1 Change of Basis

Write down the 4-by-4 rotation matrix $M \in \mathbb{R}^{4 \times 4}$, that takes the orthonormal 3D vectors $u = (x_u, y_u, z_u, 0)$, $v = (x_v, y_v, z_v, 0)$ and $w = (x_w, y_w, z_w, 0)$, to orthonormal 3D vectors $a = (x_a, y_a, z_a, 0)$, $b = (x_b, y_b, z_b, 0)$ and $c = (x_c, y_c, z_c, 0)$, such that, $Mu=a$, $Mv=b$, and $Mw=c$.

2 Ray Tracing

Find the point of intersection and the normal (at the point of intersection) for the following objects, given a ray $r = p + td$:

(a) A canonical cylinder of height $H$, radius $R$, and whose central axis is along the $z$ axis.

(b) A canonical cone of height $H$, base radius $R$, and whose central axis is along the $z$ axis.

3 Normal Transformations

3.1 Basic transformations

Consider a transformation $M = \begin{bmatrix} L & t \\
0 & 1 \end{bmatrix}$ on points in 2D ($L \in \mathbb{R}^{2 \times 2}$, $t \in \mathbb{R}^2$), with the corresponding transformation of normals $L^{-1T}$. For which of the following transformations (in 2D) represented by $M \in \mathbb{R}^{3 \times 3}$, does the linear part satisfy $L = L^{-1T}$.

(a) Translation

(b) Rotation
(c) Reflection about x-axis

(d) Uniform scale

(e) Non-uniform scale

(f) Shear

3.2 Computing the transformed normal

Given a triangle $ABC$ in 3D with vertices $A = (1, 2, 0)$, $B = (6, 4, 0)$, $C = (3, 6, 0)$, and the following sequence of transformations:

1. The triangle is rotated by an angle $\alpha = 45^\circ$ about the $y$-axis, with the center of rotation being the triangle’s center of mass.

2. The triangle is translated by the vector $t = (10, 11, 4)$.

Assume that the normal of the original triangle is defined as the one that points to the positive $z$ direction. Find the 3D unit vector, that corresponds to the normal of the transformed triangle.

4 Aligning Line Segments

Consider two line segments in 2D (homogeneous coordinates):

- $l_1$ with end points $p_1 = (2, 1, 1)$, $p_2 = (5, 3, 1)$,
- $l_2$ with end points $q_1 = (1, 3, 1)$, $q_2 = (0, 4, 1)$.

Find the 3-by-3 transformation matrix $M$, that aligns $l_1$ with $l_2$. In other words, you have to find the transformation $M$ (rotation + scale + translation), such that $Mp_1 = q_1$ and $Mp_2 = q_2$. 