CS4620 Fall 2011
HW1 - Transforms and Ray Tracing

Due: Sep 16 2011

1 Change of Basis

Write down the 4-by-4 rotation matrix $\mathbf{M} \in R^{4\times4}$, that takes the orthonormal 3D vectors **u** = $(x_u, y_u, z_u, 0)$, **v** = $(x_v, y_v, z_v, 0)$ and **w** = $(x_w, y_w, z_w, 0)$, to orthonormal 3D vectors **a** = $(x_a, y_a, z_a, 0)$, **b** = $(x_b, y_b, z_b, 0)$ and **c** = $(x_c, y_c, z_c, 0)$, such that, $\mathbf{Mu}=\mathbf{a}$, $\mathbf{Mv}=\mathbf{b}$, and $\mathbf{Mw}=\mathbf{c}$.

2 Ray Tracing

Find the point of intersection and the normal (at the point of intersection) for the following objects, given a ray $\mathbf{r} = \mathbf{p} + t\mathbf{d}$:

(a) A canonical cylinder of height H, radius R, and whose central axis is along the z axis.

(b) A canonical cone of height H, base radius R, and whose central axis is along the z axis.

3 Normal Transformations

3.1 Basic transformations

Consider a transformation $\mathbf{M} = \begin{bmatrix} \mathbf{L} & \mathbf{t} \\ \mathbf{0} & 1 \end{bmatrix}$ on points in 2D ($\mathbf{L} \in R^{2\times2}$, $\mathbf{t} \in R^2$), with the corresponding transformation of normals $\mathbf{L}^{-1T}$. For which of the following transformations (in 2D) represented by $\mathbf{M} \in \mathbf{R}^{3\times3}$, does the linear part satisfy $\mathbf{L} = \mathbf{L}^{-1T}$.

(a) Translation

(b) Rotation

(c) Reflection about x-axis

(d) Uniform scale

(e) Non-uniform scale

(f) Shear

3.2 Computing the transformed normal

Given a triangle ABC in 3D with vertices $A = (1, 2, 0)$, $B = (6, 4, 0)$, $C = (3, 6, 0)$, and the following sequence of transformations:

1. The triangle is rotated by an angle $\alpha = 45°$ about the y-axis, with the center of rotation being the triangle's center of mass.

2. The triangle is translated by the vector $\mathbf{t} = (10, 11, 4)$.

Assume that the normal of the original triangle is defined as the one that points to the positive z direction. Find the 3D unit vector, that corresponds to the normal of the transformed triangle.

4 Aligning Line Segments

Consider two line segments in 2D (homogeneous coordinates):
l_1 with end points $\mathbf{p}_1 = (2, 1, 1)$, $\mathbf{p}_2 = (5, 3, 1)$, and
l_2 with end points $\mathbf{q}_1 = (1, 3, 1)$, $\mathbf{q}_2 = (0, 4, 1)$.

Find the 3-by-3 transformation matrix $\mathbf{M}$, that aligns l_1 with l_2. In other words, you have to find the transformation $\mathbf{M}$ (rotation + scale + translation), such that $\mathbf{M}\mathbf{p}_1 = \mathbf{q}_1$ and $\mathbf{M}\mathbf{p}_2 = \mathbf{q}_2$.