

Critical Sections in Harmony

```
def thread(self):  
    while True  
        ... # code outside critical section  
        ... # code to enter the critical section  
        ... # critical section itself  
        ... # code to exit the critical section
```

```
spawn T1()  
spawn T2()  
...
```

*How do we check
mutual exclusion?*

*How do we check
progress?*

Critical Sections in Harmony

```
def thread(self):  
    while True  
        ... # code outside critical section  
        ... # code to enter the critical section  
        ... # critical section itself  
        ... # code to exit the critical section
```

```
spawn T1()  
spawn T2()  
...
```


*How do we check
mutual exclusion?*

Critical Sections in Harmony

```
def thread(self):  
    while True  
        ... # code outside critical section  
        ... # code to enter the critical section  
        cs: assert countLabel(cs) == 1  
        ... # code to exit the critical section
```

```
spawn T1()  
spawn T2()  
...
```

*# threads executing
at the label*



*How do we check
mutual exclusion?*



Critical Sections in Harmony

```
def thread(self):  
    while True  
        ... # code outside critical section  
        ... # code to enter the critical section  
        cs: assert countLabel(cs) == 1  
        ... # code to exit the critical section
```

```
spawn T1()  
spawn T2()  
...
```

*How do we check
progress?*

Critical Sections in Harmony

```
def thread(self):  
    while choose({False, True}):  
        ... # code outside critical section  
        ... # code to enter the critical section  
        cs: assert countLabel(cs) == 1  
        ... # code to exit the critical section
```

```
spawn T1()  
spawn T2()  
...
```

*thread can choose
to enter (or not)*

*How do we check
progress? ✓*

*If code to enter/exit
the critical section
cannot terminate,
Harmony will complain!*

All you need is locks (tatta-rararaaa...)

- At most one thread can hold the lock
- Acquire the lock to enter the CS
- Release the lock when exiting
- But how does one build a lock?

Try 1: A Naïve Lock

```
1  lockTaken = False
2
3  def thread(self):
4      while choose({ False, True }):
5          # Enter critical section
6          await not lockTaken
7          lockTaken = True
8
9          # Critical section
10         cs: assert countLabel(cs) == 1
11
12         # Leave critical section
13         lockTaken = False
14
15     spawn thread(0)
16     spawn thread(1)
```

Try 1: A Naïve Lock

```
1  lockTaken = False
2
3  def thread(self):
4      while choose({ False, True }):
5          # Enter critical section
6          await not lockTaken ← Wait till lock is free, then take it
7          lockTaken = True
8
9          # Critical section
10         cs: assert countLabel(cs) == 1
11
12         # Leave critical section
13         lockTaken = False ← Release the lock
14
15     spawn thread(0)
16     spawn thread(1)
```


Try 1: A Naïve Lock

```
1  lockTaken = False
2
3  def thread(self):
4      while choose({ False, True }):
5          # Enter critical section
6          await not lockTaken
7          lockTaken = True
8
9          # Critical section
10         cs: assert countLabel(cs) == 1
11
12         # Leave critical section
13         lockTaken = False
14
15     spawn thread(0)
16     spawn thread(1)
```

Testing and setting the lock is not atomic!

← Wait till lock is free, then take it

Summary: something went wrong in an execution

Here is a summary of an execution that exhibits the issue:

- Schedule thread T0: `init()`
 - Line 1: Initialize `lockTaken` to `False`
 - **Thread terminated**
- Schedule thread T2: `thread(1)`
 - Line 4: Choose `True`
 - Preempted in `thread(1)` about to store `True` into `lockTaken` in line 7
- Schedule thread T1: `thread(0)`
 - Line 4: Choose `True`
 - Line 7: Set `lockTaken` to `True` (was `False`)
 - Preempted in `thread(0)` about to execute atomic section in line 10
- Schedule thread T2: `thread(1)`
 - Line 7: Set `lockTaken` to `True` (unchanged)
 - Line 10: Harmony assertion failed

Try 2: Flags

```
1 flags = [ False, False ]
2
3 def thread(self):
  while choose({ False, True }):
```

Invariant:
Thread i in CS
 \Rightarrow
flag[i] = True

```
    # Enter critical section
```

```
    flags[self] = True
```

```
    await not flags[1 - self]
```

```
    # Critical section
```

```
    cs: assert countLabel
```

```
    # Leave critical section
```

```
    flags[self] = False
```

10

11

12

13

14

15

16

```
spawn thread(0)
```

```
spawn thread(1)
```

Summary: some execution cannot terminate

Here is a summary of an execution that exhibits the issue:

- Schedule thread T0: **init()**
 - Line 1: Initialize flags to [False, False]
 - **Thread terminated**
- Schedule thread T2: **thread(1)**
 - Line 4: Choose True
 - Line 6: Set flags[1] to True (was False)
 - Preempted in thread(1) about to load variable flags[0] in line 7
- Schedule thread T1: **thread(0)**
 - Line 4: Choose True
 - Line 6: Set flags[0] to True (was False)

Final state (all threads have terminated or are blocked):

- Threads:
 - T1: (blocked) thread(0)
 - * about to load variable flags[1] in line 7
 - T2: (blocked) thread(1)
 - * about to load variable flags[0] in line 7
- Variables:
 - flags: [True, True]

Try 3: Turns

```
1  turn = 0
```

```
2
```

```
3  def thread(self):
```

```
4      while choose({ False, True }):
```

```
5          # Enter critical section
```

```
6          turn = 1 - self
```

```
7          await turn == self
```

```
8          # Critical section
```

```
9          cs: assert countLabel(cs)
```

```
10         # Leave critical section
```

```
11
```

```
12
```

```
13
```

```
14  spawn thread(0)
```

```
15  spawn thread(1)
```

Invariant:

Thread i in CS

\Rightarrow

turn = i

\leftarrow *After you...*

\leftarrow *Wait for your turn*

Summary: some execution cannot terminate

Here is a summary of an execution that exhibits the issue:

- Schedule thread T0: `init()`
 - Line 1: Initialize turn to 0
 - **Thread terminated**
- Schedule thread T1: `thread(0)`
 - Line 4: Choose False
 - **Thread terminated**
- Schedule thread T2: `thread(1)`
 - Line 4: Choose True
 - Line 6: Set turn to 0 (unchanged)

Final state (all threads have terminated or are blocked):

- Threads:
 - T1: (terminated) `thread(0)`
 - T2: (blocked) `thread(1)`
 - * about to load variable turn in line 7
- Variables:
 - turn: 0

Peterson's Algorithm:

Flags and Turns!

```
1 sequential flags, turn ← Prevents out-of-order execution
2
3 flags = [ False, False ]
4 turn = choose({0, 1})
5
6 def thread(self):
7     while choose({ False, True }):
8         # Enter critical section
9         flags[self] = True ← I'd like to enter...
10        turn = 1 - self ← ...but you go first!
11        await (not flags[1 - self]) or (turn == self)
12        ← Wait until alone or it's my turn
13        # Critical section is here
14        cs: assert countLabel(cs) == 1
15
16        # Leave critical section
17        flags[self] = False ← Leave
18
19 spawn thread(0)
20 spawn thread(1)
```

```
[dhcp-vl2041-5018:~/Documents/harmony/code] lorenzo% harmony Peterson.hny
* Phase 1: compile Harmony program to bytecode
* Phase 2: run the model checker (nworkers = 8)
  * 104 states (time 0.00s, mem=0.000GB)
* Phase 3: analysis
  * 37 components (0.00 seconds)
  * Check for data races
  * **No issues found**
* Phase 4: write results to Peterson.hco
* Phase 5: loading Peterson.hco
```

What about a proof?

- To understand **why** it works...
- We need to show that, for any execution, all states reached satisfy mutual exclusion
 - i.e., that mutual exclusion is an **invariant**
- **See the Harmony book for a proof!**
 - or come talk to me!

Once More unto the Breach: Taking Turns

Thread T_0

$in_0 := true$

await $\neg in_1$

Thread T_1

$in_1 := true$

await $\neg in_0$

The above condition for entering CS_i is too strong: we weaken it by adding **turns**

Even if in_1 if it is T_0 's turn, then T_0 is allowed to enter CS_0

Invariant I: $turn = 0 \vee turn = 1$

The new entry code then is

Thread T_0

$in_0 := true$

await $\neg in_1 \vee (turn = 0)$

Thread T_1

$in_1 := true$

await $\neg in_0 \vee (turn = 1)$

Critical Section: Taking Turns

Thread T_0

while(!terminate) {
 $in_0 := true \{in_0 \wedge I\}$

 await $\neg in_1 \vee (turn = 0)$
 $\{in_0 \wedge (\neg in_1 \vee turn = 0) \wedge I\}$

CS_0

$in_0 := false \{\neg in_0 \wedge I\}$

NCS_0

}

Thread T_1

while(!terminate) {
 $in_1 := true \{in_1 \wedge I\}$

 await $\neg in_0 \vee (turn = 1)$
 $\{in_1 \wedge (\neg in_0 \vee turn = 1) \wedge I\}$

CS_1

$in_1 := false \{\neg in_1 \wedge I\}$

NCS_1

}

Critical Section: Taking Turns

Thread T_0

while(!terminate) {
 $in_0 := true \{in_0 \wedge I\}$

 while ($in_1 \wedge turn \neq 0$);
 $\{in_0 \wedge (\neg in_1 \vee turn = 0) \wedge I\}$

CS_0

$in_0 := false \{\neg in_0 \wedge I\}$

NCS_0

}

Thread T_1

while(!terminate) {
 $in_1 := true \{in_1 \wedge I\}$

 while ($in_0 \wedge turn \neq 1$);
 $\{in_1 \wedge (\neg in_0 \vee turn = 1) \wedge I\}$

CS_1

$in_1 := false \{\neg in_1 \wedge I\}$

NCS_1

}

Critical Section: Taking Turns

Thread T_0

```
while(!terminate) {
   $in_0 := true \{in_0 \wedge I\}$ 
```

Thread T_1

```
while(!terminate) {
   $in_1 := true \{in_1 \wedge I\}$ 
```

```
while ( $in_1 \wedge turn \neq 0$ );
 $\{in_0 \wedge (\neg in_1 \vee turn = 0) \wedge I\}$ 
```

```
while ( $in_0 \wedge turn \neq 1$ );
 $\{in_1 \wedge (\neg in_0 \vee turn = 1) \wedge I\}$ 
```

CS_0

```
 $in_0 := false \{\neg in_0 \wedge I\}$ 
```

NCS_0

}

CS_1

```
 $in_1 := false \{\neg in_1 \wedge I\}$ 
```

NCS_1

}

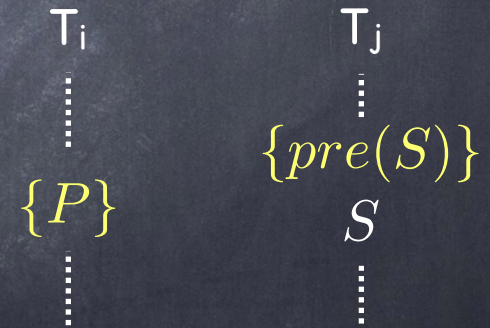
Interference Freedom

- By executing $in_1 := true$, T_1 can **interfere** on the truth of T_0 's assertion! (and symmetrically for T_0)

$$\{in_0 \wedge (\neg in_1 \vee turn = 0) \wedge I\}$$

- In general, **interference freedom** requires to establish

$$\{pre(S) \wedge P\} \quad S \quad \{P\}$$



for all S in one thread and P in the other

Establishing Interference Freedom

Thread T_0

while(!terminate) {
 $in_0 := true \{in_0 \wedge I\}$

 while ($in_1 \wedge turn \neq 0$);
 $\{in_0 \wedge (\neg in_1 \vee turn = 0) \wedge I\}$

CS_0

$in_0 := false \{\neg in_0 \wedge I\}$

NCS_0

}

Thread T_1

while(!terminate) {
 $in_1 := true \{in_1 \wedge I\}$

 while ($in_0 \wedge turn \neq 1$);
 $\{in_1 \wedge (\neg in_0 \vee turn = 1) \wedge I\}$

CS_1

$in_1 := false \{\neg in_1 \wedge I\}$

NCS_1

}

Establishing Interference Freedom

Thread T_0

while(!terminate) {
 $in_0 := true \{in_0 \wedge I\}$ ✓

Thread T_1

while(!terminate) {
 $in_1 := true \{in_1 \wedge I\}$

while ($in_1 \wedge turn \neq 0$);
 $\{in_0 \wedge (\neg in_1 \vee turn = 0) \wedge I\}$ ✗

while ($in_0 \wedge turn \neq 1$);
 $\{in_1 \wedge (\neg in_0 \vee turn = 1) \wedge I\}$

CS_0

$in_0 := false \{\neg in_0 \wedge I\}$ ✓

CS_1

$in_1 := false \{\neg in_1 \wedge I\}$

NCS_0

}

NCS_1

}

Establishing Interference Freedom

Thread T_0

while(!terminate) {
 $in_0 := true \{in_0 \wedge I\}$ ✓ ✓

while ($in_1 \wedge turn \neq 0$);
 $\{in_0 \wedge (\neg in_1 \vee turn = 0) \wedge I\}$ ✗ ✓

CS_0

$in_0 := false \{\neg in_0 \wedge I\}$ ✓ ✓

NCS_0

}

Thread T_1

while(!terminate) {
 $in_1 := true \{in_1 \wedge I\}$

while ($in_0 \wedge turn \neq 1$);
 $\{in_1 \wedge (\neg in_0 \vee turn = 1) \wedge I\}$

CS_1

$in_1 := false \{\neg in_1 \wedge I\}$

NCS_1

}

Establishing Interference Freedom

Thread T_0

$$\text{while}(!\text{terminate}) \left\{ \begin{array}{l} \langle in_0 := \text{true} \quad \{in_0 \wedge I\} \rangle \\ \langle \text{turn} = 1 \quad \quad \{in_0 \wedge I\} \rangle \end{array} \right.$$

$$\text{while } (in_1 \wedge \text{turn} \neq 0); \\ \{in_0 \wedge (\neg in_1 \vee \text{turn} = 0) \wedge I\}$$

CS_0

$$in_0 := \text{false} \quad \{\neg in_0 \wedge I\}$$

NCS_0

}

Thread T_1

$$\text{while}(!\text{terminate}) \left\{ \begin{array}{l} \langle in_1 := \text{true} \quad \{in_1 \wedge I\} \rangle \\ \langle \text{turn} = 0 \quad \quad \{in_1 \wedge I\} \rangle \end{array} \right.$$

Operations
execute
atomically

$$\text{while } (in_0 \wedge \text{turn} \neq 1); \\ \{in_1 \wedge (\neg in_0 \vee \text{turn} = 1) \wedge I\}$$

CS_1

$$in_1 := \text{false} \quad \{\neg in_1 \wedge I\}$$

NCS_1

}

Taking stock

- ◉ We solved the critical section problem, as long as we **know how to execute multiple operations atomically**
 - in other words, we can solve the CS problem as long as we can solve the CS problem... sigh...
 - besides, no machine instruction allows for those operations to execute atomically...
- ◉ But what if we don't execute the entry code atomically? Where is the problem?

Establishing Interference Freedom

Thread T_0

$$\text{while}(!\text{terminate}) \left\{ \begin{array}{l} \langle in_0 := \text{true} \quad \{in_0 \wedge I\} \rangle \\ \langle \text{turn} = 1 \quad \quad \{in_0 \wedge I\} \rangle \end{array} \right.$$

$$\text{while } (in_1 \wedge \text{turn} \neq 0); \\ \{in_0 \wedge (\neg in_1 \vee \text{turn} = 0) \wedge I\}$$

CS_0

$$in_0 := \text{false} \quad \{\neg in_0 \wedge I\}$$

NCS_0

}

Thread T_1

$$\text{while}(!\text{terminate}) \left\{ \begin{array}{l} \langle in_1 := \text{true} \quad \{in_1 \wedge I\} \rangle \\ \langle \text{turn} = 0 \quad \quad \{in_1 \wedge I\} \rangle \end{array} \right.$$

Operations
execute
atomically

$$\text{while } (in_0 \wedge \text{turn} \neq 1); \\ \{in_1 \wedge (\neg in_0 \vee \text{turn} = 1) \wedge I\}$$

CS_1

$$in_1 := \text{false} \quad \{\neg in_1 \wedge I\}$$

NCS_1

}

Establishing Interference Freedom

Thread T_0

```
while(!terminate) {
  in0 := true {in0 ∧ I}
  turn = 1    {in0 ∧ I}
```

```
while (in1 ∧ turn ≠ 0);
{in0 ∧ (¬in1 ∨ turn = 0) ∧ I}
```

CS_0

```
in0 := false {¬in0 ∧ I}
```

NCS_0

}

Thread T_1

```
while(!terminate) {
  PCT1 → in1 := true {in1 ∧ I} No problem!
  turn = 0    {in1 ∧ I}
```

```
while (in0 ∧ turn ≠ 1);
{in1 ∧ (¬in0 ∨ turn = 1) ∧ I}
```

CS_1

```
in1 := false {¬in1 ∧ I}
```

NCS_1

}

Establishing Interference Freedom

Thread T_0

```
while(!terminate) {
  in0 := true {in0 ∧ I}
  turn = 1    {in0 ∧ I}
```

```
while (in1 ∧ turn ≠ 0);
{in0 ∧ (¬in1 ∨ turn = 0) ∧ I}
```

CS_0

```
in0 := false {¬in0 ∧ I}
```

NCS_0

}

Thread T_1

```
while(!terminate) {
  in1 := true {in1 ∧ I}
  turn = 0    {in1 ∧ I}
```

$\xrightarrow{PC_{T_1}}$ while (in₀ ∧ turn ≠ 1); No problem!
 {in₁ ∧ (¬in₀ ∨ turn = 1) ∧ I}

CS_1

```
in1 := false {¬in1 ∧ I}
```

NCS_1

}

Establishing Interference Freedom

Thread T_0

```
while(!terminate) {
  in0 := true {in0 ∧ I}
  turn = 1    {in0 ∧ I}
```

Thread T_1

```
while(!terminate) {
  in1 := true {in1 ∧ I}
  turn = 0    {in1 ∧ I} Problem!
```

$\xrightarrow{PC_{T_1}}$

We weaken the assertion

```
while (in1 ∧ turn ≠ 0);
{in0 ∧ (¬in1 ∨ turn = 0) ∧ I}
```

```
while (in0 ∧ turn ≠ 1);
{in1 ∧ (¬in0 ∨ turn = 1) ∧ I}
```

CS_0

```
in0 := false {¬in0 ∧ I}
```

NCS_0

}

CS_1

```
in1 := false {¬in1 ∧ I}
```

NCS_1

}

Establishing Interference Freedom

Thread T_0

```
while(!terminate) {
  in0 := true {in0 ∧ I}
  turn = 1    {in0 ∧ I}
```

```
while (in1 ∧ turn ≠ 0);
{in0 ∧ (¬in1 ∨ turn = 0 ∨ at(turn = 0)) ∧ I}
```

CS_0

```
in0 := false {¬in0 ∧ I}
```

NCS_0

}

Thread T_1

```
while(!terminate) {
  in1 := true {in1 ∧ I}
  turn = 0    {in1 ∧ I} No problem!
```

$\xrightarrow{PC_{T_1}}$

We weaken the assertion

```
while (in0 ∧ turn ≠ 1);
{in1 ∧ (¬in0 ∨ turn = 1) ∧ I}
```

CS_1

```
in1 := false {¬in1 ∧ I}
```

NCS_1

}

Establishing Interference Freedom

Thread T_0

```
while(!terminate) {
  in0 := true {in0 ∧ I}
  turn = 1    {in0 ∧ I}
```

```
while (in1 ∧ turn ≠ 0);
{in0 ∧ (¬in1 ∨ turn = 0 ∨ at(turn = 0)) ∧ I}
```

CS_0

```
in0 := false {¬in0 ∧ I}
```

NCS_0

}

Thread T_1

```
while(!terminate) {
  in1 := true {in1 ∧ I}
  turn = 0    {in1 ∧ I} No problem!
```

$\xrightarrow{PC_{T_1}}$

We weaken the assertion

```
while (in0 ∧ turn ≠ 1);
{in1 ∧ (¬in0 ∨ turn = 1 ∨ at(turn = 1)) ∧ I}
```

CS_1

```
in1 := false {¬in1 ∧ I}
```

NCS_1

}

Peterson's Algorithm

Thread T_0

```
while(!terminate) {  
   $in_0 := true \{in_0 \wedge I\}$   
   $turn = 1 \quad \{in_0 \wedge I\}$ 
```

```
  while ( $in_1 \wedge turn \neq 0$ );  
   $\{in_0 \wedge (\neg in_1 \vee turn = 0 \vee at(turn = 0)) \wedge I\}$ 
```

CS_0

```
 $in_0 := false \{\neg in_0 \wedge I\}$ 
```

NCS_0

```
}
```

Thread T_1

```
while(!terminate) {  
   $in_1 := true \{in_1 \wedge I\}$   
   $turn = 0 \quad \{in_1 \wedge I\}$ 
```

```
  while ( $in_0 \wedge turn \neq 1$ );  
   $\{in_1 \wedge (\neg in_0 \vee turn = 1 \vee at(turn = 1)) \wedge I\}$ 
```

CS_1

```
 $in_1 := false \{\neg in_1 \wedge I\}$ 
```

NCS_1

```
}
```

Peterson's Algorithm: Safety

Thread T_0	Thread T_1
while(!terminate) {	while(!terminate) {
$in_0 := true \{in_0 \wedge I\}$	$in_1 := true \{in_1 \wedge I\}$
$turn = 1 \{in_0 \wedge I\}$	$turn = 0 \{in_1 \wedge I\}$
while ($in_1 \wedge turn \neq 0$);	while ($in_0 \wedge turn \neq 1$);
$\{in_0 \wedge (\neg in_1 \vee turn = 0 \vee at(turn = 0)) \wedge I\}$	$\{in_1 \wedge (\neg in_0 \vee turn = 1 \vee at(turn = 1)) \wedge I\}$
CS_0	CS_1
$in_0 := false \{\neg in_0 \wedge I\}$	$in_1 := false \{\neg in_1 \wedge I\}$
NCS_0	NCS_1
}	}

Mutual exclusion?

$$\begin{aligned}
 in(CS_0) &\Rightarrow \{in_0 \wedge (\neg in_1 \vee turn = 0 \vee at(turn = 0)) \wedge I\} \wedge \\
 &\quad \wedge \quad \neg at(turn = 1) \wedge \\
 in(CS_1) &\Rightarrow \{in_1 \wedge (\neg in_0 \vee turn = 1 \vee at(turn = 1)) \wedge I\} \wedge \\
 &\quad \neg at(turn = 0) = \\
 &\quad in_0 \wedge in_1 \wedge turn = 0 \wedge turn = 1 = false \quad \checkmark
 \end{aligned}$$

Peterson: Non-blocking

```

while(!terminate) {
  {R1 : ¬in0 ∧ (turn = 1 ∨ turn = 0)}
  in0 = true
  {R2 : in0 ∧ (turn = 1 ∨ turn = 0)}
α0 turn = 1
  {R2}
  while(in1 ∧ turn ≠ 0);
  {R3 : in0 ∧ (¬in1 ∨ turn = 0 ∨ at(α1))}
  CS0
  {R3}
  in0 = false
  {R1}
  NCS0
}

```

T₀'s PC →

```

while(!terminate) {
  {S1 : ¬in1 ∧ (turn = 1 ∨ turn = 0)}
  in1 := true
  {S2 : in1 ∧ (turn = 1 ∨ turn = 0)}
α1 turn := 0
  {S2}
  while(in0 ∧ turn ≠ 1);
  {S3 : in1 ∧ (¬in0 ∨ turn = 1 ∨ at(α0))}
  CS1
  {S3}
  in1 = false
  {S1}
  NCS1
}

```

T₁'s PC →

Blocking Scenario: T₀ before NCS₀, T₁ stuck at while loop

$$R_1 \wedge S_2 \wedge in_0 \wedge (turn = 0) = \neg in_0 \wedge in_1 \wedge in_0 \wedge (turn = 0) = false$$

Peterson: Deadlock-free

```

while(!terminate) {
  {R1 : ¬in0 ∧ (turn = 1 ∨ turn = 0)}
  in0 = true
  {R2 : in0 ∧ (turn = 1 ∨ turn = 0)}
  α0 turn = 1
  {R2}
  T0's PC → while(in1 ∧ turn ≠ 0);
  {R3 : in0 ∧ (¬in1 ∨ turn = 0 ∨ at(α1))}
  CS0
  {R3}
  in0 = false
  {R1}
  NCS0
}

```

```

while(!terminate) {
  {S1 : ¬in1 ∧ (turn = 1 ∨ turn = 0)}
  in1 := true
  {S2 : in1 ∧ (turn = 1 ∨ turn = 0)}
  α1 turn := 0
  {S2}
  T1's PC → while(in0 ∧ turn ≠ 1);
  {S3 : in1 ∧ (¬in0 ∨ turn = 1 ∨ at(α0))}
  CS1
  {S3}
  in1 = false
  {S1}
  NCS1
}

```

Blocking Scenario: T₀ and T₁ at the while loop, before entering critical section

$$R_2 \wedge S_2 \wedge in_1 \wedge (turn = 1) \wedge in_0 \wedge (turn = 0) \Rightarrow (turn = 0) \wedge (turn = 1) = false$$