Surfaces and Solids

CS 4620 Lecture 30
Administration

• A4 and PPA2 demos
  – Today

• A5 due on Friday

• Dreamworks visiting Thu/Fri

• Rest of class
  – Surfaces, Animation, Rendering
Modeling in 3D

• Representing subsets of 3D space
  – volumes (3D subsets)
  – surfaces (2D subsets)
  – curves (1D subsets)
  – points (0D subsets)
Representing geometry

• In order of dimension…
• Points: trivial case
• Curves
  – normally use parametric representation
  – line—just a point and a vector (like ray in ray tracer)
    • polylines (approximation scheme for drawing)
  – more general curves: usually use splines
• \( p(t) \) is from \( \mathbb{R} \) to \( \mathbb{R}^3 \)
• \( p \) is defined by piecewise polynomial functions
Representing geometry

- Surfaces
  - implicit and parametric representations both useful
  - example: plane
    - implicit: vector from point perpendicular to normal
    - parametric: point plus scaled tangents
  - example: sphere
    - implicit: distance from center equals $r$
    - parametric: write out in spherical coordinates
      - messiness of parametric form not unusual
Specific surface representations

• Parametric spline surfaces
  – extrusions
  – surfaces of revolution
  – generalized cylinders
  – spline patches
From curves to surfaces

• So far have discussed spline curves in 2D
  – it turns out that this already provides mathematical machinery for several ways of building curved surfaces
• Building surfaces from 2D curves
  – extrusions and surfaces of revolution
• Building surfaces from 2D and 3D curves
  – generalized swept surfaces
• Building surfaces from spline patches
  – generalizing spline curves to spline patches
Extrusions

• Given a spline curve $C \in \mathbb{R}^2$, define $S \in \mathbb{R}^3$ by
  \[ S = C \times [a, b] \]

• This produces a “tube” with the given cross section
  – Circle: cylinder; “L”: shelf bracket; “I”: I beam

• It is parameterized by the spline $t$ and the interval $[a, b]$
  \[ s(t, s) = [c_x(t), c_y(t), s]^T \]
Surfaces of revolution

- Take a 2D curve and spin it around an axis
- Given curve $c(t)$ in the plane, the surface is defined easily in cylindrical coordinates:

$$s(t, s) = (r, \phi, z) = (c_x(t), s, c_y(t))$$

- the torus is an example in which the curve $c$ is a circle
Swept surfaces

- Surface defined by a cross section moving along a spine
- Simple version: a single 3D curve for spine and a single 2D curve for the cross section
Generalized cylinders

- General swept surfaces
  - varying radius
  - varying cross-section
  - curved axis

[Snyder 1992]
From curves to surface patches

• Curve was sum of weighted 1D basis functions
• Surface is sum of weighted 2D basis functions
  – construct them as separable products of 1D fns.
  – choice of different splines
    • spline type
    • order
    • closed/open (B-spline)
Separable product construction
Bilinear patch

• Simplest case: 4 points, cross product of two linear segments
Bicubic Bézier patch

- Cross product of two cubic Bézier segments

\[ p(u, v) = \sum_{i=0}^{n} \sum_{j=0}^{m} B^n_i(u) B^m_j(v) \ P_{ij} \]
Bicubic Bézier patch

- Cross product of two cubic Bézier segments
  - properties that carry over
    - interpolation at corners, edges
    - tangency at corners, edges
    - convex hull
Biquadratic Bézier patch

- Cross product of quadratic Bézier curves
3x5 Bézier patch

- Cross product of quadratic and quartic Béziers
Cylindrical B-spline surfaces

• Cross product of closed and open cubic B-splines
Approximating spline surfaces

• Like curves, approximate with simple primitives
  – in surface case, triangles or quads
  – quads widely used because they fit in parameter space
• generally eventually rendered as pairs of triangles

• adaptive subdivision
  – basic approach: recursively test flatness
• if the patch is not flat enough, subdivide into four using curve subdivision twice, and recursively process each piece
  – as with curves, convex hull property is useful for termination testing (and is inherited from the curves)
Approximating spline surfaces

- With adaptive subdivision, must take care with cracks
  - (at the boundaries between degrees of subdivision)
Geri’s Game

- Pixar short film to test subdivision in production
  - Catmull-Clark (quad mesh) surfaces
  - complex geometry
  - extensive use of creases
  - subdivision surfaces to support cloth dynamics

[DeRose et al. SIGGRAPH 1998]
Specific surface representations

- Subdivision surfaces
  - based on polygon meshes (quads or triangles)
  - rules for subdividing surface by adding new vertices
  - converges to continuous limit surface
Subdivision curves

- Key idea: let go of the polynomials as the definition of the curve, and let the refinement rule define the curve
- Curve is defined as the \textit{limit of a refinement process}
  - properties of curve depend on the rules
  - some rules make polynomial curves, some don’t
  - complexity shifts from implementations to proofs
Subdivision surfaces

Figure 2.2: Example of subdivision for a surface, showing 3 successive levels of refinement. On the left an initial triangular mesh approximating the surface. Each triangle is split into 4 according to a particular subdivision rule (middle). On the right the mesh is subdivided in this fashion once again.