# CS4450 Problem Set \#2 

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## 1 CSMA/CD: Random Access

Let $A$ and $B$ be two stations attempting to transmit on an Ethernet. Each has a steady queue of frames ready to send; $A$ 's frames are denoted $A_{1}, A_{2}, \ldots$, and B's are defined similarly.

Recall the random access protocol discussed in class. In case of a collision, $A$ and $B$ back off for $d \times T$ time where $T$ is the back off unit time and $d \in D=\left\{0, \ldots, 2^{k}-1\right\}$, where $k$ is the number of collisions so far. You can think of selecting a $d$ from $D$ as choosing a time slot to transmit the packet from $2^{k}$ future slots.

Suppose $A$ and $B$ simultaneously attempt to send their first frame, collide, and happen to choose back off times of $0 \times T$ and $1 \times T$, respectively, meaning $A$ wins the race and transmits $A_{1}$ while $B$ waits.
a) At the end of the first transmission, $B$ will attempt to retransmit $B_{1}$ while $A$ will attempt to transmit $A_{2}$. These attempts will collide. Now A will choose a waiting time in $\{0 \times$ $T, 1 \times T\}$, while $B$ will choose a waiting time in $\{0 \times T, \ldots, 3 \times T\}$. What is the probability that A wins this second back off race?
b) Suppose $A$ wins the second back off race in (a). $A$ transmits $A_{2}$, and when it is finished, $A$ and $B$ collide again as $A$ tries to transmit $A_{3}$ and $B$ tries once more to transmit $B_{1}$. What is the probability that $A$ wins this third back off race?
c) Given that $A$ wins the first three back off races, what is a lower bound for the probability that A wins all of the remaining back off races?
d) In the case that (c) holds, what happens to the frame $B_{1}$ ?

## 2 CSMA/CD: Random Access

Let $A$ and $B$ be two stations attempting to transmit a single packet on an Ethernet.
Recall that in case of the $k$ th collision, $A$ and $B$ choose a $d \in D=\left\{0, \ldots, 2^{k}-1\right\}$ and wait for $d \times T$ time. Here, $d$ is chosen randomly from the set $D$, where the probability of selecting any element in $D$ is distributed uniformly.
a) Let $P_{k}$ be the probability of success after the $k^{t h}$ collision in the $(k+1)^{t h}$ attempt. Write $P_{k}$ in terms of $k$.
b) Let $S_{k}$ be the probability of success in $(k+1)$ attempts given there is a collision to start with. Write $S_{k}$ in terms of $k$.
c) Let $S$ be the probability of success after $k$ collisions, at some point in the future. Calculate $S$.

Now, we will consider when the probability of selecting elements from $D$, i.e. selecting a time slot, is not distributed uniformly.

Specifically, let

$$
D=\left\{0,1,2, \ldots, d_{2^{k}-1}\right\} \text { and } P=\left\{p, 2 p, 3 p, \ldots, 2^{k} p\right\}
$$

where $p$ is the solution to

$$
p+2 p+3 p+\ldots+2^{k} p=1
$$

Let $p_{i}$, the $i^{\text {th }}$ element of $P$, be the probability of choosing $d_{i}$, the $i^{\text {th }}$ element of $D$. Let $P_{k}$ and $S_{k}$ be defined as before.
d) Given the probability distribution above, calculate the probability of success in the second attempt, i.e. $P_{1}$.
e) Calculate the probability of success in the third attempt, i.e. $P_{2}$. Calculate $S_{2}$.
f) Write $P_{k}$ and $S_{k}$ in terms of $k$.

Now, assume there are 3 stations, $A, B \& C$, and a uniform probability distribution in choosing slots.
g) Can we use the same method as we used in (a) \& (b) to calculate $P_{k}$ and $Q_{k}$ ? Why/why not?

## 3 The Spanning Tree Algorithm



Above an extended LAN and its corresponding network graph is given.
a) Which ports are selected by the spanning tree algorithm?
b) Assume that the bridge $B 1$ fails. Which ports are selected by the spanning tree algorithm after the recovery process and a new tree has been formed?

## 4 Programming

Suppose $N$ stations are waiting for another packet to finish on an Ethernet. All transmit at once when the packet is finished and collide.

Write a program to implement the simulation of this case up until the point when one of the $N$ waiting stations succeeds. Model time as an integer, $T$, in units of slot times and treat collisions as taking one slot time (e.g. a collision at time T followed by a backoff of $\mathrm{k}=0$ should result in a retransmission attempt at time $\mathrm{T}+1$ ).
a) Find the average delay before one station transmits successfully, for $\mathrm{N}=5, \mathrm{~N}=10, \mathrm{~N}=20$, $\mathrm{N}=40$, and $\mathrm{N}=100$.
b) Plot the average delay against the number of stations. How is delay related to the number of stations?

