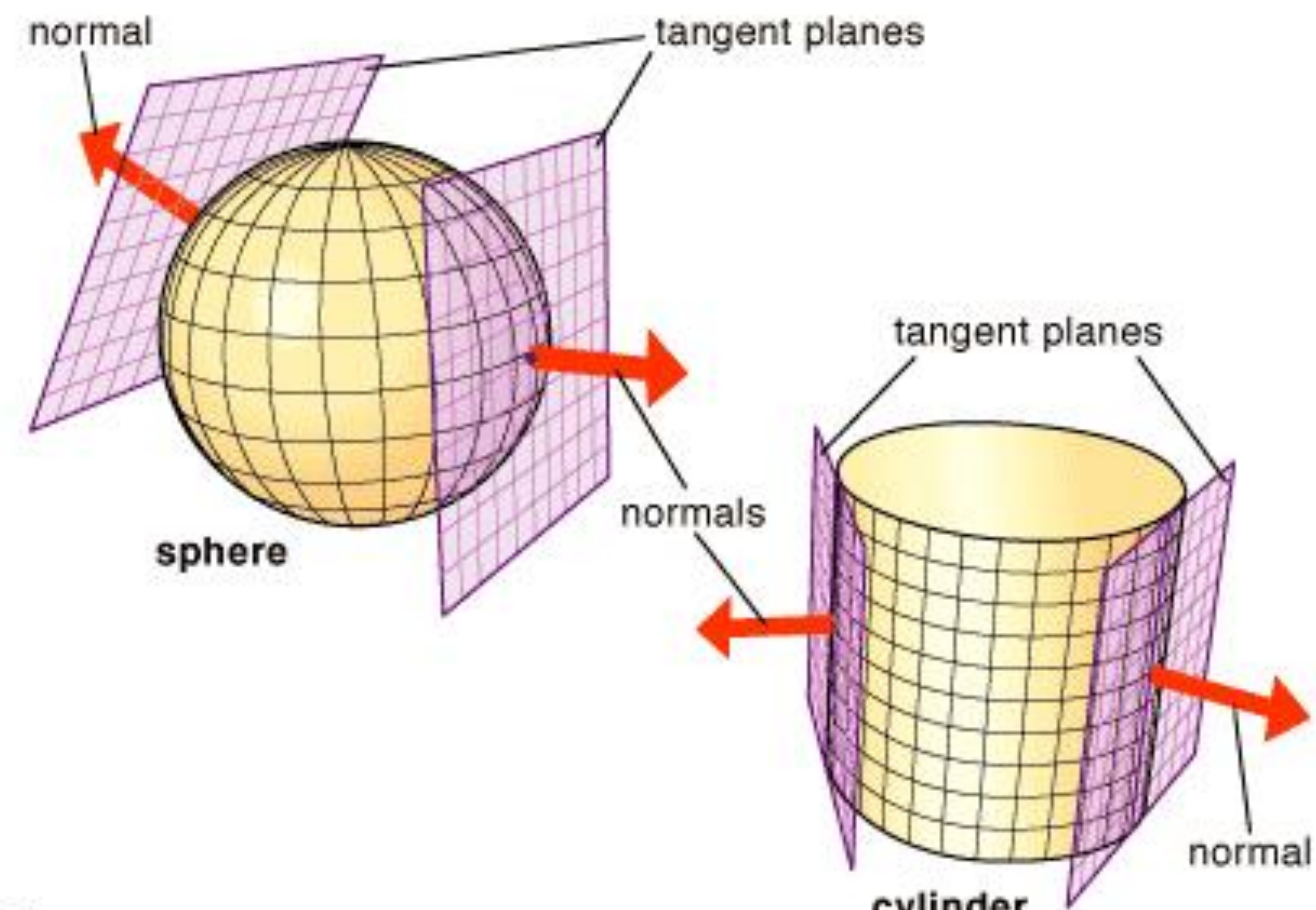
- **Often need to store additional information besides just the geometry**
- **Can store additional data at faces, vertices, or edges**
- **Examples**
  – colors stored on faces, for faceted objects
  – information about sharp creases stored at edges
  – any quantity that varies *continuously* (without sudden changes, or *discontinuities*) gets stored at vertices

Key types of vertex data

- **Surface normals**
  - when a mesh is approximating a curved surface, store normals at vertices
- **Texture coordinates**
  - 2D coordinates that tell you how to paste images on the surface
- **Positions**
  - at some level this is just another piece of data
  - position varies continuously between vertices

Differential geometry 101

- **Tangent plane**
  - at a point on a smooth surface in 3D, there is a unique plane tangent to the surface, called the *tangent plane*
- **Normal vector**
  - vector perpendicular to a surface (that is, to the tangent plane)
  - only unique for smooth surfaces (not at corners, edges)



Surface parameterization

- **A surface in 3D is a two-dimensional thing**
- **Sometimes we need 2D coordinates for points on the surface**
- **Defining these coordinates is *parameterizing* the surface**
- **Examples:**
  - cartesian coordinates on a rectangle (or other planar shape)
  - cylindrical coordinates (θ, y) on a cylinder
  - latitude and longitude on the Earth's surface
  - spherical coordinates (θ, ϕ) on a sphere

Example: unit sphere

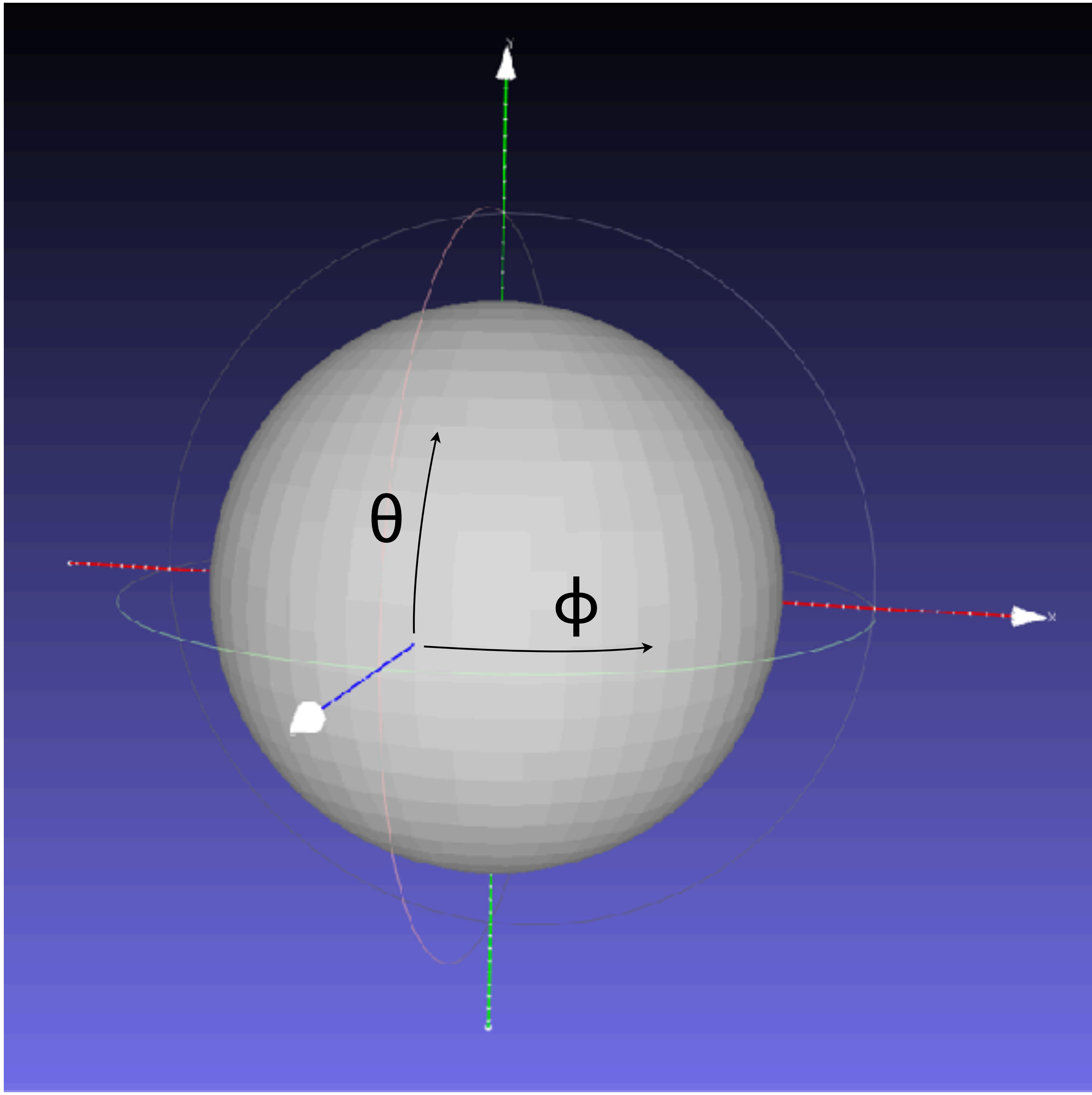
- **position:**

$$x = \cos\theta \sin\phi$$

$$y = \sin\theta$$

$$z = \cos\theta \cos\phi$$

- **normal is position (easy!)**

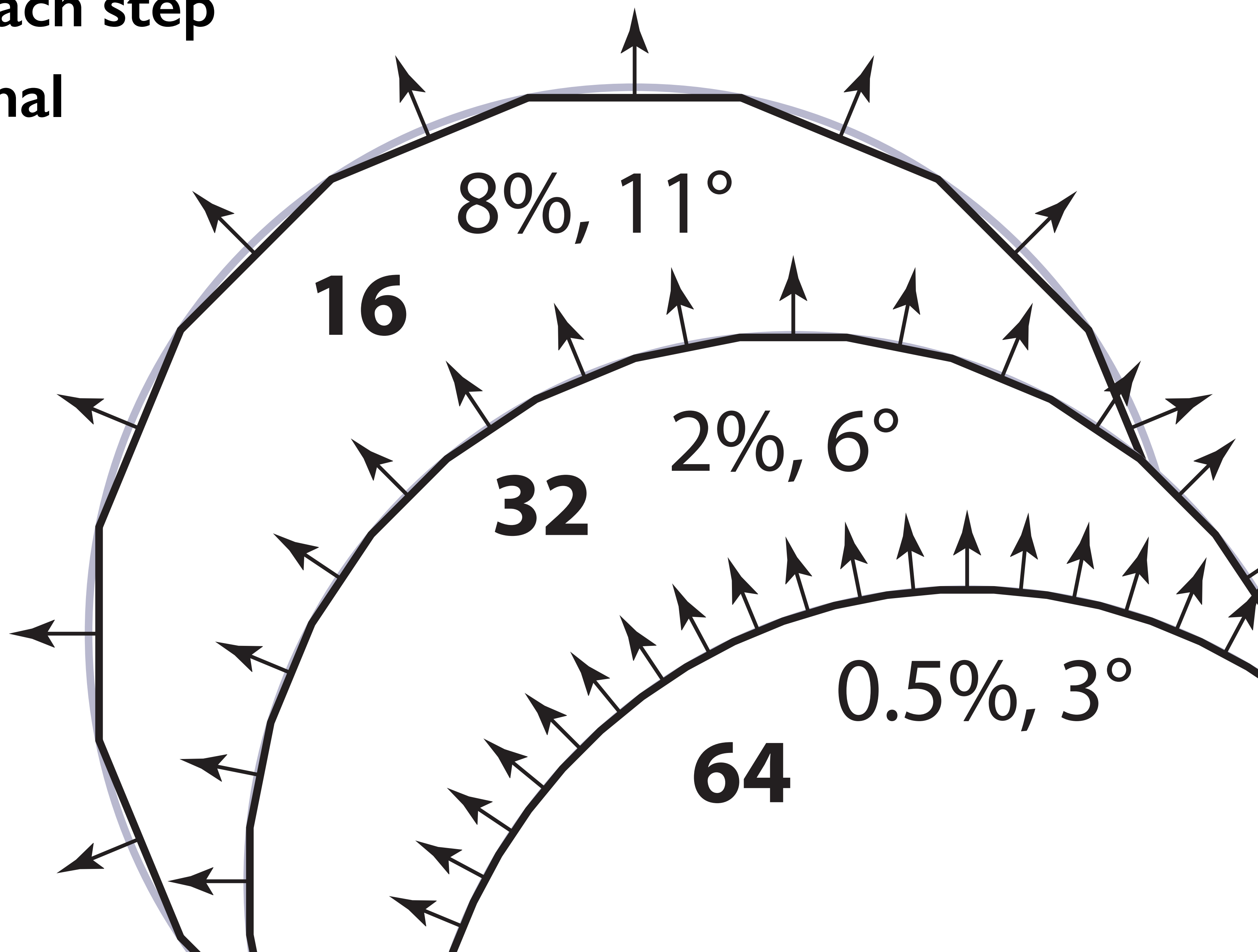


How to think about vertex normals

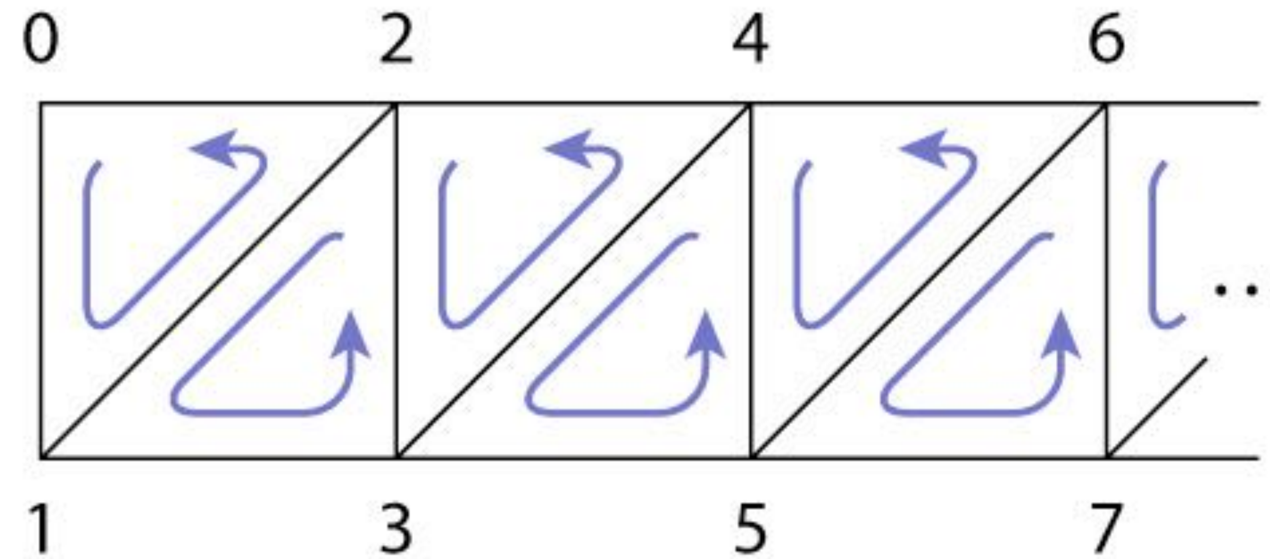
- **Piecewise planar approximation converges pretty quickly to the smooth geometry as the number of triangles increases**
  - for mathematicians: error is $O(h^2)$
- **But the surface normals don't converge so well**
  - normal is constant over each triangle, with discontinuous jumps across edges
  - for mathematicians: error is only $O(h)$
- **Better: store the "real" normal at each vertex, and *interpolate* to get normals that vary gradually across triangles**

Interpolated normals—2D example

- **Approximating circle with increasingly many segments**
- **Max error in position error drops by factor of 4 at each step**
- **Max error in normal only drops by factor of 2**

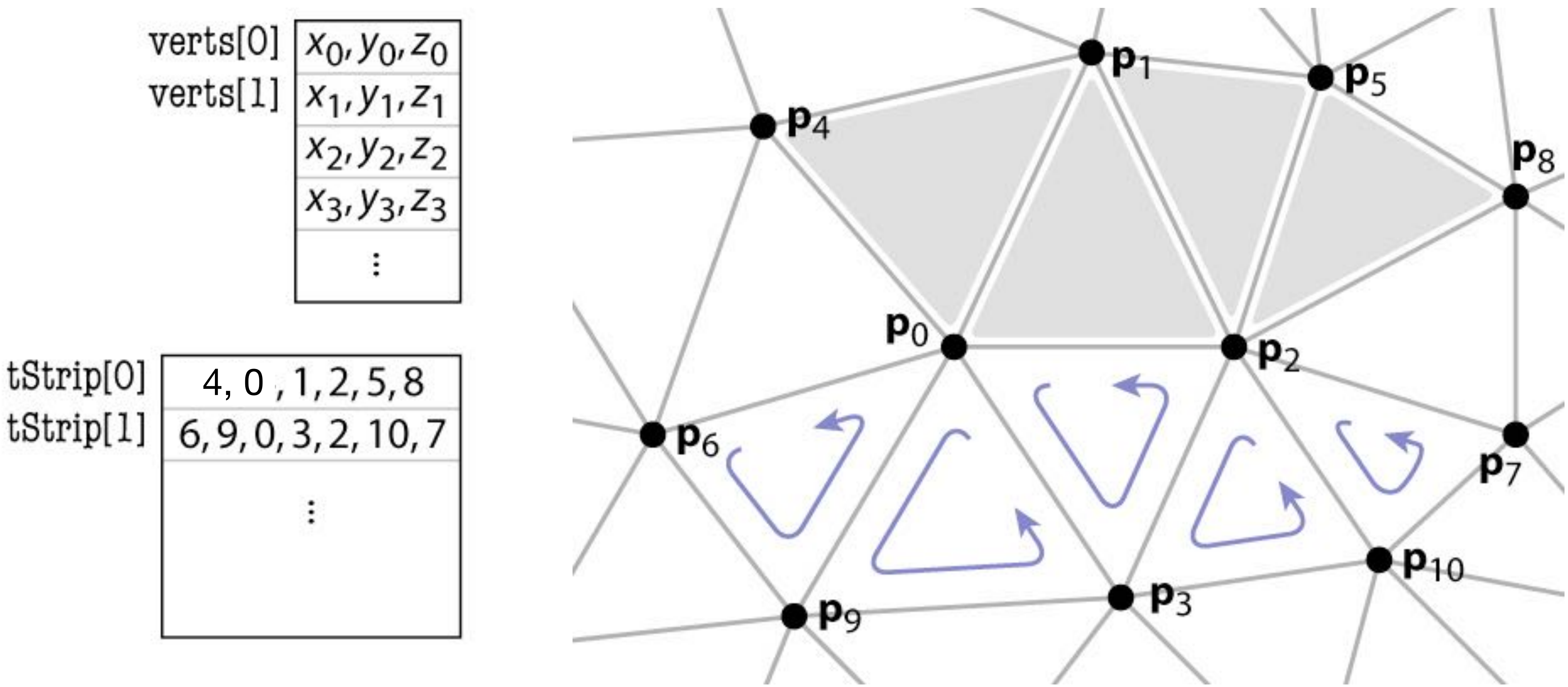


Triangle strips



- **Take advantage of the mesh property**
  - each triangle is usually adjacent to the previous
  - let every vertex create a triangle by reusing the second and third vertices of the previous triangle
  - every sequence of three vertices produces a triangle (but not in the same order)
  - e. g., 0, 1, 2, 3, 4, 5, 6, 7, … leads to
    (0 1 2), (2 1 3), (2 3 4), (4 3 5), (4 5 6), (6 5 7), …
  - for long strips, this requires about one index per triangle

Triangle strips

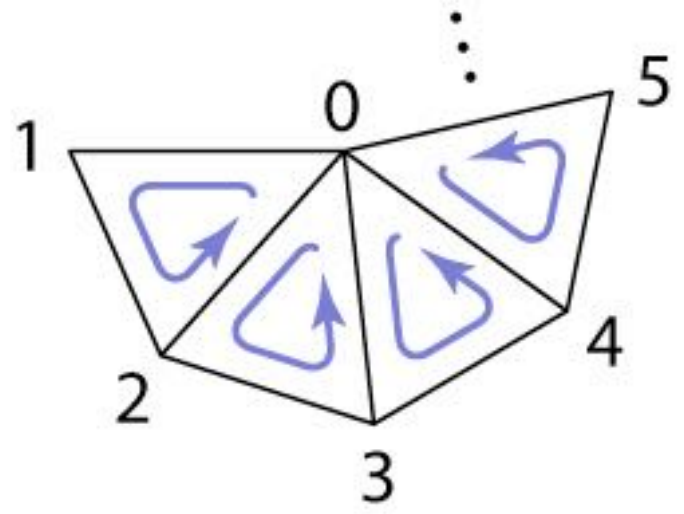


Triangle strips

- **array of vertex positions**
  - float[n_V][3]: 12 bytes per vertex
    - (3 coordinates x 4 bytes) per vertex
- **array of index lists**
  - int[n_S][***variable***]: 2 + ***n*** indices per strip
  - on average, (1 + ε) indices per triangle (assuming long strips)
    - 2 triangles per vertex (on average)
    - about 4 bytes per triangle (on average)
- **total is 20 bytes per vertex (limiting best case)**
  - factor of 3.6 over separate triangles; 1.8 over indexed mesh

Triangle fans

- **Same idea as triangle strips, but keep oldest rather than newest**
  - every sequence of three vertices produces a triangle
  - e. g., 0, 1, 2, 3, 4, 5, … leads to
    (0 1 2), (0 2 3), (0 3 4), (0 4 5), …
  - for long fans, this requires about one index per triangle
- **Memory considerations exactly the same as triangle strip**

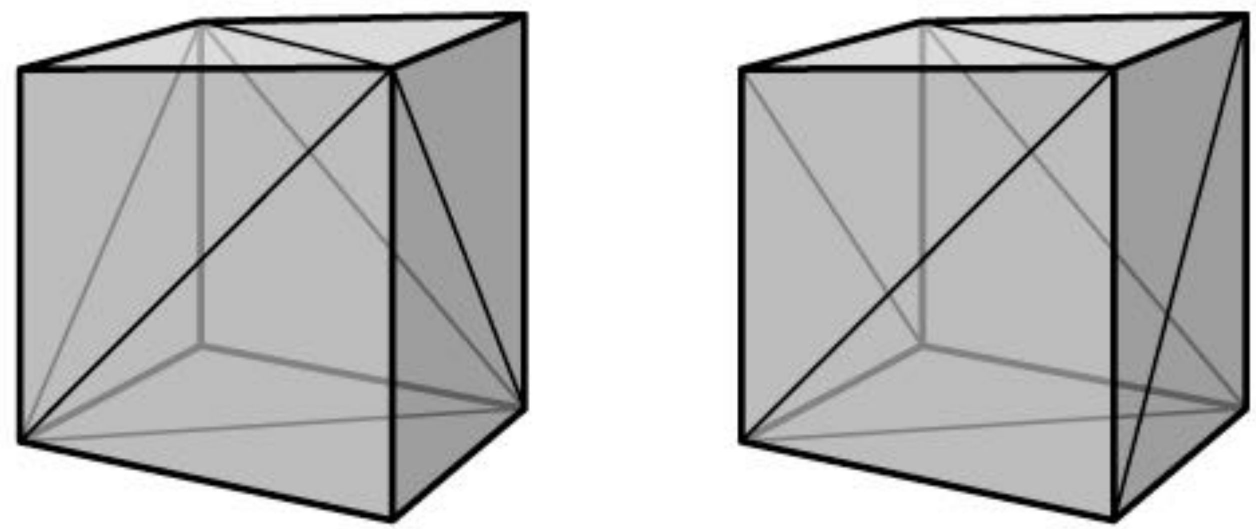


Validity of triangle meshes

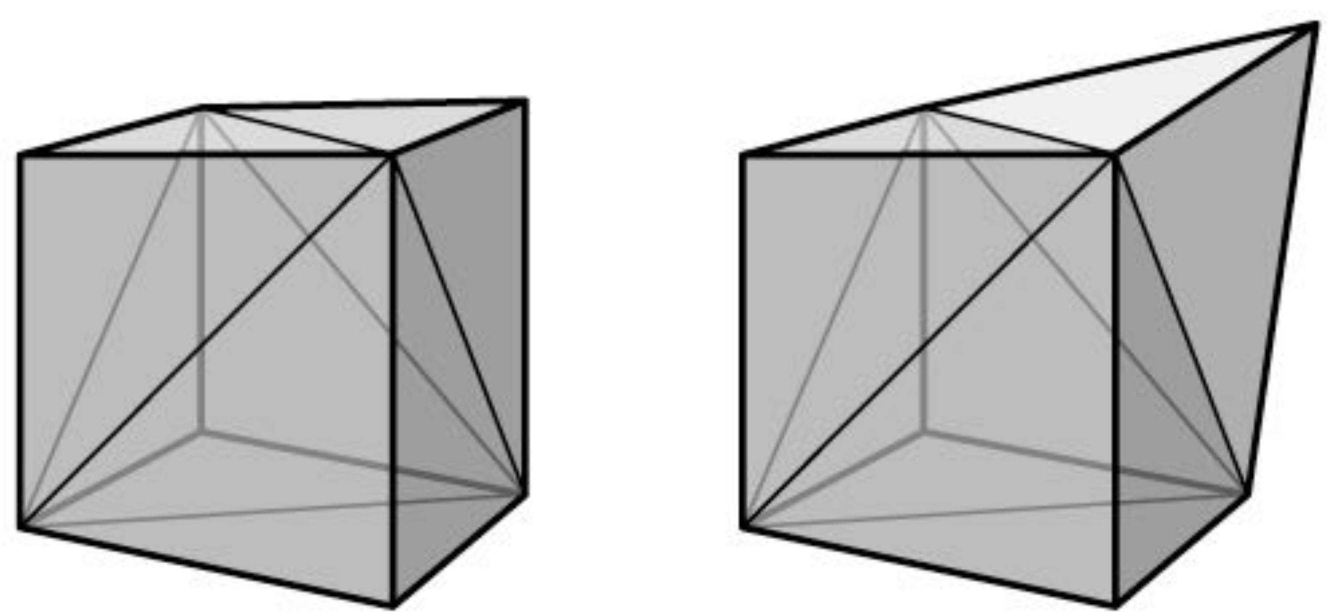
- **in many cases we care about the mesh being able to bound a region of space nicely**
- **in other cases we want triangle meshes to fulfill assumptions of algorithms that will operate on them (and may fail on malformed input)**
- **two completely separate issues:**
  - **mesh topology**: how the triangles are connected (ignoring the positions entirely)
  - **geometry**: where the triangles are in 3D space

Topology/geometry examples

- **same geometry, different mesh topology:**

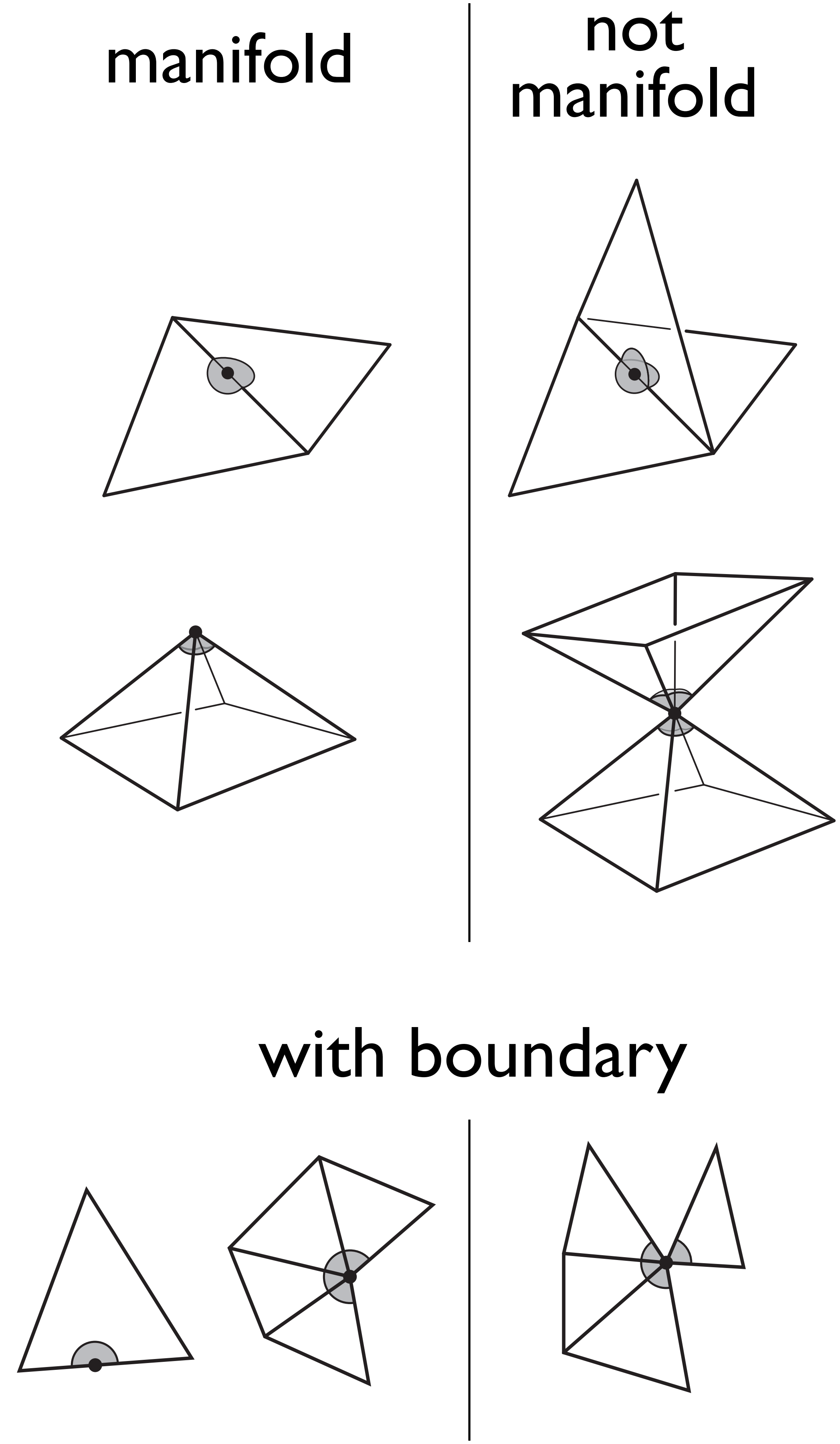


- **same mesh topology, different geometry:**



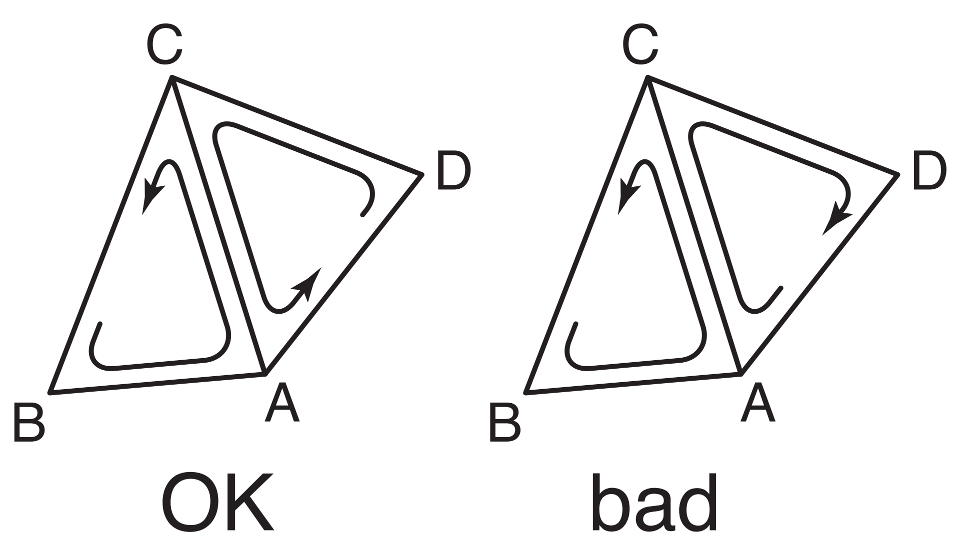
Topological validity

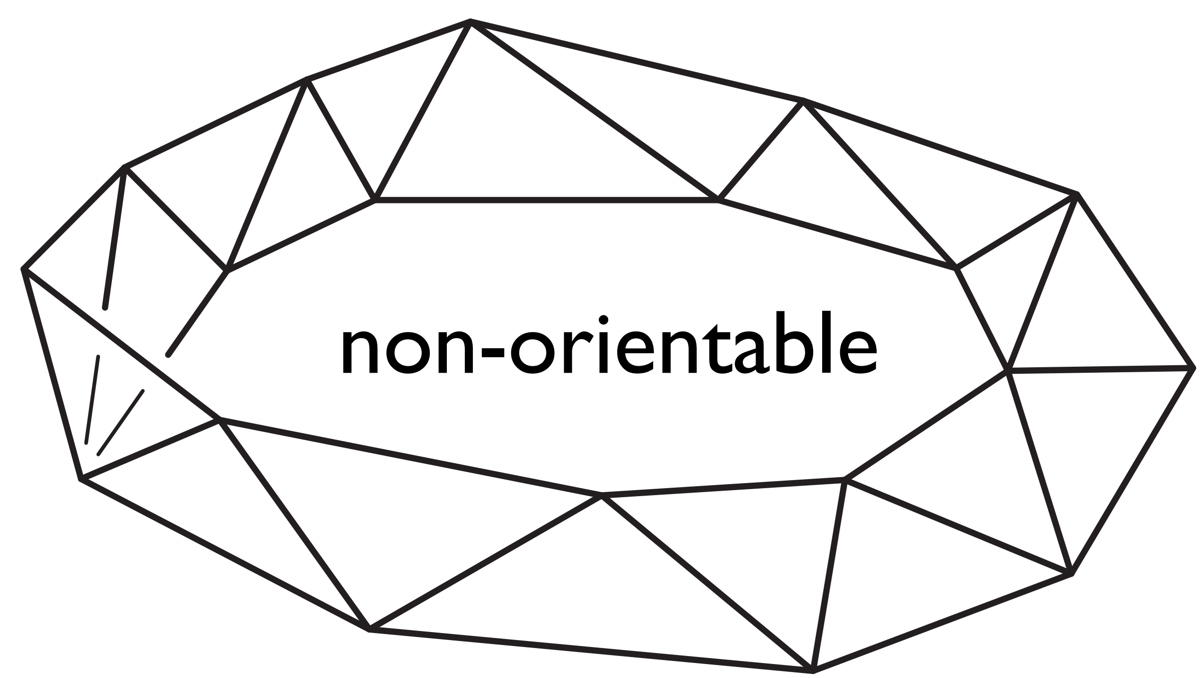
- **strongest property: be a manifold**
  - this means that no points should be "special"
  - interior points are fine
  - edge points: each edge must have exactly 2 triangles
  - vertex points: each vertex must have one loop of triangles
- **slightly looser: manifold with boundary**
  - weaken rules to allow boundaries



Topological validity

- **Consistent orientation**
  - Which side is the "front" or "outside" of the surface and which is the "back" or "inside?"
  - rule: you are on the outside when you see the vertices in counter-clockwise order
  - in mesh, neighboring triangles should agree about which side is the front!
  - caution: not always possible





Geometric validity

- **generally want non-self-intersecting surface**
- **hard to guarantee in general**
  - because far-apart parts of mesh might intersect