Animation

CS 4620 Lecture 19

What is animation?

- Modeling = specifying shape
 - using all the tools we've seen: hierarchies, meshes, curved surfaces...
- Animation = specifying shape as a function of time
 - just modeling done once per frame?
 - yes, but need smooth, concerted movement

Keyframes in hand-drawn animation

- End goal: a drawing per frame, with nice smooth motion
- "Straight ahead" is drawing frames in order (using a lightbox to see the previous frame or frames)
 - but it is hard to get
 a character to land
 at a particular pose
 at a particular time
- Instead use key frames to plan out the action
 - draw important poses first, then fill in the in-betweens

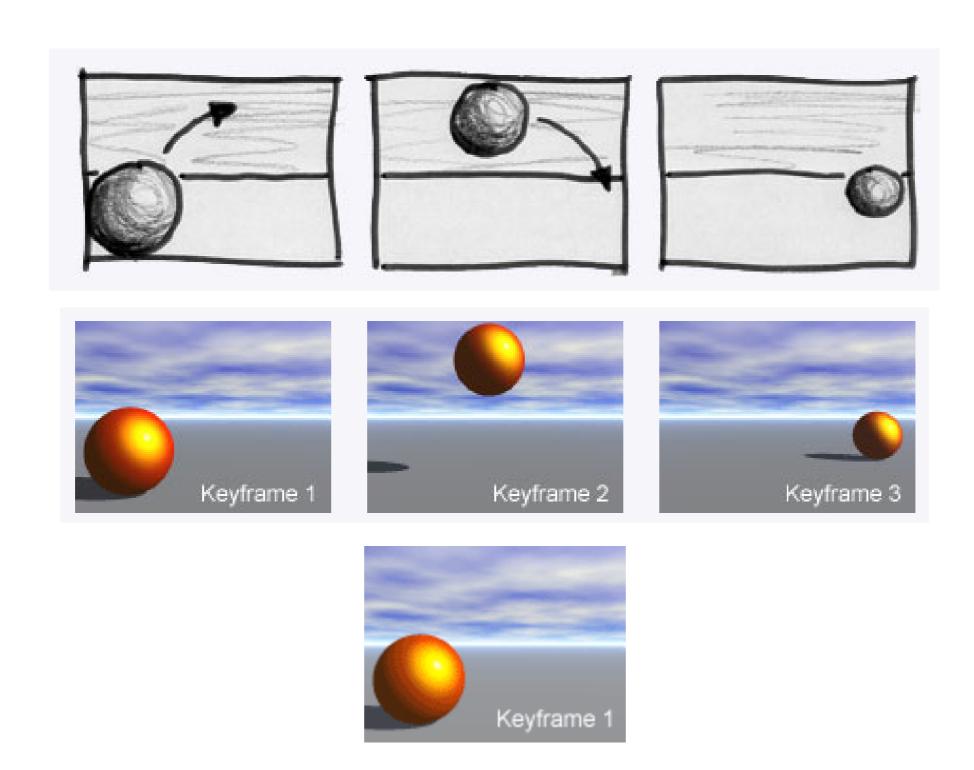
Keyframes in computer animation

- Just as with hand-drawn animation, adjusting the model from scratch for every frame would be tedious and difficult
- Same solution: animator establishes the keyframes, software fills in the in-betweens
- Two key ideas of computer animation:
 - create high-level controls for adjusting geometry
 - interpolate these controls over time between keyframes

The most basic animation control

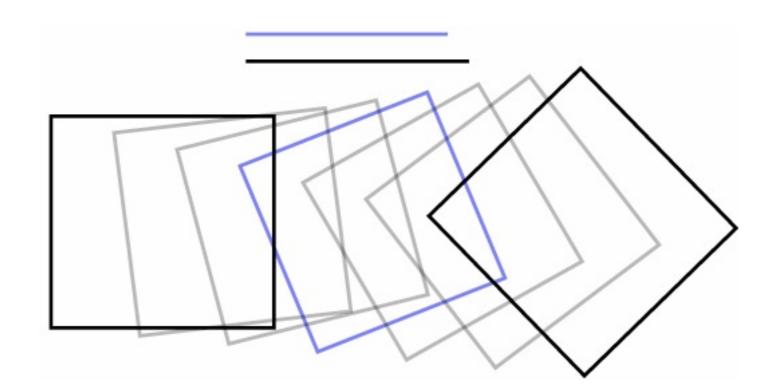
- Affine transformations position things in modeling
- Time-varying affine transformations move things around in animation
- A hierarchy of time-varying transformations is the main workhorse of animation
 - and the basic framework within which all the more sophisticated techniques are built

Keyframe animation



Interpolating transformations

- Move a set of points by applying an affine transformation
- How to animate the transformation over time?
 - Interpolate the matrix entries from keyframe to keyframe?
 - This is fine for translations but bad for rotations



What is a rotation?

- Think of the set of possible orientations of a 3D object
 - you get from one orientation to another by rotating
 - if we agree on some starting orientation, rotations and orientations are pretty much the same thing
- It is a smoothly connected three-dimensional space
 - how can you tell? For any orientation, I can make a small rotation around any axis (pick axis = 2D, pick angle = ID)
- This set is a subset of linear transformations called SO(3)
 - O for orthogonal, S for "special" (determinant +1), 3 for 3D

Calculating with rotations

Representing rotations with numbers requires a function

$$f: \mathbb{R}^n \to SO(3)$$

- The situation is analogous to representing directions in 3space
 - there we are dealing with the set S^2 , the two-dimensional sphere (I mean the sphere is a 2D surface)
 - like SO(3) it is very symmetric; no directions are specially distinguished

Warm-up: spherical coordinates

- We can use latitude and longitude to parameterize the 2-sphere (aka. directions in 3D), but with some annoyances
 - the poles are special, and are represented many times
 - if you are at the pole, going East does nothing
 - near the pole you have to change longitude a lot to get anywhere
 - traveling along straight lines in (latitude, longitude) leads to some pretty weird paths on the globe
 - you are standing one mile from the pole, facing towards it; to get to the point 2 miles ahead of you the map tells you to turn right and walk 3.14 miles along a latitude line...
 - Conclusion: use unit vectors instead

Warm-up: unit vectors

- When we want to represent directions we use unit vectors: points that are literally on the unit sphere in R³
 - now no points are special
 - every point has a unique representation
 - equal sized changes in coordinates are equal sized changes in direction
- Down side: one too many coordinates
 - have to maintain normalization
 - but normalize() is a simple and easy operation

Warm-up: interpolating directions

- Interpolating in the space of 3D vectors is well behaved
- Simple computation: interpolate linearly and normalize
 - this is what we do all the time, e.g. with normals for fragment shading

$$\hat{\mathbf{v}}(t) = \text{normalize}((1-t)\mathbf{v}_0 + t\mathbf{v}_1)$$

- but for far-apart endpoints the speed is uneven (faster towards the middle)
- For constant speed: spherical linear interpolation
 - build basis $\{\mathbf v_0, \mathbf w\}$ from $\mathbf v_0$ and $\mathbf v_1 \ \mathbf w = \hat{\mathbf v}_1 (\hat{\mathbf v}_0 \cdot \hat{\mathbf v}_1)\hat{\mathbf v}_0$
 - interpolate angle from 0 to θ

$$\hat{\mathbf{w}} = \mathbf{w}/\|\mathbf{w}\|$$

$$\theta = a\cos(\hat{\mathbf{v}}_0 \cdot \hat{\mathbf{v}}_1)$$

$$\hat{\mathbf{v}}(t) = (\cos t\theta) \,\hat{\mathbf{v}}_0 + (\sin t\theta) \,\hat{\mathbf{w}}$$

Warm-up: rays vs. lines

- The set of directions (unit vectors) describes the set of rays leaving a point
- The set of lines through a point is a bit different
 - no notion of "forward" vs. "backward"
- Would probably still represent using unit vectors
 - but every line has exactly two representations, v and -v

Parameterizing rotations

Euler angles

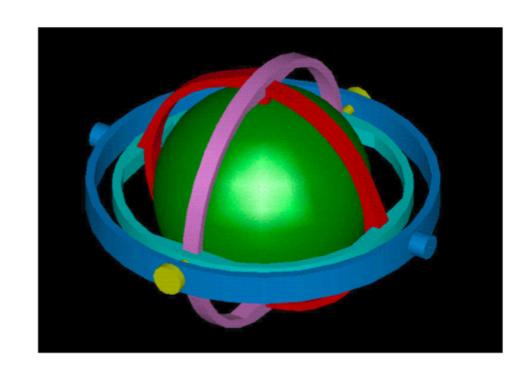
- rotate around x, then y, then z
- nice and simple

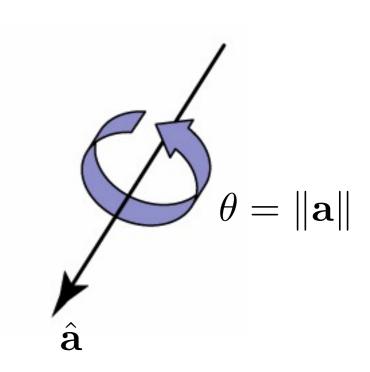
$$f(\alpha, \beta, \gamma) = R_{\mathbf{z}}(\gamma) R_{\mathbf{y}}(\beta) R_{\mathbf{x}}(\alpha)$$

Axis/angle

- specify axis to rotate around, then angle by which to rotate
- multiply axis and angle to get a more compact form

$$f(\mathbf{a}) = R_{\hat{\mathbf{a}}}(\|\mathbf{a}\|)$$





Problems

- Euler angles
 - gimbal lock (saw this before)
 - some rotations have many representations
- Axis/angle
 - with separate rotation angle, multiple representations for identity rotation
 - even with combined rotation angle, making small changes near 180 degree rotations requires larger changes to parameters
- These resemble the problems with polar coordinates on the sphere
- as with choosing poles, choosing the reference orientation for an object changes how the representation works

Quaternions for Rotation

A quaternion is an extension of complex numbers

$$q = (s, v) = (s, v_1, v_2, v_3)$$

Review of complex numbers

$$z = a + bi$$

$$z' = a - bi$$

$$||z|| = \sqrt{z \cdot z'} = \sqrt{a^2 + b^2}$$

Complex numbers to quaternions

 Rather than one imaginary unit i, there are three such symbols i, j, and k, with the properties

$$i^2 + j^2 + k^2 = ijk = -1$$

Multiplication of these units acts like the cross product

$$ij = k$$
 $ji = -k$
 $jk = i$ $kj = -i$
 $ki = j$ $ik = -j$

• Combining multiples of *i*, *j*, *k* with a scalar gives the general form of a quaternion:

$$\mathbf{H} = \{ a + bi + cj + dk \mid (a, b, c, d) \in \mathbb{R}^4 \}$$

Complex numbers to quaternions

Like complex numbers, quaternions have conjugates and magnitudes

$$q = a + bi + cj + dk$$

$$\bar{q} = a - bi - cj - dk$$

$$|q| = (q\bar{q})^{\frac{1}{2}} = \sqrt{a^2 + b^2 + c^2 + d^2} = ||(a, b, c, d)||$$

 Also like complex numbers, quaternions have reciprocals of the form

$$q^{-1} = \frac{1}{q} = \frac{\bar{q}}{|q|}$$

Quaternion Properties

Associative

$$q_1(q_2q_3) = q_1q_2q_3 = (q_1q_2)q_3$$

Not commutative

$$q_1q_2 \not\equiv q_2q_1$$

Magnitudes multiply

$$|q_1q_2| = |q_1| |q_2|$$

• For unit quaternions:

$$|q| = 1$$
$$q^{-1} = \bar{q}$$

Unit quaternions

 The set of unit-magnitude quaternions is called the "unit quaternions"

$$S^3 = \{ q \in \mathbf{H} \mid |q| = 1 \}$$

- as a subset of 4D space, it is the unit 3-sphere
- multiplying unit quaternions produces more unit quaternions

$$|q_1| = |q_2| = 1 \implies |q_1q_2| = 1$$

 $q_1, q_2 \in S^3 \implies q_1q_2 \in S^3$

Quaternion as scalar plus vector

• Write q as a pair of a scalar $s \in \mathbf{R}$ and vector $\mathbf{v} \in \mathbf{R}^3$

$$q = a + bi + cj + dk$$

 $q = (s, \mathbf{v}) \text{ where } s = a \text{ and } \mathbf{v} = (b, c, d)$

Multiplication

$$(s_1, \mathbf{v}_1)(s_2, \mathbf{v}_2) = (s_1 s_2 - \mathbf{v}_1 \cdot \mathbf{v}_2, s_1 \mathbf{v}_2 + s_2 \mathbf{v}_1 + \mathbf{v}_1 \times \mathbf{v}_2)$$

- For a unit quaternion, $|s|^2 + ||\mathbf{v}||^2 = 1$
 - so think of these as the sine and cosine of an angle ψ :

$$q = (\cos \psi, \hat{\mathbf{v}} \sin \psi) \text{ or } \cos \psi + \hat{\mathbf{v}} \sin \psi$$

– this is a lot like writing a complex number as $\cos \theta + i \sin \theta$

[Wikimedia Commons user Geek3]

Quaternions and rotations

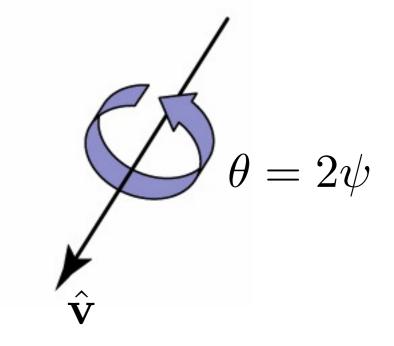
There is a natural association between the unit quaternion

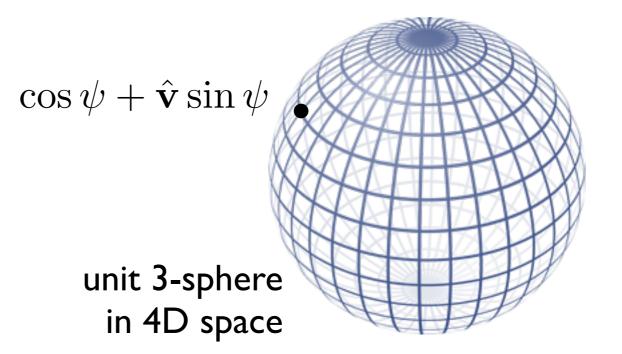
$$\cos \psi + \hat{\mathbf{v}} \sin \psi \in S^3 \subset \mathbf{H}$$

and the 3D axis-angle rotation

$$R_{\hat{\mathbf{v}}}(\theta) \in SO(3)$$

where $\theta = 2\psi$.





Rotation and quaternion multiplication

Represent a point in space by a pure-imaginary quaternion

$$\mathbf{x} = (x, y, z) \in \mathbb{R}^3 \leftrightarrow X = xi + yj + zk \in \mathbf{H}$$

Can compute rotations using quaternion multiplication

$$X_{\text{rotated}} = qX\bar{q}$$

- note that q and -q correspond to the same rotation
- you can verify this is a rotation by multiplying out...
- Multiplication of quaternions corresponds to composition of rotations

$$q_1(q_2X\bar{q_2})\bar{q_1} = (q_1q_2)X(\bar{q_2}\bar{q_1}) = q_1q_2X\bar{q_1}q_2$$

- the quaternion q_1q_2 corresponds to "rotate by q_2 , then rotate by q_1 "

Rotation and quaternion multiplication

If we write a unit quaternion in the form

$$q = \cos \psi + \hat{\mathbf{v}} \sin \psi$$

then the operation

$$X_{\text{rotated}} = qX\bar{q} = (\cos\psi + \hat{\mathbf{v}}\sin\psi)X(\cos\psi - \hat{\mathbf{v}}\sin\psi)$$

is a rotation by 2Ψ around the axis \mathbf{v} .

So an alternative explanation is, "All this algebraic mumbo-jumbo aside, a quaternion is just a slightly different way to encode an axis and an angle in four numbers: rather than the number θ and the unit vector \mathbf{v} , we store the number cos $(\theta/2)$ and the vector sin $(\theta/2)$ \mathbf{v} ."

Unit quaternions and axis/angle

 With this in hand, we can write down a parameterization of 3D rotations using unit quaternions (points on the 3-sphere)

$$f: S^{3} \subset \mathbf{H} \to SO(3)$$

$$: \cos \psi + \hat{\mathbf{v}} \sin \psi \mapsto R_{\hat{\mathbf{v}}}(2\psi)$$

$$: (w, x, y, z) \mapsto \begin{bmatrix} w^{2} + x^{2} - y^{2} - z^{2} & 2(xy - wz) & 2(xz + wy) \\ 2(xy + wz) & w^{2} - x^{2} + y^{2} - z^{2} & 2(yz - wx) \\ 2(xz - wy) & 2(yz + wx) & w^{2} - x^{2} - y^{2} + z^{2} \end{bmatrix}$$

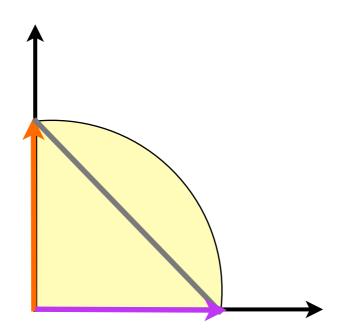
- This mapping is wonderfully uniform:
 - is exactly 2-to-I everywhere
 - has constant speed in all directions
 - has constant Jacobian (does not distort "volume")
 - maps shortest paths to shortest paths
 - and... it comes with a nice multiplication operation!

Why Quaternions?

- Fast, few operations, not redundant
- Numerically stable for incremental changes
- Composes rotations nicely
- Convert to matrices at the end
- Biggest reason: spherical interpolation

Interpolating between quaternions

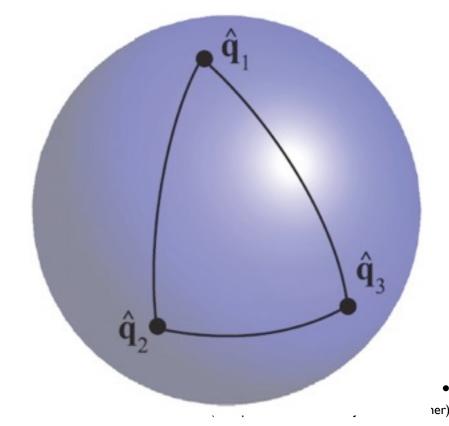
- Why not linear interpolation?
 - Need to be normalized
 - Does not have constant rate of rotation



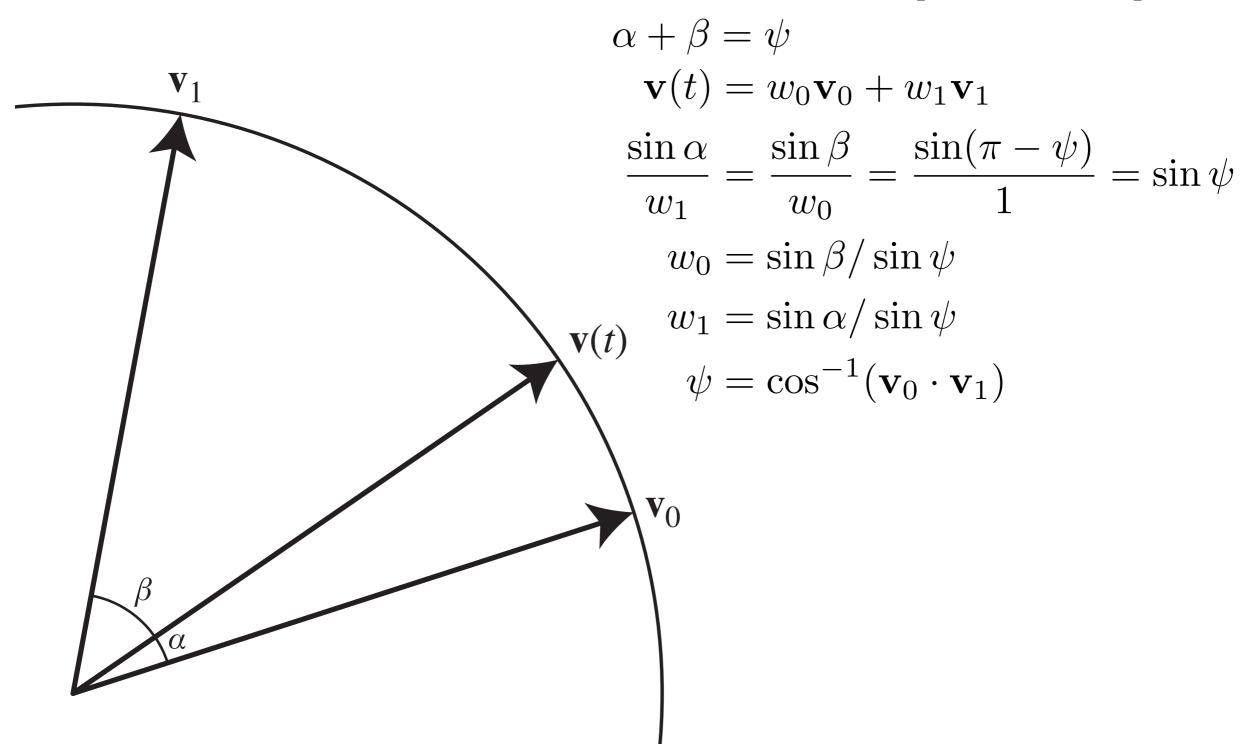
$$\frac{(1-\alpha)x + \alpha y}{||(1-\alpha)x + \alpha y||}$$

Spherical Linear Interpolation

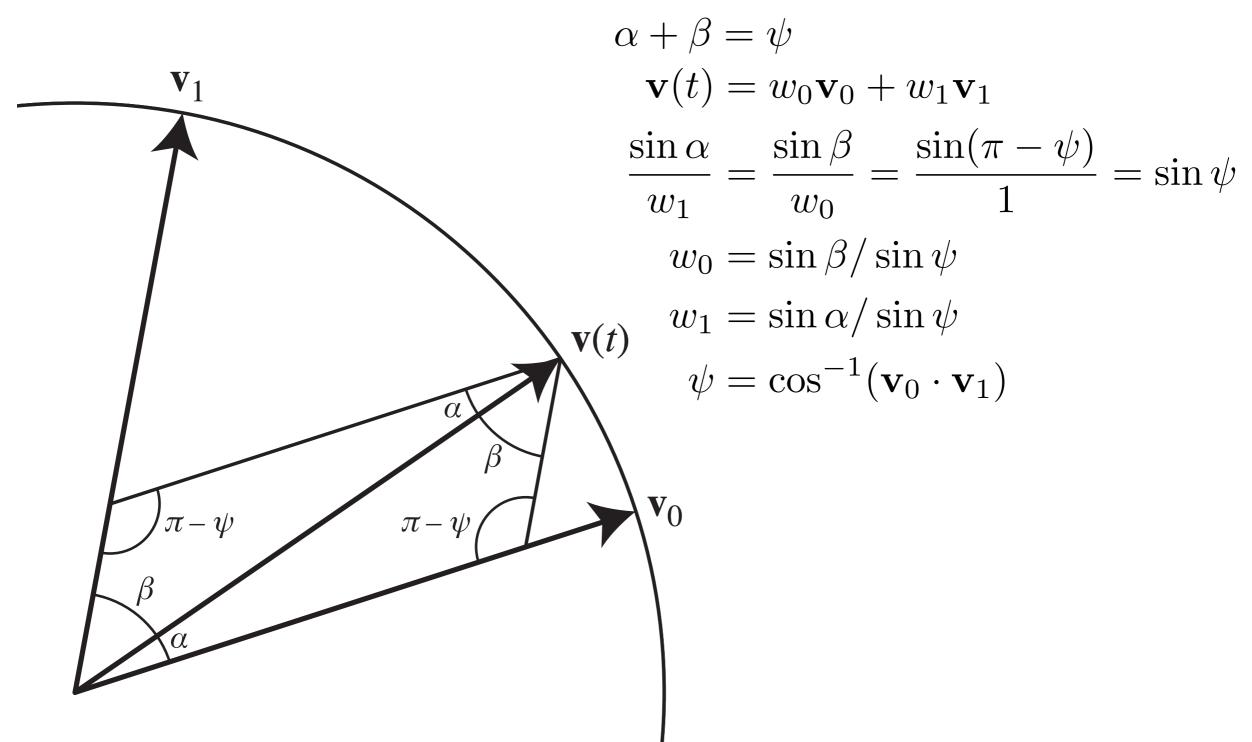
- Intuitive interpolation between different orientations
 - Nicely represented through quaternions
 - Useful for animation
 - Given two quaternions, interpolate between them
 - Shortest path between two points on sphere
 - Geodesic, on Great Circle



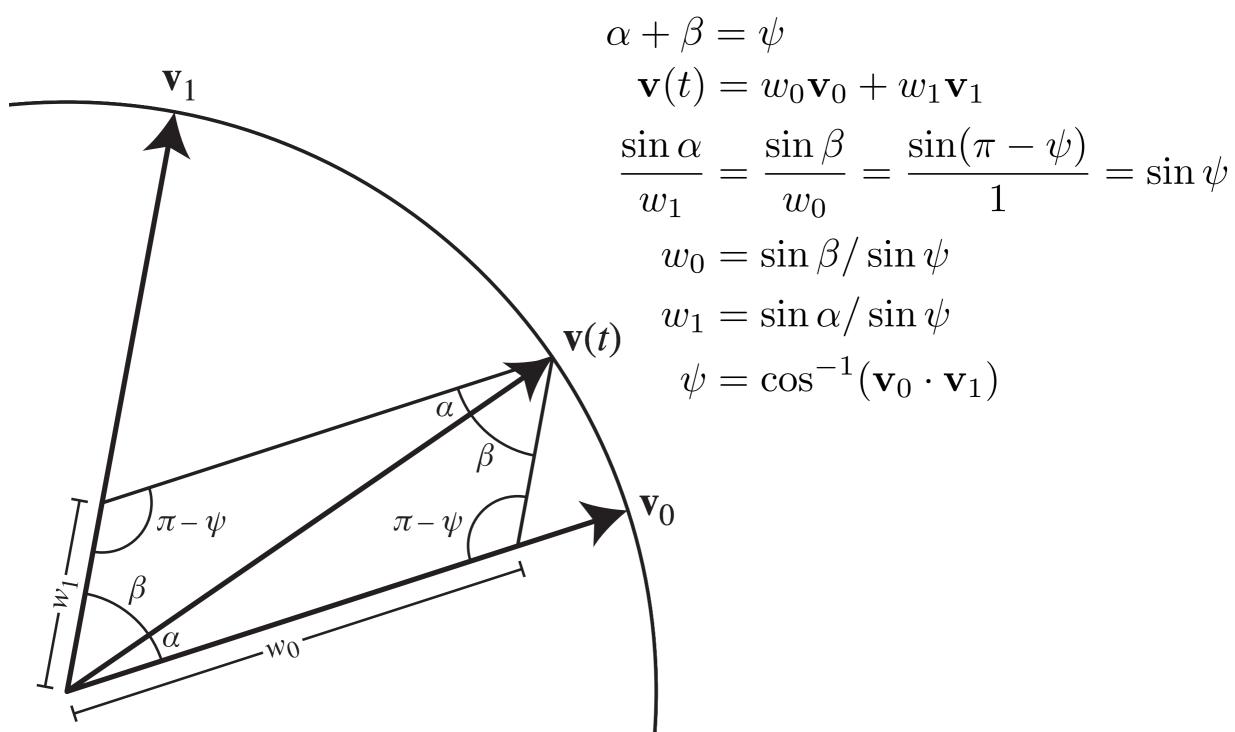
Spherical linear interpolation ("slerp")



Spherical linear interpolation ("slerp")



Spherical linear interpolation ("slerp")



Quaternion Interpolation

- Spherical linear interpolation naturally works in any dimension
- Traverses a great arc on the sphere of unit quaternions
 - Uniform angular rotation velocity about a fixed axis

$$\psi = \cos^{-1}(q_0 \cdot q_1)$$

$$q(t) = \frac{q_0 \sin(1 - t)\psi + q_1 \sin t\psi}{\sin \psi}$$

Practical issues

- When angle gets close to zero, estimation of ψ is inaccurate
 - slerp naturally approaches linear interpolation for small Ψ
 - so switch to linear interpolation when $q_0 \approx q_1$.
- q is same rotation as –q
 - if $q_0 \cdot q_1 > 0$, slerp between them
 - else, slerp between q_0 and $-q_1$

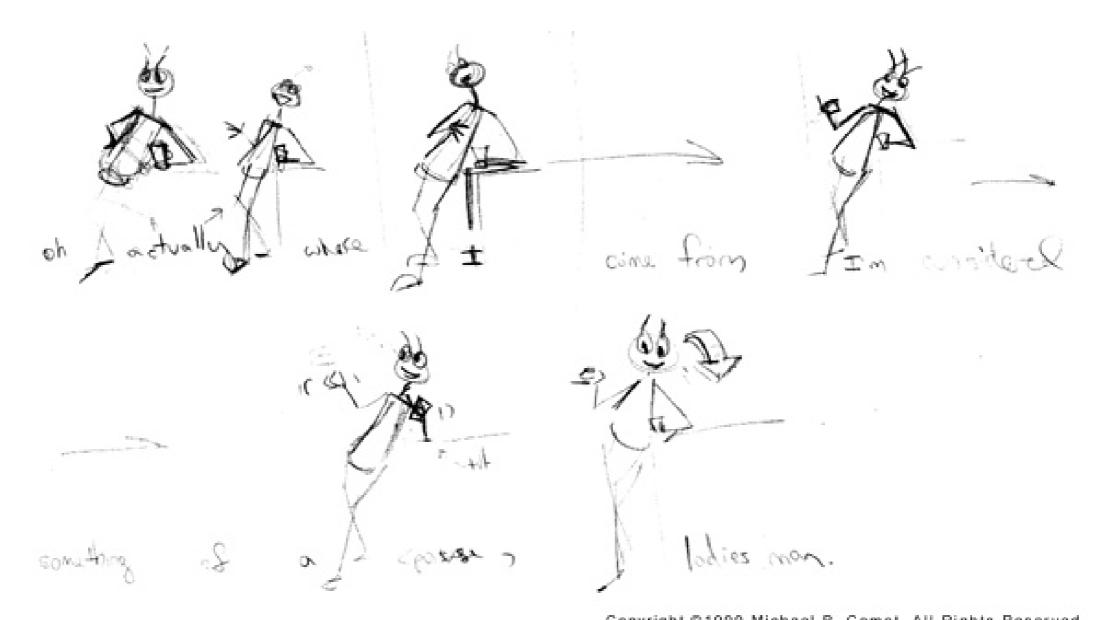
Animation

- Industry production process leading up to animation
- What animation is
- How animation works (very generally)
- Artistic process of animation
- Further topics in how it works

Approaches to animation

- Straight ahead
 - Draw/animate one frame at a time
 - Can lead to spontaneity, but is hard to get exactly what you want
- Pose-to-pose
 - Top-down process:
 - Plan shots using storyboards
 - Plan key poses first
 - Finally fill in the in-between frames

Pose-to-pose animation planning

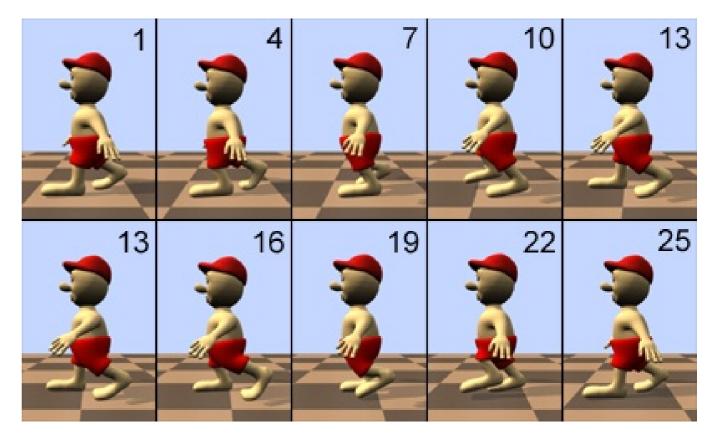


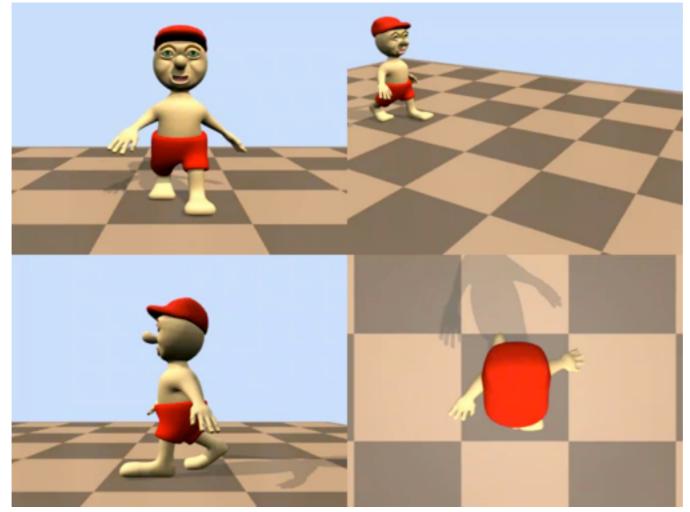
- First work out poses that are key to the story
- Next fill in animation in between

Keyframe animation

- Keyframing is the technique used for pose-to-pose animation
 - Head animator draws key poses—just enough to indicate what the motion is supposed to be
 - Assistants do "in-betweening" and draws the rest of the frames
 - In computer animation substitute "user" and "animation software"
 - Interpolation is the principal operation

Walk cycle

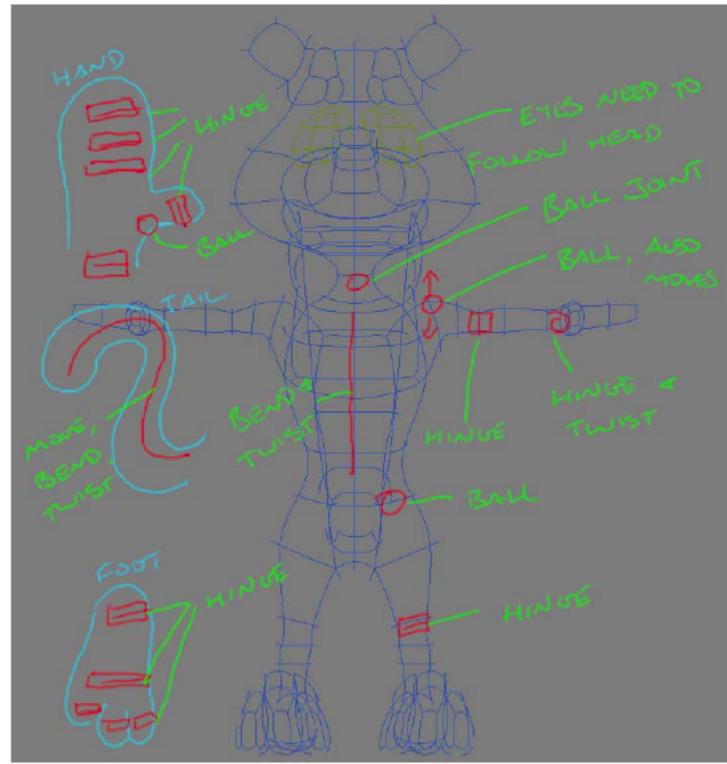




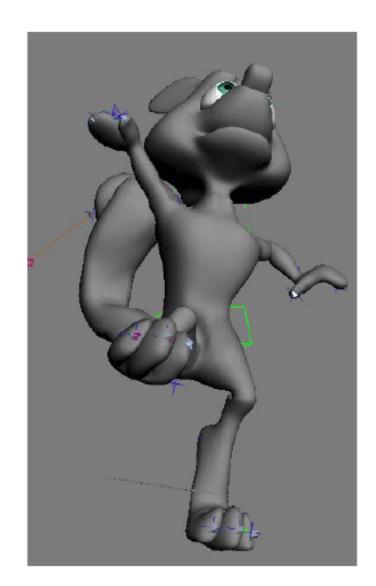
Controlling geometry conveniently

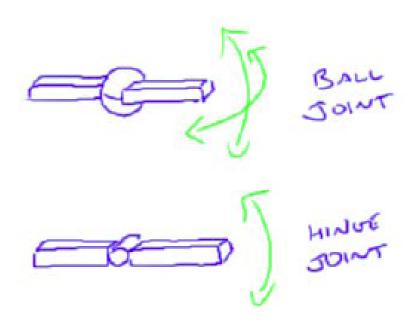
- Could animate by moving every control point at every keyframe
 - This would be labor intensive
 - It would also be hard to get smooth, consistent motion
- Better way: animate using smaller set of meaningful degrees of freedom (DOFs)
 - Modeling DOFs are inappropriate for animation
 - E.g. "move one square inch of left forearm"
 - Animation DOFs need to be higher level
 - E.g. "bend the elbow"

Character with DOFs

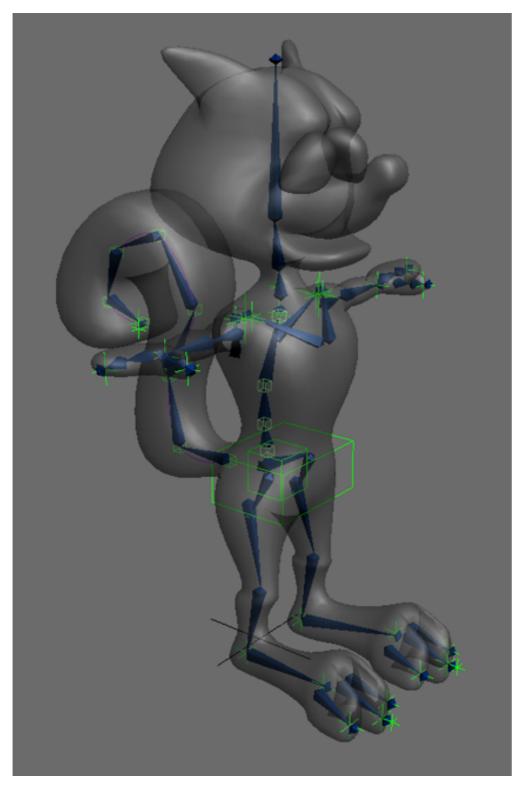


A visual description of the possible movements for the squirrel





Rigged character



- Surface is deformed by a set of bones
- Bones are in turn controlled by a smaller set of controls
- The controls are useful, intuitive DOFs for an animator to use

The artistic process of animation

- What are animators trying to do?
 - Important to understand in thinking about what tools they need
- Basic principles are universal across media
 - 2D hand-drawn animation
 - 2D and computer animation
 - 3D computer animation
- Widely cited set of principles laid out by Frank Thomas and Ollie Johnston in The Illusion of Life (1981)
- The following slides follow Michael Comet's examples: www.comet-cartoons.com

Animation principles: timing

- Speed of an action is crucial to the impression it makes
 - examples with same keyframes, different times:



60 fr: looking around

30 fr:"no"

5 fr: just been hit

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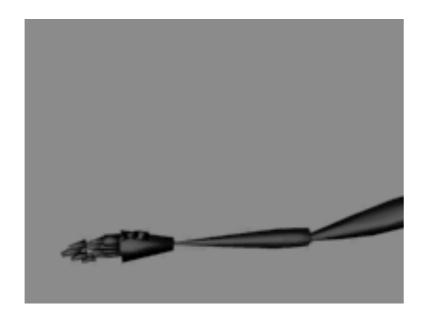
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Animation principles: ease in/out

- Real objects do not start and stop suddenly
 - animation parameters shouldn't either



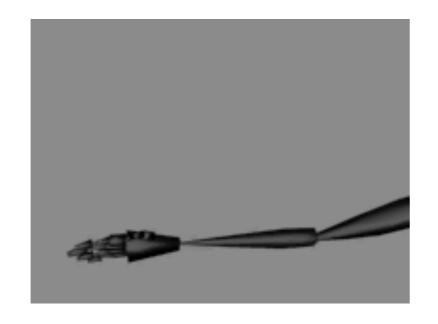
straight linear interp.

ease in/out

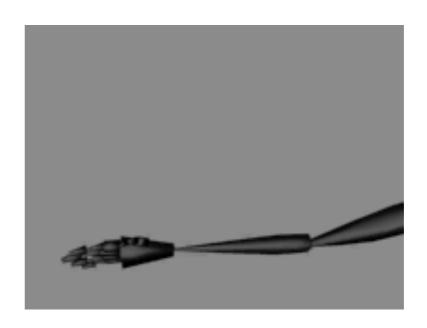
 a little goes a long way (just a few frames acceleration or deceleration for "snappy" motions)

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ease in/out

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Animation principles: moving in arcs

- Real objects also don't move in straight lines
 - generally curves are more graceful and realistic



Animation principles: moving in arcs

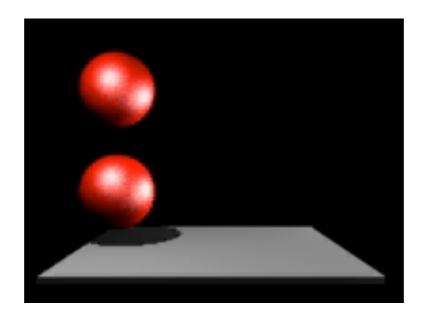
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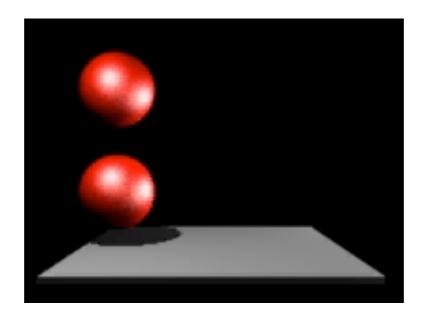
Animation principles: anticipation

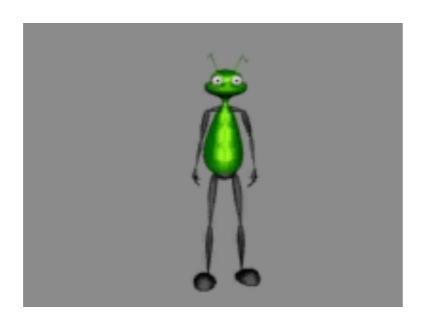
Most actions are preceded by some kind of "wind-up"

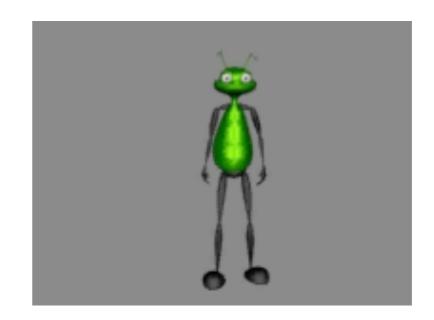


Animation principles: anticipation

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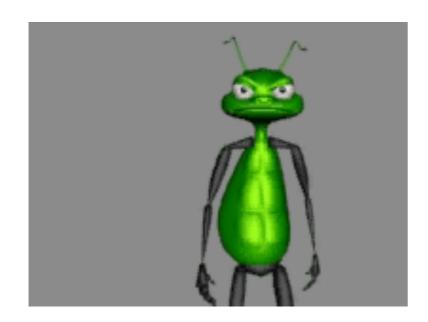






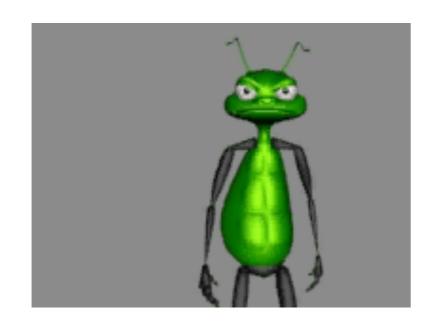
Animation principles: exaggeration

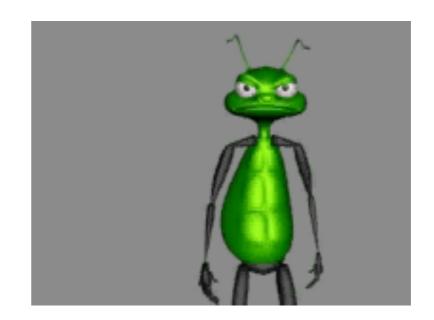
- Animation is not about exactly modeling reality
- Exaggeration is very often used for emphasis



Animation principles: exaggeration

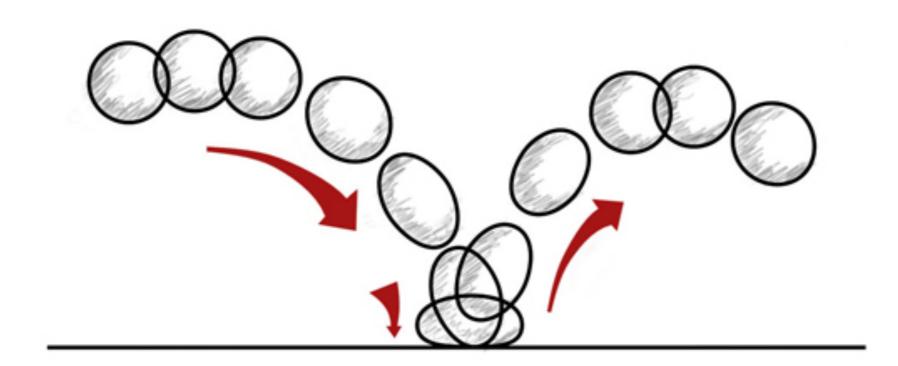
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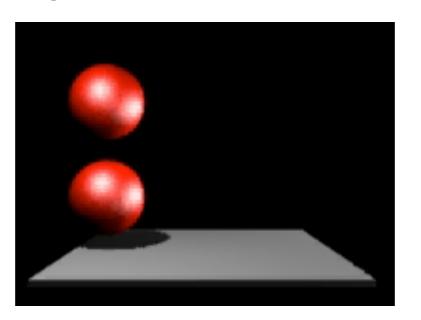
Animation principles: squash & stretch

- Objects do not remain perfectly rigid as they move
- Adding stretch with motion and squash with impact:
 - models deformation of soft objects
 - indicates motion by simulating exaggerated "motion blur"



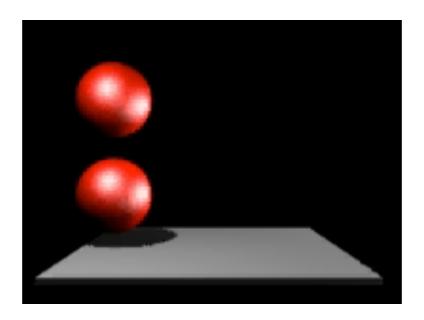
Animation principles: follow through

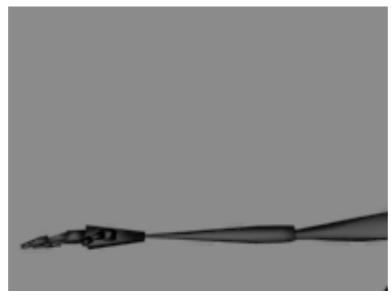
- We've seen that objects don't start suddenly
- They also don't stop on a dime



Animation principles: follow through

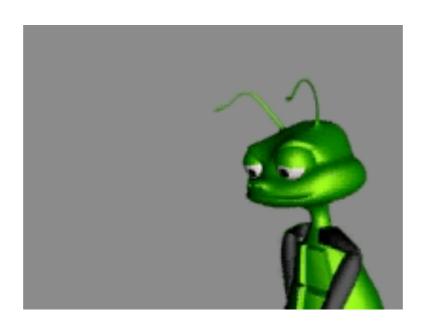
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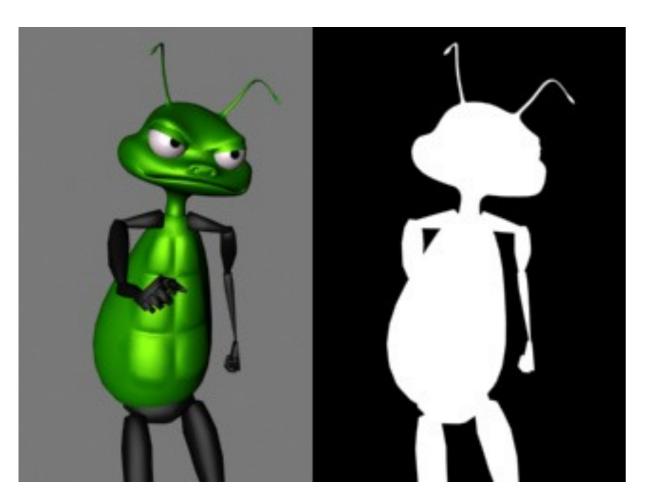
Anim. principles: overlapping action

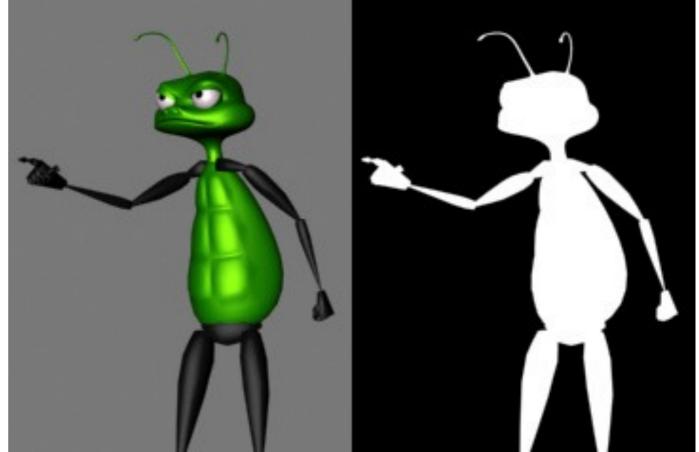
Usually many actions are happening at once



lichael B. Come

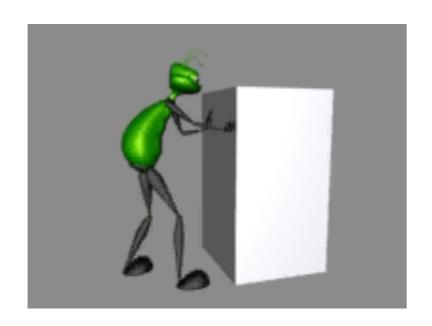
Animation principles: staging



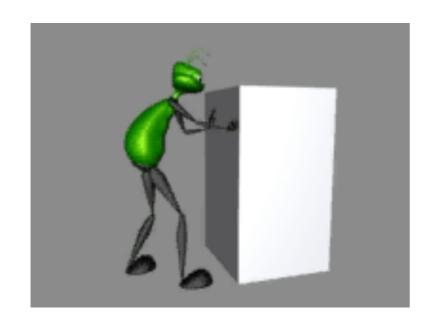


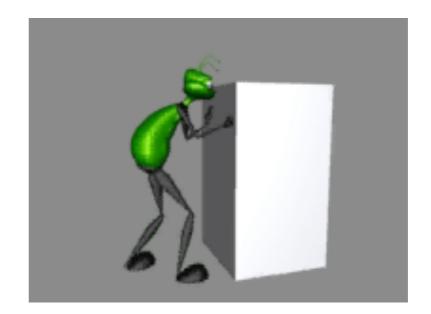
- Want to produce clear, good-looking 2D images
 - need good camera angles, set design, and character positions

Principles at work: weight



Principles at work: weight

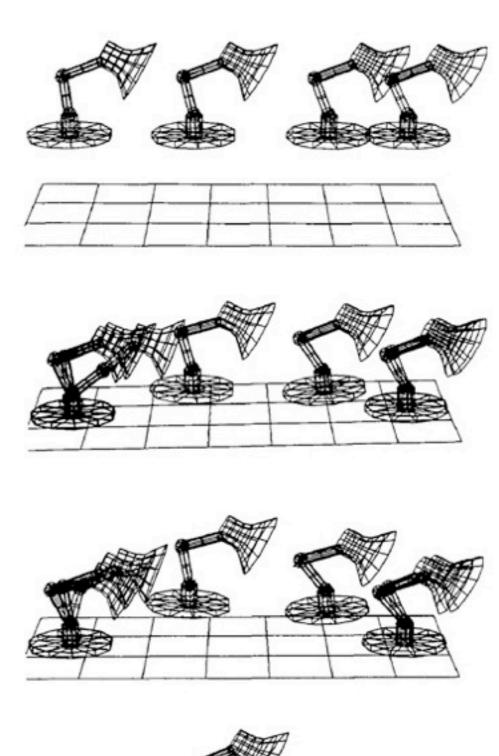


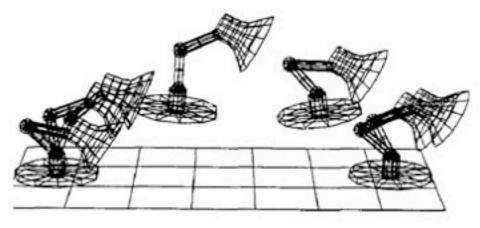


Extended example: Luxo, Jr.

Computer-generated motion

- Interesting aside: many principles of character animation follow indirectly from physics
- Anticipation, follow-through, and many other effects can be produced by simply minimizing physical energy
- Seminal paper: "Spacetime Constraints" by Witkin and Kass in SIGGRAPH 1988



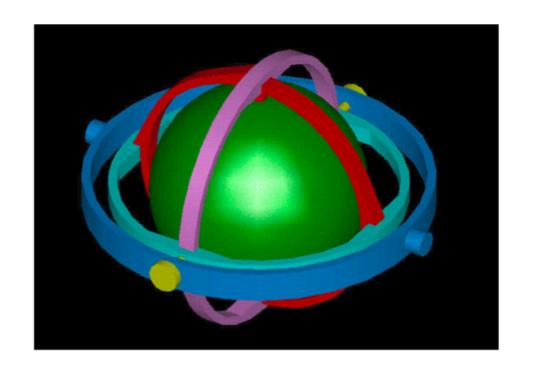


Controlling shape for animation

- Start with modeling DOFs (control points)
- Deformations control those DOFs at a higher level
 - Example: move first joint of second finger on left hand
- Animation controls control those DOFs at a higher level
 - Example: open/close left hand
- Both cases can be handled by the same kinds of deformers

Parameterizing rotations

- Euler angles
 - Rotate around x, then y, then z
 - Problem: gimbal lock
 - If two axes coincide, you lose one DOF
- Unit quaternions
 - A 4D representation (like 3D unit vectors for 2D sphere)
 - Good choice for interpolating rotations
- These are first examples of motion control
 - Matrix = deformation
 - Angles/quaternion = animation controls



Hierarchies and articulated figures

- Luxo as an example
 - small number of animation controls control many transformations
 - constraint: the joints hold together
- Some operations are tricky with hierarchies
 - how to ensure lampshade touches ball?
- In mechanics, the relationship between DOFs and 3D pose is kinematics
- Robotics as source of math. Methods
 - robots are transformation hierarchies
 - forward kinematics
 - inverse kinematics



Forward Kinematics

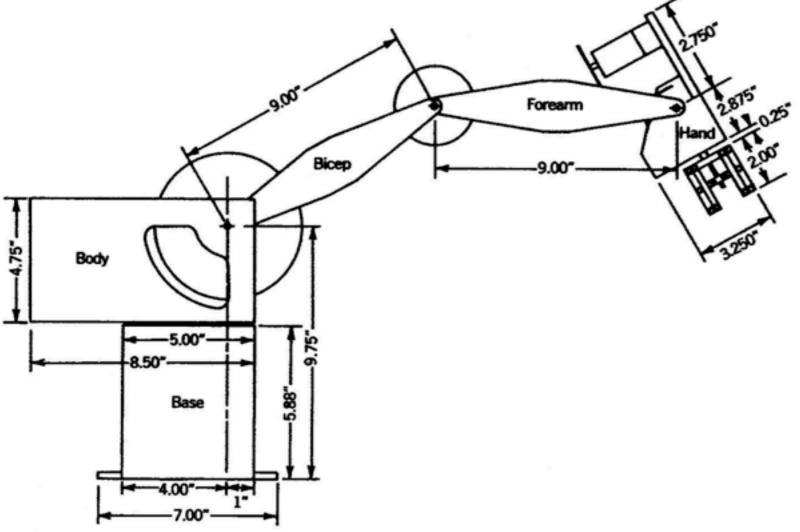


Inverse Kinematics

Forward Kinematics

- Articulated body
 - Hierarchical transforms

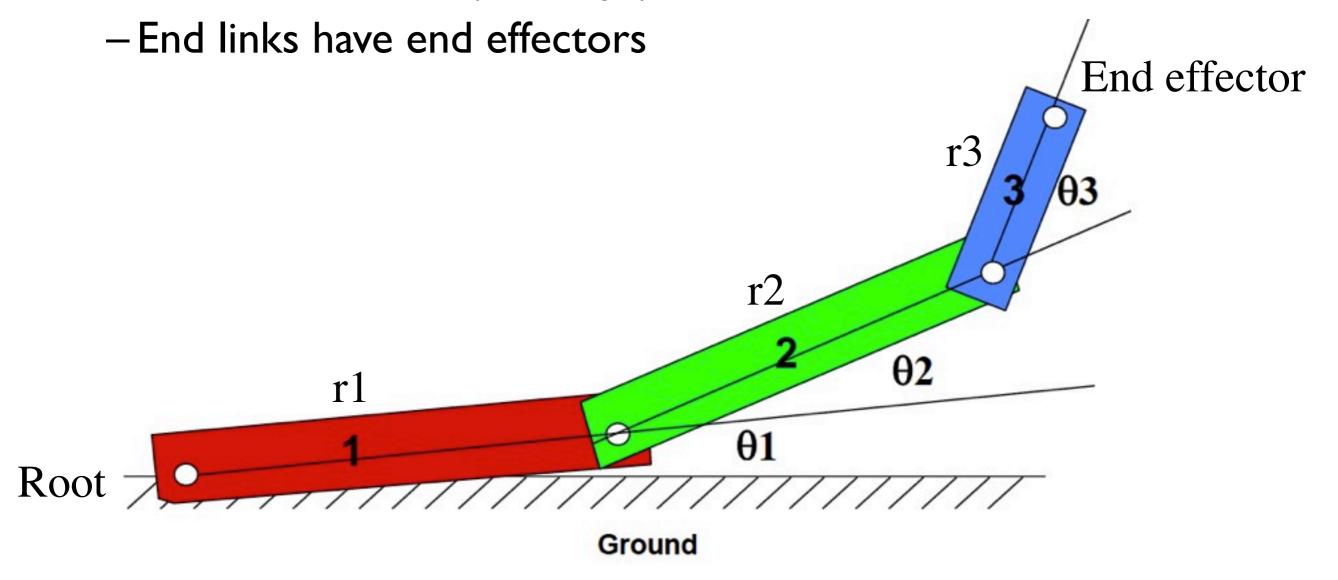
Comes from robotics



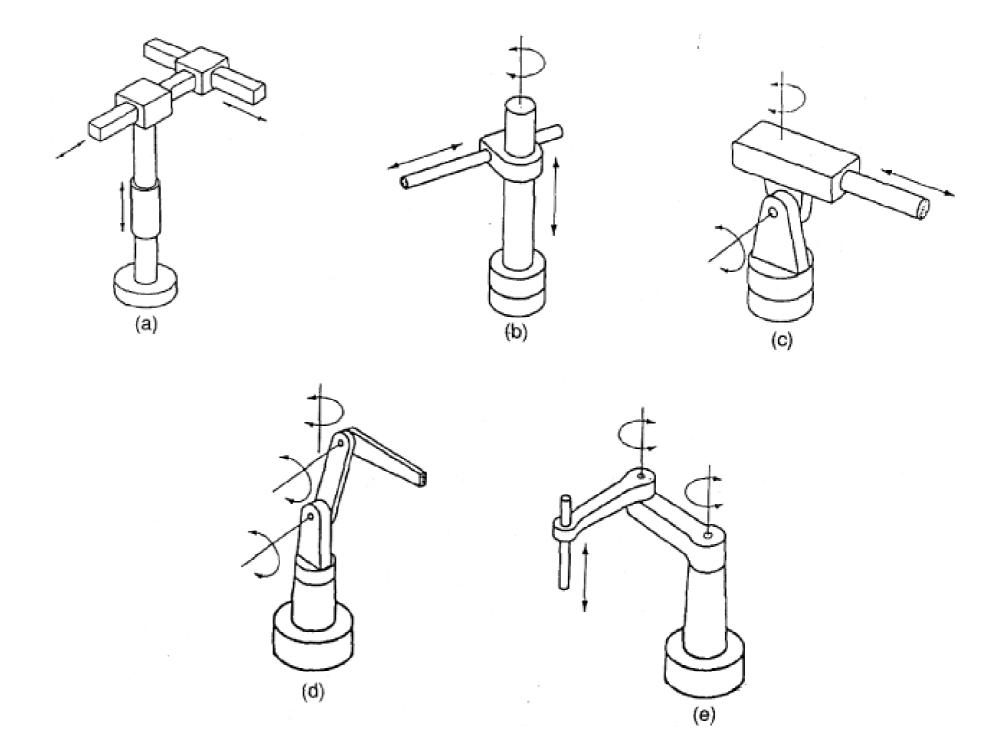


Rigid Links and Joint Structure

- Links connected by joints
 - -Joints are purely rotational (single DOF)
 - -Links form a tree (no loops)



Articulation in robotics



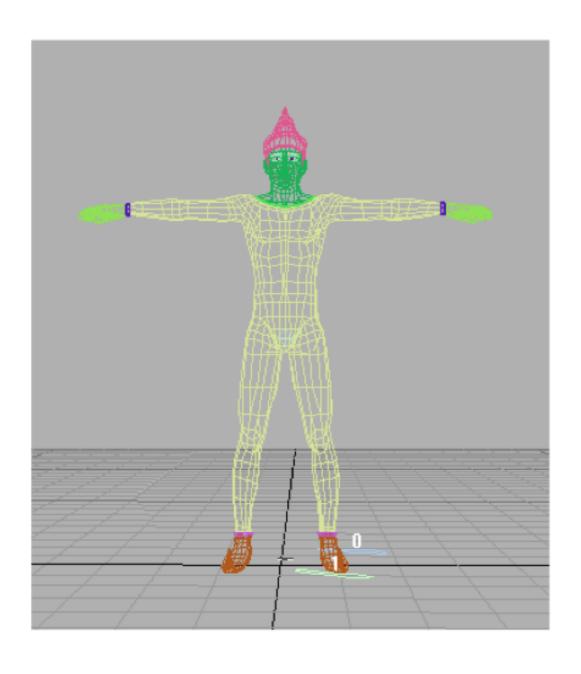
- a. rectangular or cartesian
- b. cylindrical or post-type
- c. spherical or polar
- d. joint-arm or articulated
- e. SCARA (selective compliance assembly robot arm)

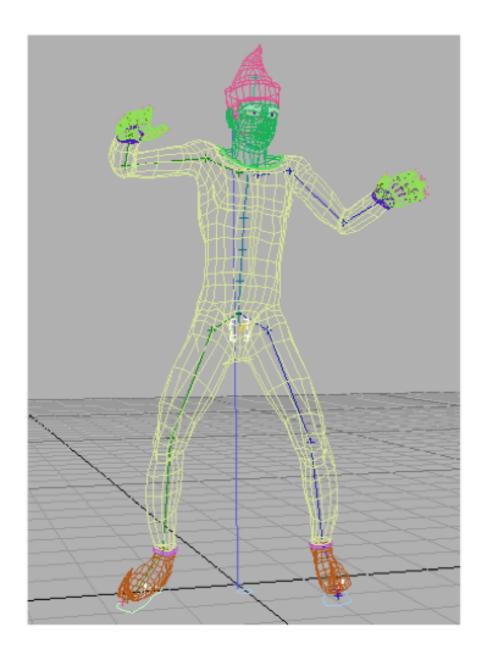
Basic surface deformation methods

- Mesh skinning: deform a mesh based on an underlying skeleton
- Blend shapes: make a mesh by combining several meshes
- Both use simple linear algebra
 - Easy to implement—first thing to try
 - Fast to run—used in games
- The simplest tools in the offline animation toolbox

Mesh skinning

A simple way to deform a surface to follow a skeleton





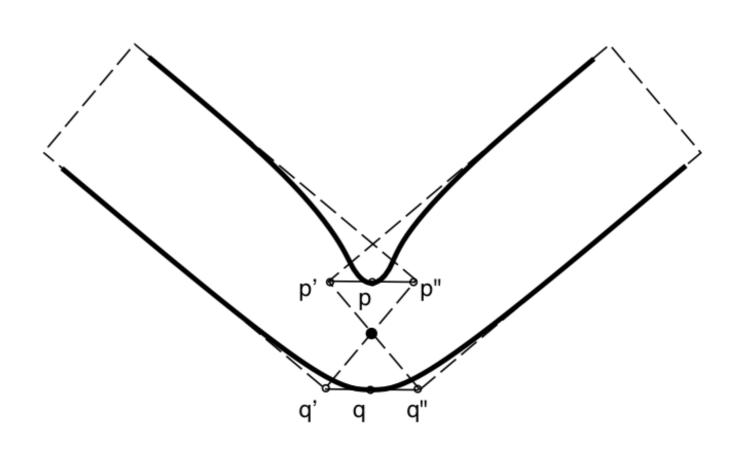
Mesh skinning math: setup

- Surface has control points p_i
 - Triangle vertices, spline control points, subdiv base vertices
- Each bone has a transformation matrix M_j
 - Normally a rigid motion
- Every point—bone pair has a weight w_{ij}
 - In practice only nonzero for small # of nearby bones
 - The weights are provided by the user

Mesh skinning math

- Deformed position of a point is a weighted sum
 - of the positions determined by each bone's transform alone
 - weighted by that vertex's weight for that bone

$$\mathbf{p}_i' = \sum_j w_{ij} M_j \mathbf{p}_i$$



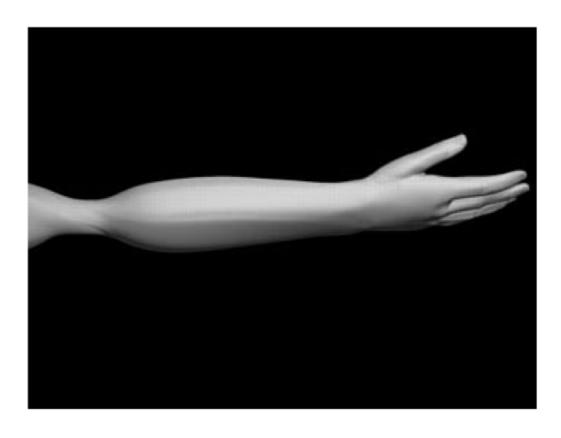
Mesh skinning

- Simple and fast to compute
 - Can even compute in the vertex stage of a graphics pipeline
- Used heavily in games
- One piece of the toolbox for offline animation
 - Many other deformers also available

Mesh skinning: classic problems

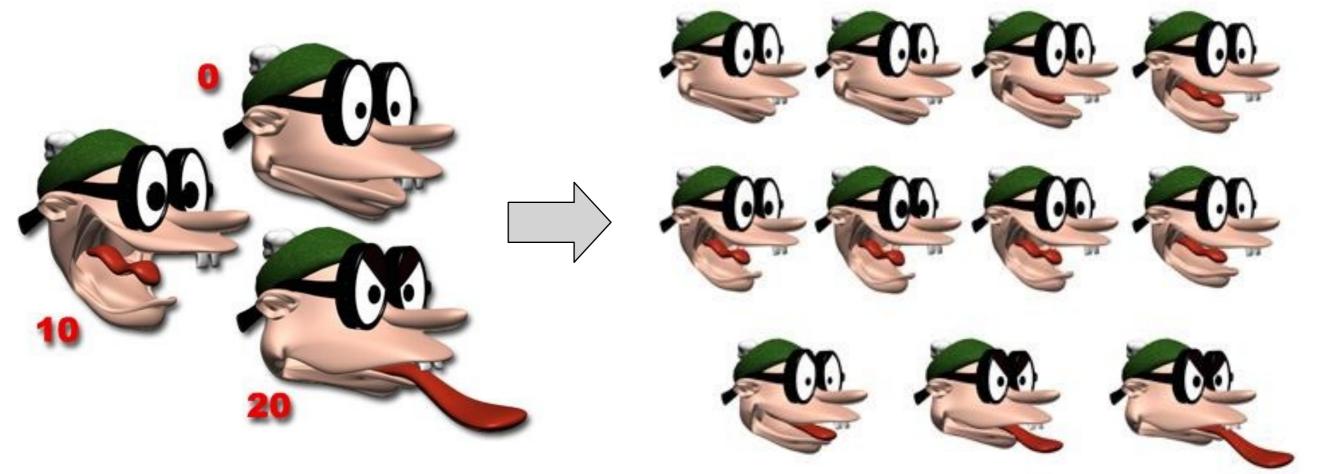
- Surface collapses on the inside of bends and in the presence of strong twists
 - Average of two rotations is not a rotation!
 - Add more bones to keep adjacent bones from being too different, or change the blending rules.





Blend shapes

- Another very simple surface control scheme
- Based on interpolating among several key poses
 - Aka. blend shapes or morph targets



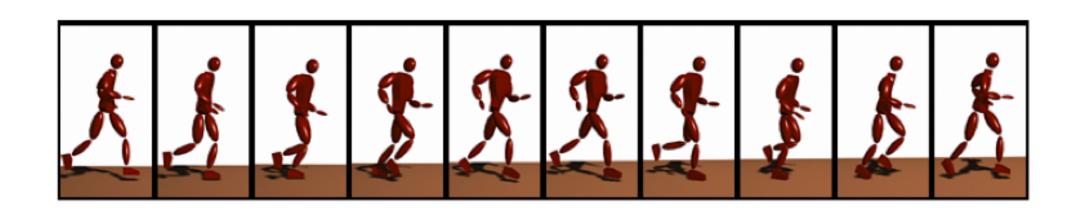
Blend shapes math

- Simple setup
 - User provides key shapes—that is, a position for every control point in every shape: \mathbf{p}_{ij} for point i, shape j
 - Per frame: user provides a weight w_j for each key shape
 - Must sum to 1.0
- Computation of deformed shape

$$\mathbf{p}_i' = \sum_j w_j \mathbf{p}_{ij}$$

- Works well for relatively small motions
 - Often used for for facial animation
 - Runs in real time; popular for games

Motion capture



 A method for creating complex motion quickly: measure it from the real world

[thanks to Zoran Popović for many visuals]

Motion capture in movies



[Final Fanatsy]

Motion capture in movies

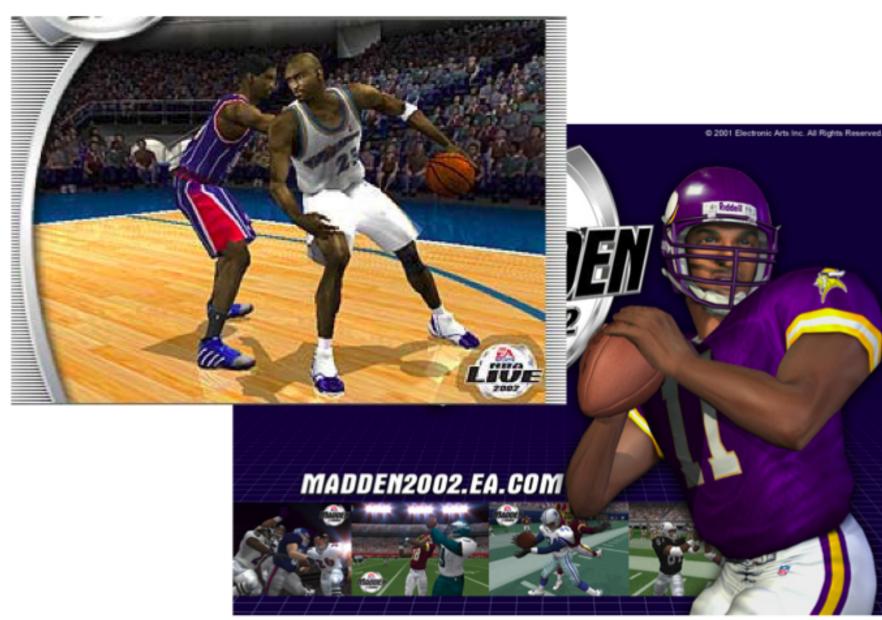




[The Two Towers | New Line Productions]

Motion capture in games





Magnetic motion capture

- Tethered
- Nearby metal objects cause distortions
- Low freq. (60Hz)



Mechanical motion capture

- Measures joint angles directly
- Works in any environment
- Restricts motion



Optical motion capture

 Passive markers on subject



Retroreflective markers

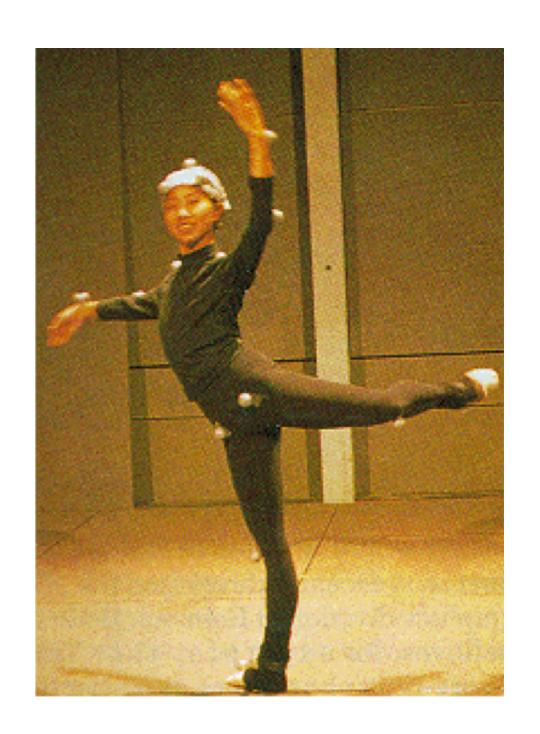


Cameras with IR illuminators

- Markers observed by cameras
 - Positions via triangulation

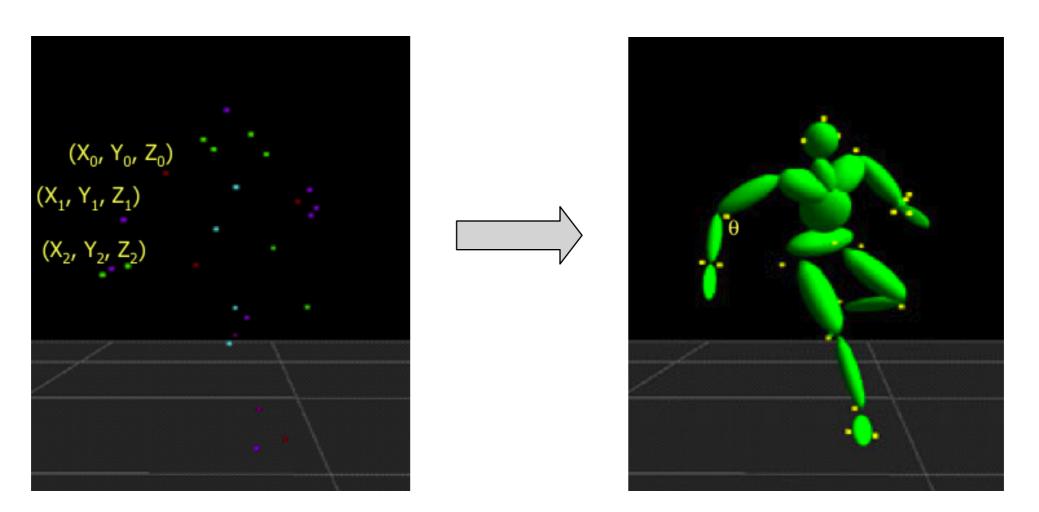
Optical motion capture

- 8 or more cameras
- Restricted volume
- High frequency (240Hz)
- Occlusions are troublesome



From marker data to usable motion

- Motion capture system gives inconvenient raw data
 - Optical is "least information" case: accurate position but:
 - Which marker is which?
 - Where are the markers are relative to the skeleton?



Motion capture data processing

- Marker identification: which marker is which
 - Start with standard rest pose
 - Track forward through time (but watch for markers dropping out due to occlusion!)
- Calibration: match skeleton, find offsets to markers
 - Use a short sequence that exercises all DOFs of the subject
 - A nonlinear minimization problem
- Computing joint angles: explain data using skeleton DOFs
 - A inverse kinematics problem per frame!

Motion capture in context

- Mocap data is very realistic
 - Timing matches performance exactly
 - Dimensions are exact
- But it is not enough for good character animation
 - Too few DOFs
 - Noise, errors from nonrigid marker mounting
 - Contains no exaggeration
 - Only applies to human-shaped characters
- Therefore mocap data is generally a starting point for skilled animators to create the final product