CS4620/5620: Lecture 16

Rasterization

Rasterizing triangles

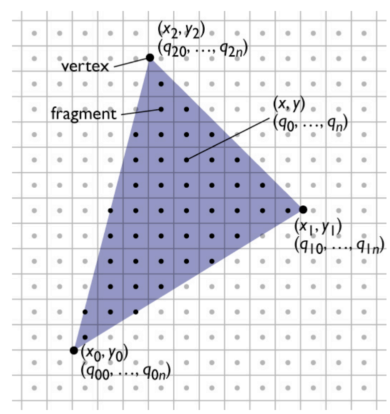
- The most common case in most applications
  - with good antialiasing can be the only case
  - some systems render a line as two skinny triangles
- Triangle represented by three vertices
- Simple way to think of algorithm follows the pixel-walk interpretation of line rasterization
  - walk from pixel to pixel over (at least) the polygon's area
  - evaluate linear functions as you go
  - use those functions to decide which pixels are inside

Rasterizing triangles

- Input:
  - three 2D points (the triangle's vertices in pixel space)
    - $(x_0, y_0); (x_1, y_1); (x_2, y_2)$
  - parameter values at each vertex
    - $q_{00}, \ldots, q_{0n}; q_{10}, \ldots, q_{1n}; q_{20}, \ldots, q_{2n}$
- Output: a list of fragments, each with
  - the integer pixel coordinates (x, y)
  - interpolated parameter values $q_0, \ldots, q_n$

Rasterizing triangles

- Summary
  1. evaluation of linear functions on pixel grid
  2. functions defined by parameter values at vertices
  3. using extra parameters to determine fragment set



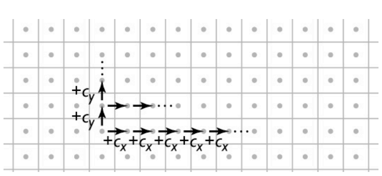
Incremental linear evaluation

- A linear (affine, really) function on the plane is:

$$q(x, y) = c_x x + c_y y + c_k$$

- Linear functions are efficient to evaluate on a grid:

$$q(x+1, y) = c_x(x+1) + c_y y + c_k = q(x, y) + c_x$$
$$q(x, y+1) = c_x x + c_y(y+1) + c_k = q(x, y) + c_y$$

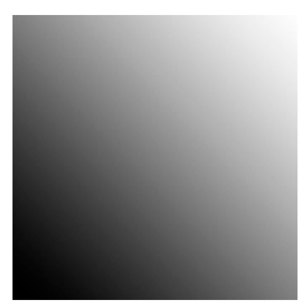


Incremental linear evaluation

```
linEval(xl, xh, yl, yh, cx, cy, ck) {

  // setup
  qRow = cx*xl + cy*yl + ck;

  // traversal
  for y = yl to yh {
    qPix = qRow;
    for x = xl to xh {
      output(x, y, qPix);
      qPix += cx;
    }
    qRow += cy;
  }
}
```



$c_x = .005; c_y = .005; c_k = 0$

(image size 100x100)

Defining parameter functions

- To interpolate parameters across a triangle we need to find the c_x, c_y, and c_k that define the (unique) linear function that matches the given values at all 3 vertices
  - this is 3 constraints on 3 unknown coefficients:

$$c_x x_0 + c_y y_0 + c_k = q_0$$
$$c_x x_1 + c_y y_1 + c_k = q_1$$
$$c_x x_2 + c_y y_2 + c_k = q_2$$

(each states that the function agrees with the given value at one vertex)

  - leading to a 3x3 matrix equation for the coefficients:

$$\begin{bmatrix} x_0 & y_0 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{bmatrix} \begin{bmatrix} c_x \\ c_y \\ c_k \end{bmatrix} = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \end{bmatrix}$$

(singular iff triangle is degenerate)

Defining parameter functions

- More efficient version: shift origin to (x_0, y_0)

$$q(x, y) = c_x(x - x_0) + c_y(y - y_0) + q_0$$
$$q(x_1, y_1) = c_x(x_1 - x_0) + c_y(y_1 - y_0) + q_0 = q_1$$
$$q(x_2, y_2) = c_x(x_2 - x_0) + c_y(y_2 - y_0) + q_0 = q_2$$

  - now this is a 2x2 linear system (since q_0 falls out):

$$\begin{bmatrix} (x_1 - x_0) & (y_1 - y_0) \\ (x_2 - x_0) & (y_2 - y_0) \end{bmatrix} \begin{bmatrix} c_x \\ c_y \end{bmatrix} = \begin{bmatrix} q_1 - q_0 \\ q_2 - q_0 \end{bmatrix}$$

  - solve using Cramer's rule (see Shirley):

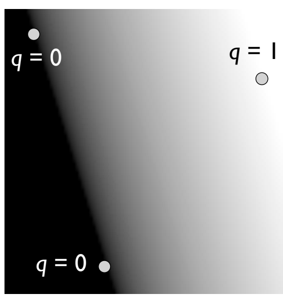
$$c_x = (\Delta q_1 \Delta y_2 - \Delta q_2 \Delta y_1)/(\Delta x_1 \Delta y_2 - \Delta x_2 \Delta y_1)$$
$$c_y = (\Delta q_2 \Delta x_1 - \Delta q_1 \Delta x_2)/(\Delta x_1 \Delta y_2 - \Delta x_2 \Delta y_1)$$

Defining parameter functions

```
linInterp(xl, xh, yl, yh, x0, y0, q0,
        x1, y1, q1, x2, y2, q2) {

   // setup
   det = (x1-x0)*(y2-y0) - (x2-x0)*(y1-y0);
   cx = ((q1-q0)*(y2-y0) - (q2-q0)*(y1-y0)) / det;
   cy = ((q2-q0)*(x1-x0) - (q1-q0)*(x2-x0)) / det;
   qRow = cx*(xl-x0) + cy*(yl-y0) + q0;

   // traversal (same as before)
   for y = yl to yh {
      qPix = qRow;
      for x = xl to xh {
         output(x, y, qPix);
         qPix += cx;
      }
      qRow += cy;
   }
}
```

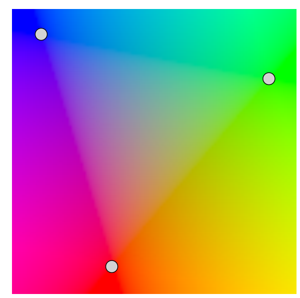


Interpolating several parameters

```
linInterp(xl, xh, yl, yh, n, x0, y0, q0[],
        x1, y1, q1[], x2, y2, q2[]) {

   // setup
   for k = 0 to n-1
      // compute cx[k], cy[k], qRow[k]
      // from q0[k], q1[k], q2[k]

   // traversal
   for y = yl to yh {
      for k = 1 to n, qPix[k] = qRow[k];
      for x = xl to xh {
         output(x, y, qPix);
         for k = 1 to n, qPix[k] += cx[k];
      }
      for k = 1 to n, qRow[k] += cy[k];
   }
}
```



Rasterizing triangles

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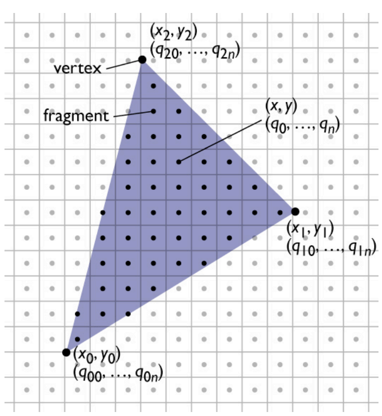
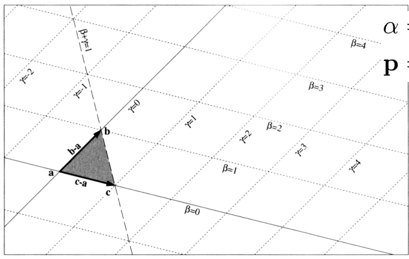


Clipping to the triangle

- Interpolate three *barycentric coordinates* across the plane
  - each barycentric coord is 1 at one vert. and 0 at the other two
- Output fragments only when all three are > 0.

Barycentric coordinates

- Basis: a coordinate system for trinagles



$$\alpha = 1 - \beta - \gamma$$
$$\mathbf{p} = \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a})$$

[Shirley 2000]

  – in this view, the triangle interior test is just

$$\beta > 0; \quad \gamma > 0; \quad \beta + \gamma < 1$$

Barycentric coordinates

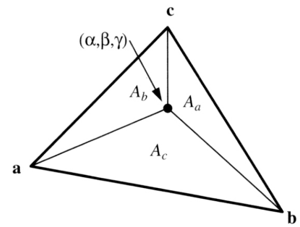
- Geometric viewpoint
  – algebraic viewpoint:

$$\mathbf{p} = \alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c}$$
$$\alpha + \beta + \gamma = 1$$

  – geometric viewpoint (areas):

- Triangle interior test:

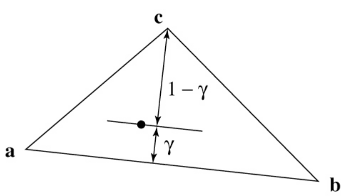
$$\alpha > 0; \quad \beta > 0; \quad \gamma > 0$$

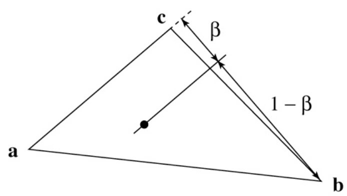


[Shirley 2000]

Barycentric coordinates

- A coordinate system for triangles
  – geometric viewpoint: distances



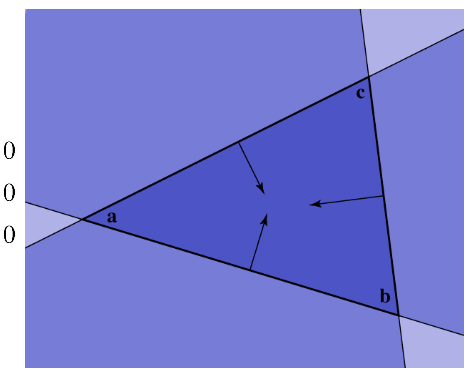


  – linear viewpoint: basis of edges

Edge equations

- In plane, triangle is the intersection of 3 half spaces

$$(\mathbf{x} - \mathbf{a}) \cdot (\mathbf{b} - \mathbf{a})^\perp > 0$$
$$(\mathbf{x} - \mathbf{b}) \cdot (\mathbf{c} - \mathbf{b})^\perp > 0$$
$$(\mathbf{x} - \mathbf{c}) \cdot (\mathbf{a} - \mathbf{c})^\perp > 0$$

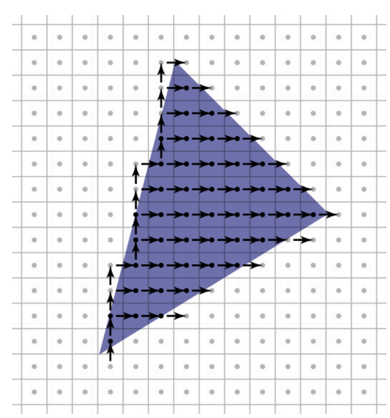


Walking edge equations

- We need to update values of the three edge equations with single-pixel steps in x and y
- Edge equation already in form of dot product
- components of vector are the increments

Pixel-walk (Pineda) rasterization

- Conservatively visit a superset of the pixels you want
- Interpolate linear functions
- Use those functions to determine when to emit a fragment



Rasterizing triangles

- Exercise caution with rounding and arbitrary decisions
  - need to visit these pixels once
  - but it's important not to visit them twice!