CS4620/5620: Lecture 7

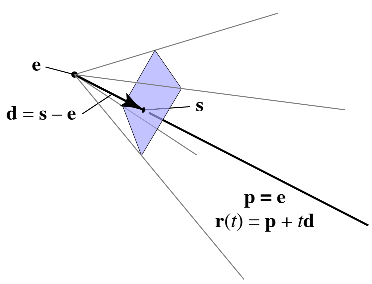
Ray Tracing Basics

Announcements

- HW 1 is out
  - Piazza updates
- PA1 will be out tonight

Generating eye rays—perspective

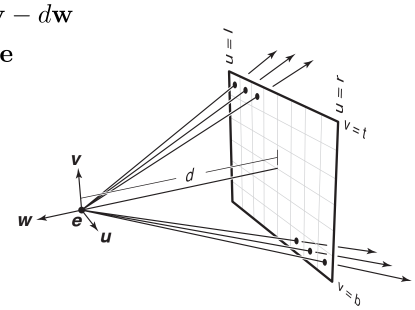
- View rectangle needs to be away from viewpoint
- Distance is important: "focal length" of camera
  - still use camera frame but position view rect away from viewpoint
  - ray origin always **e**
  - ray direction now controlled by **s**



Generating eye rays—perspective

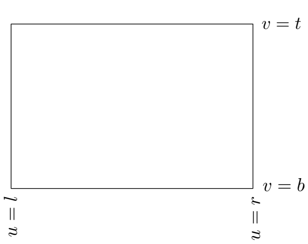
- Compute **s** in the same way; just subtract *d***w**
  - coordinates of **s** are (*u*, *v*, –*d*)

$$\mathbf{s} = \mathbf{e} + u\mathbf{u} + v\mathbf{v} - d\mathbf{w}$$
$$\mathbf{p} = \mathbf{e};\ \mathbf{d} = \mathbf{s} - \mathbf{e}$$
$$\mathbf{r}(t) = \mathbf{p} + t\mathbf{d}$$



Pixel-to-image mapping

- One last detail: (*u*, *v*) coords of a pixel

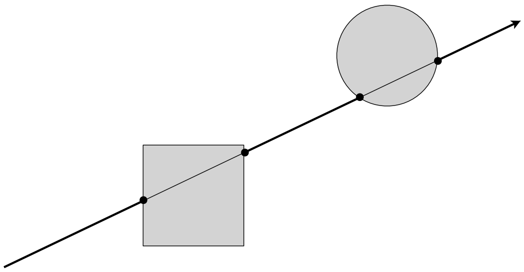


$$u = l + (r - l)(i + 0.5)/n_x$$
$$v = b + (t - b)(j + 0.5)/n_y$$

PA 1 camera

- viewPoint == e
- projNormal == w, viewUp == up
  - Compute u, v from the above
- l = -viewWidth/2
- r = +viewWidth/2
- n_x = imageWidth

Ray intersection



Ray-sphere intersection: algebraic

- Solution for t by quadratic formula:

$$t = \frac{-\mathbf{d} \cdot \mathbf{p} \pm \sqrt{(\mathbf{d} \cdot \mathbf{p})^2 - (\mathbf{d} \cdot \mathbf{d})(\mathbf{p} \cdot \mathbf{p} - 1)}}{\mathbf{d} \cdot \mathbf{d}}$$

$$t = -\mathbf{d} \cdot \mathbf{p} \pm \sqrt{(\mathbf{d} \cdot \mathbf{p})^2 - \mathbf{p} \cdot \mathbf{p} + 1}$$

  – simpler form holds when **d** is a unit vector
  but we won't assume this in practice (reason later)
  – discriminant intuition?

  – use the unit-vector form to make the geometric interpretation

Ray-triangle intersection

- Condition 1: point is on ray

$$\mathbf{r}(t) = \mathbf{p} + t\mathbf{d}$$

- Condition 2: point is on plane

$$(\mathbf{x} - \mathbf{a}) \cdot \mathbf{n} = 0$$

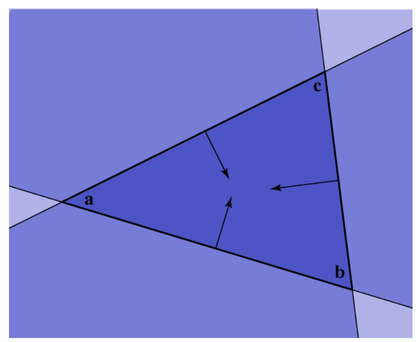
- Condition 3: point is on the inside of all three edges
- First solve 1&2 (ray–plane intersection)
  – substitute and solve for t:

$$(\mathbf{p} + t\mathbf{d} - \mathbf{a}) \cdot \mathbf{n} = 0$$

$$t = \frac{(\mathbf{a} - \mathbf{p}) \cdot \mathbf{n}}{\mathbf{d} \cdot \mathbf{n}}$$

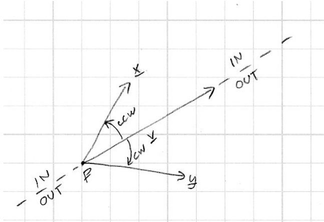
Ray-triangle intersection

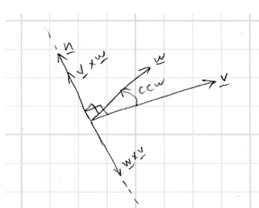
- In plane, triangle is the intersection of 3 half spaces



Inside-edge test

- Need outside vs. inside
- Reduce to clockwise vs. counterclockwise
  – vector of edge to vector to **x**
- Use cross product to decide



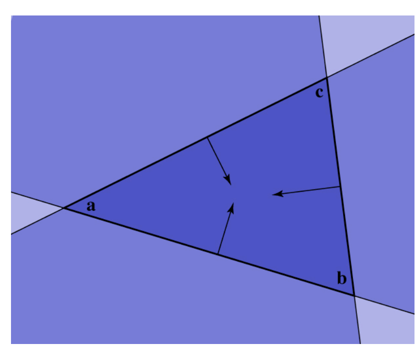


Ray-triangle intersection

$$(\mathbf{b} - \mathbf{a}) \times (\mathbf{x} - \mathbf{a}) \cdot \mathbf{n} > 0$$

$$(\mathbf{c} - \mathbf{b}) \times (\mathbf{x} - \mathbf{b}) \cdot \mathbf{n} > 0$$

$$(\mathbf{a} - \mathbf{c}) \times (\mathbf{x} - \mathbf{c}) \cdot \mathbf{n} > 0$$

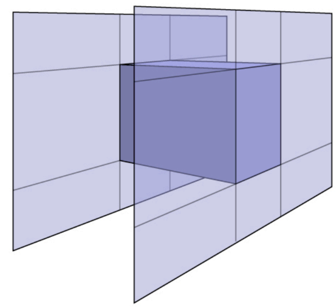


Ray-triangle intersection

- See book for a more efficient method based on linear systems
  - (don't need this for (PA1) Ray 1 anyhow—but stash away for (PA3) Ray 2)

Ray-box intersection

- Could intersect with 6 faces individually
- Better way: box is the intersection of 3 slabs

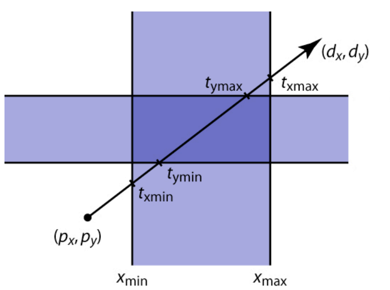


Ray-slab intersection

- 2D example
- 3D is the same!

$$p_x + t_{x\min}\, d_x = x_{\min}$$
$$t_{x\min} = (x_{\min} - p_x)/d_x$$

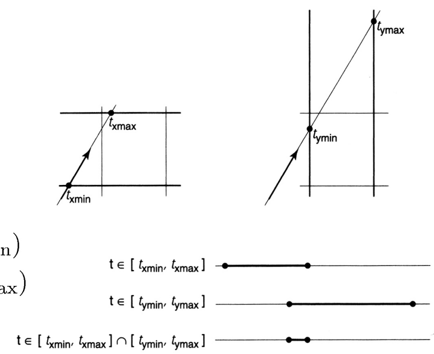
$$p_y + t_{y\min}\, d_y = y_{\min}$$
$$t_{y\min} = (y_{\min} - p_y)/d_y$$



Intersecting intersections

- Each intersection is an interval
- Want last entry point and first exit point

$$t_{\min} = \max(t_{x\min}, t_{y\min})$$
$$t_{\max} = \min(t_{x\max}, t_{y\max})$$



Shirley fig. 10.16

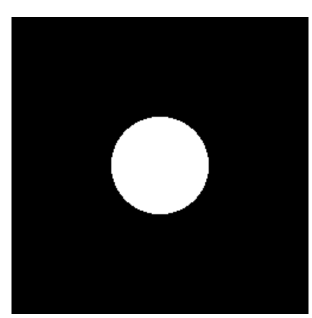
Does it work?

- d_x positive or negative

Image so far

- With eye ray generation and sphere intersection

```
Surface s = new Sphere((0.0, 0.0, 0.0), 1.0);
for 0 <= iy < ny
  for 0 <= ix < nx {
    ray = camera.getRay(ix, iy);
    hitSurface, t = s.intersect(ray, 0, +inf)
    if hitSurface is not null
      image.set(ix, iy, white);
  }
```



Intersection against many shapes

- The basic idea is:

```
Group.intersect (ray, tMin, tMax) {
    tBest = +inf; firstSurface = null;
    for surface in surfaceList {
        hitSurface, t = surface.intersect(ray, tMin, tBest);
        if (hitSurface is not null) {
            tBest = t;
            firstSurface = hitSurface;
        }
    }
return hitSurface, tBest;
}
```

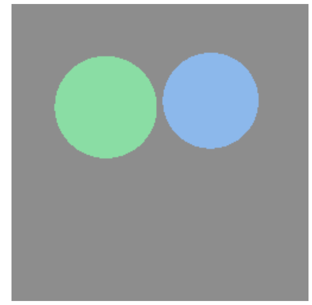
  – this is linear in the number of shapes
    but there are sublinear methods (acceleration structures)

Image so far

- With eye ray generation and scene intersection

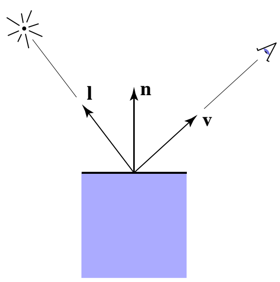
```
for 0 <= iy < ny
    for 0 <= ix < nx {
        ray = camera.getRay(ix, iy);
        c = scene.trace(ray, 0, +inf);
        image.set(ix, iy, c);
    }
...

Scene.trace(ray, tMin, tMax) {
    surface, t = surfs.intersect(ray, tMin, tMax);
    if (surface != null) return surface.color();
    else return black;
}
```



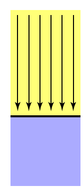
Shading

- Compute light reflected toward camera
- Inputs:
  – eye direction
  – light direction
    (for each of many lights)
  – surface normal
  – surface parameters
    (color, shininess, …)

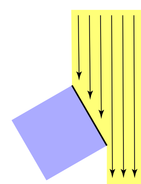


Diffuse reflection

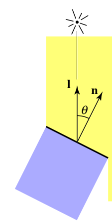
- Light is scattered uniformly in all directions
  – the surface color is the same for all viewing directions
- Lambert's cosine law



Top face of cube
receives a certain
amount of light



Top face of
60° rotated cube
intercepts half the light



In general, light per unit
area is proportional to
cos θ = **l** • **n**