CS4620/5620: Lecture 30

Animation

Keyframe animation

- Keyframing is the technique used for pose-to-pose animation
  - User creates key poses—just enough to indicate what the motion is supposed to be
  - Interpolate between the poses

Controlling shape for animation

- Start with *modeling DOFs* (control points)
- *Deformations* control those DOFs at a higher level
  - Example: move first joint of second finger on left hand
- *Animation controls* control *those* DOFs at a higher level
  - Example: open/close left hand
- Both cases can be handled by the same kinds of deformers

Rigid motion: the simplest deformation

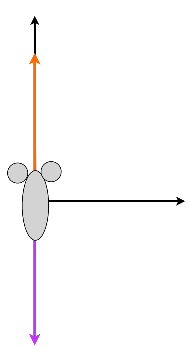
- Move a set of points by applying an affine transformation
- How to animate the transformation over time?
  - Interpolate the matrix entries from keyframe to keyframe?
    - Translation: ok
      - start location, end location, interpolate
    - Rotation: not so good

Rigid motion: the simplest deformation

$$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

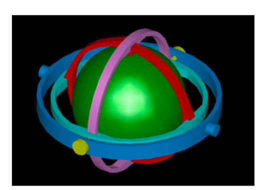

start


end



Parameterizing rotations

- Euler angles
  - Rotate around x, then y, then z
  - Problem: gimbal lock
    - If two axes coincide, you lose one DOF



- Unit quaternions
  - A 4D representation (like 3D unit vectors for 2D sphere)
  - Good choice for interpolating rotations
- These are first examples of motion control
  - Matrix = deformation
  - Angles/quaternion = animation controls

Quaternions

- Remember that
  - Orientations can be expressed as rotation
    - Why?
      - Start in a default position (say aligned with z axis)
      - New orientation is rotation from default position
  - Rotations can be expressed as (axis, angle)
- Quaternions let you express (axis, angle)

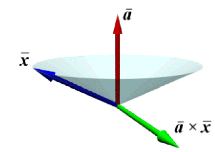
Quaternion for Rotation

- Rotate about axis a by angle θ

$$q = (s, v) = (s, v_1, v_2, v_3)$$

$$s = cos\left(\frac{\theta}{2}\right)$$

$$v = sin\left(\frac{\theta}{2}\right)\hat{a}$$



Quaternions for Rotation

- A quaternion is an extension of complex numbers

Review complex numbers

$$z = a + bi$$
$$z' = a - bi$$
$$||z|| = \sqrt{z.z'} = \sqrt{a^2 + b^2}$$

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- Each of i, j and k are three square roots of –1

$$i^2 = j^2 = k^2 = ijk = -1$$

- Cross-multiplication is like cross product

$$ij = -ji = k$$
$$jk = -kj = i$$
$$ki = -ik = -j$$

ONB in quaternions

- Quaternion is extension of complex number in 4D space

$$q = w + xi + yj + zk$$
$$q' = w - xi - yj - zk$$
$$||q|| = \sqrt{w^2 + x^2 + y^2 + z^2}$$

Quaternion Properties

- Linear combination of 1, *i*, *j*, *k*

$$q = w + xi + yj + zk = (s, v)$$
$$s = w, v = [x, y, z]$$

- Multiplication

$$q_1 = (s_1, v_1), q_2 = (s_2, v_2)$$
$$q_1 * q_2 = (s_1 s_2 - v_1 \cdot v_2, s_1 v_2 + s_2 v_1 + v_1 \times v_2)$$

Quaternion Properties

- Associative

$$q_1 * (q_2 * q_3) = (q_1 * q_2) * q_3$$

- Not commutative

$$q_1 * q_2 \neq q_2 * q_1$$

- Unit quaternion

$$||q|| = 1$$
$$q^{-1} = q'$$

Quaternion for Rotation

- Rotate about axis a by angle θ

$$q = (s, v) = (s, v_1, v_2, v_3)$$
$$s = cos\left(\frac{\theta}{2}\right)$$
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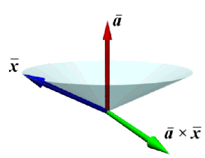

- Note: unit quaternion

Rotation Using Quaternion

- A point in space is a quaternion with 0 scalar

$$X = (0, \vec{x})$$

Rotation Using Quaternion

- A point in space is a quaternion with 0 scalar

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- Rotation is computed as follows

$$x_{rotated} = qXq^{-1} = qXq'$$

- See Buss 3D CG: A mathematical introduction with OpenGL, Chapter 7

Matrix for quaternion

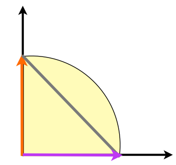
$$\begin{bmatrix} w^2+x^2-y^2-z^2 & 2xy-2wz & 2xz+2wy & 0 \\ 2xy+2wz & w^2-x^2+y^2-z^2 & 2yz-2wx & 0 \\ 2xz-2wy & 2yz+2wx & w^2-x^2-y^2+z^2 & 0 \\ 0 & 0 & 0 & w^2+x^2+y^2+z^2 \end{bmatrix}$$

Why Quaternions?

- Fast, few operations, not redundant
- Numerically stable for incremental changes
- Composes rotations nicely
- Convert to matrices at the end
- Biggest reason: spherical interpolation

Interpolating between quaternions

- Why not linear interpolation?
  - Need to be normalized
  - Does not have constant rate of rotation

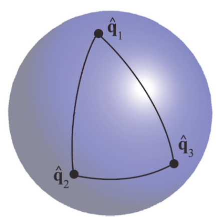


$$\frac{(1-\alpha)x + \alpha y}{||(1-\alpha)x + \alpha y||}$$

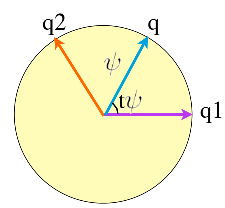
Spherical Linear Interpolation

- Intuitive interpolation between different orientations
  - Nicely represented through quaternions
  - Useful for animation
  - Given two quaternions, interpolate between them

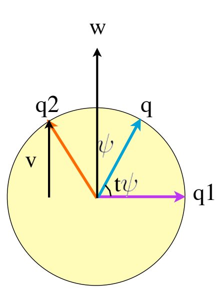
  - Shortest path between two points on sphere
    - Geodesic, on Great Circle



Spherical Linear Interpolation



Spherical Linear Interpolation



Quaternion Interpolation

- Shortest arc on the 4D unit sphere between q1 and q2
  - Path is spherical geodesic
  - Uniform angular rotation velocity about a fixed axis

$$slerp(q_1, q_2, t) = \frac{sin((1-t)\psi)}{sin\psi} q_1 + \frac{sin(t\psi)}{sin\psi} q_2$$

$$cos(\psi) = q_1 . q_2 = s_1 s_2 + v_1 . v_2$$

Practical issues

- When angle gets close to zero, use small angle approximation
  - degenerate to linear interpolation between q1 and q2
- When angle close to 180, there is no shortest geodesic. Be careful
- q is same rotation as -q
  - if q1 and q2 angle < 90, slerp between them
  - else, slerp between q1 and -q2

Rotation Using Quaternion

- Composing rotations
  - q1 and q2 are two rotations
  - First, q1 then q2

$$x_{rotated} = q_2(q_1 X q_1^{-1})q_2^{-1}$$

$$x_{rotated} = (q_2 q_1) X (q_1^{-1} q_2^{-1})$$

$$x_{rotated} = (q_2 q_1) X (q_2 q_1)^{-1}$$