CS4620/5620: Lecture 13

Viewing, Pipeline

Announcements

- PA 1
  - Hopefully not a Reed Hastings
- Cone normals
- Lessons
  - Dissemination of information
    - Piazza, email, FAQ, Staff attendance, office hours
  - We are human
  - Extensions
- Other issues
- Grading slots on next Thursday
  - Please sign up as a group

Perspective transformation chain

- Transform into world coords (modeling transform, M_m)
- Transform into eye coords (camera xf., $M_{cam} = F_c^{-1}$)
- Perspective matrix, P
- Orthographic projection, M_{orth}
- Viewport transform, M_{vp}

$$\mathbf{p}_s = \mathbf{M}_{\text{vp}}\mathbf{M}_{\text{orth}}\mathbf{P}\mathbf{M}_{\text{cam}}\mathbf{M}_{\text{m}}\mathbf{p}_o$$

$$\begin{bmatrix} x_s \\ y_s \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{n_x}{2} & 0 & 0 & \frac{n_x-1}{2} \\ 0 & \frac{n_y}{2} & 0 & \frac{n_y-1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix} \mathbf{M}_{\text{cam}}\mathbf{M}_{\text{m}} \begin{bmatrix} x_o \\ y_o \\ z_o \\ 1 \end{bmatrix}$$

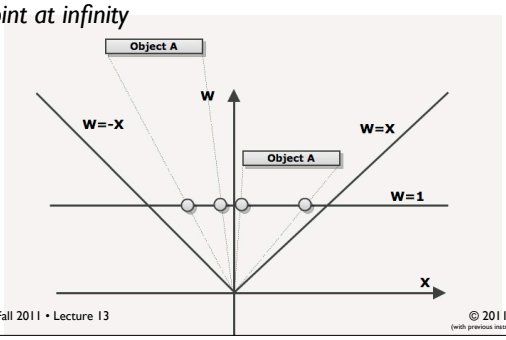
(xw,w)

x = a + m t (line through a along direction m)

if t = 1/w; x = (a w + m)/w

in homogeneous coordinates (aw+m, w)

w = 0, point at infinity

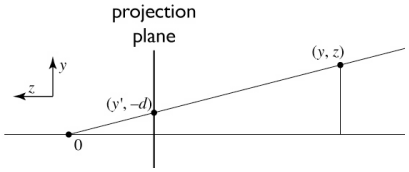


Implications of *w*

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \sim \begin{bmatrix} wx \\ wy \\ wz \\ w \end{bmatrix}$$

- All scalar multiples of a 4-vector are equivalent
- When *w* is not zero, can divide by *w*
  - therefore these points represent "normal" affine points
- When *w* is zero, it's a point at infinity, a.k.a. a direction
  - this is the point where parallel lines intersect
  - can also think of it as the vanishing point

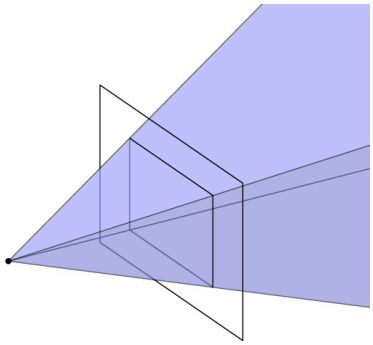
Perspective projection



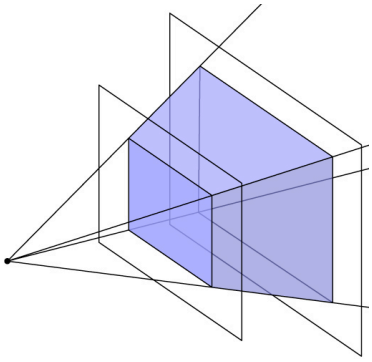
to implement perspective, just move z to w:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -dx/z \\ -dy/z \\ 1 \end{bmatrix} \sim \begin{bmatrix} dx \\ dy \\ -z \end{bmatrix} = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

View volume: perspective



View volume: perspective (clipped)



Carrying depth through perspective

- Perspective has a varying denominator—can't preserve depth!
- Compromise: preserve depth on near and far planes

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} \sim \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \\ -z \end{bmatrix} = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

  - that is, choose a and b so that $z'(n) = n$ and $z'(f) = f$.

$$\tilde{z}(z) = az + b$$

$$z'(z) = \frac{\tilde{z}}{-z} = \frac{az+b}{-z}$$

$$\text{want } z'(n) = n \text{ and } z'(f) = f$$

$$\text{result: } a = -(n+f) \text{ and } b = nf \text{ (try it)}$$

Official perspective matrix

- Use near plane distance as the projection distance
  - i.e., $d = -n$
- Scale by –1 to have fewer minus signs
  - scaling the matrix does not change the projective transformation

$$\mathbf{P} = \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Perspective projection matrix

- Product of perspective matrix with orth. projection matrix

$$\mathbf{M}_{\text{per}} = \mathbf{M}_{\text{orth}}\mathbf{P}$$

$$= \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{l+r}{l-r} & 0 \\ 0 & \frac{2n}{t-b} & \frac{b+t}{b-t} & 0 \\ 0 & 0 & \frac{f+n}{n-f} & \frac{2fn}{f-n} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

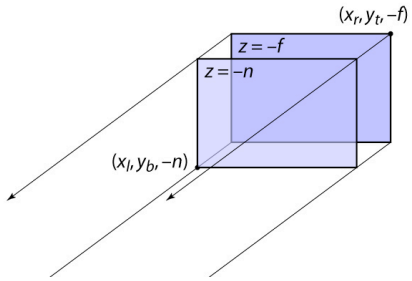
Perspective transformation chain

- Transform into world coords (modeling transform, M_m)
- Transform into eye coords (camera xf., $M_{\text{cam}} = F_c^{-1}$)
- Perspective matrix, P
- Orthographic projection, M_{orth}
- Viewport transform, M_{vp}

$$\mathbf{p}_s = \mathbf{M}_{\text{vp}}\mathbf{M}_{\text{orth}}\mathbf{P}\mathbf{M}_{\text{cam}}\mathbf{M}_{\text{m}}\mathbf{p}_o$$

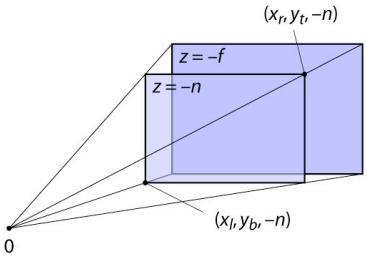
$$\begin{bmatrix} x_s \\ y_s \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{n_x}{2} & 0 & 0 & \frac{n_x-1}{2} \\ 0 & \frac{n_y}{2} & 0 & \frac{n_y-1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix} \mathbf{M}_{\text{cam}}\mathbf{M}_{\text{m}} \begin{bmatrix} x_o \\ y_o \\ z_o \\ 1 \end{bmatrix}$$

OpenGL view frustum: orthographic



Note OpenGL puts the near and far planes at $-n$ and $-f$ so that the user can give positive numbers

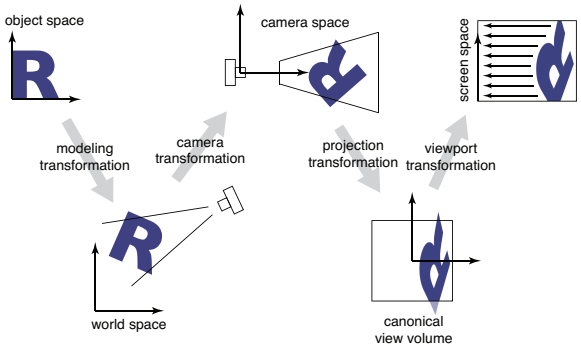
OpenGL view frustum: perspective



Note OpenGL puts the near and far planes at $-n$ and $-f$ so that the user can give positive numbers

Pipeline of transformations

- Standard sequence of transforms



Pipeline and Rasterization

CS4620 Lecture 14

The graphics pipeline

- The standard approach to object-order graphics
- Many versions exist
  - software, e.g. Pixar's REYES architecture
    - many options for quality and flexibility
  - hardware, e.g. graphics cards in PCs
    - amazing performance: millions of triangles per frame
- We'll focus on an abstract version of hardware pipeline

The graphics pipeline

- "Pipeline" because of the many stages
  - very parallelizable
  - leads to remarkable performance of graphics cards (many times the flops of the CPU at ~1/5 the clock speed)
  - gigaflops (10 to the 9th power), teraflop (12th power), petaflops (15th power)
- GeForce GTX590, 600MHz, 1024 stream processors

Supercomputers!

- Tianhe-1A, Tesla: 14,336 Xeon + 7,1688 Tesla boards
  - 448 cores

Pipeline overview