Problem 1: 2D Transformations (15 pts)

(i) Estimate the 2D affine transformation matrix, \( T = \begin{bmatrix} F & v \\ 0^T & 1 \end{bmatrix} \in \mathbb{R}^{3 \times 3} \), given its action on three homogeneous points:
\[
\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}.
\]

(ii) What kind of transformation does this matrix represent?

Problem 2: Affine Transformations (10 pts)

Show that affine transformations preserve parallel lines. 
(Hint: Recall the explicit parameterization of a line.)

Problem 3: Quaternions (15 pts)

Rotate the point \( p = (1, 1, 1) \) using the rotation specified by the quaternion \( q = \langle d; u \rangle = \langle 1; 1, 1, 1 \rangle \).

Problem 4: SLERP (10 pts)

When interpolating with SLERP between two unit quaternions, \( x \) and \( y \), we use:
\[
\text{SLERP}(x, y, \alpha), \text{ if } x \cdot y > 0, \text{ and SLERP}(x, -y, \alpha) \text{ otherwise}.
\]

(i) Why is this method better than just \( \text{SLERP}(x, y, \alpha) \)? What is the difference between \(+y\) and \(-y\) here?

(ii) When interpolating unit normal vectors, \( n_1 \) and \( n_2 \), for lighting calculations, should we also use \( \text{SLERP}(n_1, n_2, \alpha) \), if \( n_1 \cdot n_2 > 0 \), and \( \text{SLERP}(n_1, -n_2, \alpha) \) otherwise?