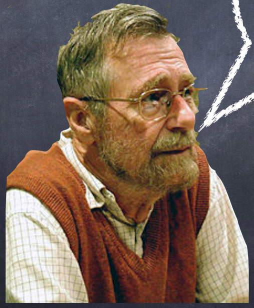
Thread Synchronization: Foundations

Edsger's perspective



Testing can only prove the presence of bugs…

…not their absence!

Properties

Property: a predicate that is evaluated over a run of the program (a trace)

> "every message that is received was previously sent"

Not everything you may want to say about a program is a property:

> "the program sends an average of 50 messages in a run"

Safety properties

- "Nothing bad happens"
    - No more than k processes are simultaneously in the critical section
    - Messages that are delivered are delivered in FIFO order
    - No patient is ever given the wrong medication
    - Windows never crashes
- A safety property is "prefix closed":
    - if it holds in a run, it holds in its every prefix

Liveness properties

- "Something good eventually happens"
  - A process that wishes to enter the critical section eventually does so
  - Some message is eventually delivered
  - Medications are eventually distributed to patients
  - Windows eventually boots
- Every run can be extended to satisfy a liveness property
  - if it does not hold in a prefix of a run, it does not mean it may not hold eventually

A really cool theorem

Every property is a combination of a safety property and a liveness property

(Alpern & Schneider)

Critical Section

- A segment of code involved in reading and writing a shared data area
- Used profusely in an OS to protect data structures (e.g., queues, shared variables, lists, ...)
- Key assumptions:
  - Finite Progress Axiom: Processes execute at a finite, positive, but otherwise unknown, speed.
  - Processes can halt only outside of the critical section (by failing, or just terminating)
    - wait-free synchronization (Herlihy, 1991)

Critical Section

- Mutual Exclusion: At most k threads are concurrently in the critical section (Safety)

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- Mutual Exclusion: At most k threads are concurrently in the critical section (Safety)
- Access Opportunity: A thread that wants to enter the critical section will eventually succeed (Liveness)
- Bounded waiting: If a thread i is in its entry section, then there is a bound on the number of times that other threads are allowed to enter the critical section before i's request is granted (Safety)

Critical Section: General Program Structure

- Entry section
  - ☐ "Lock" before entering critical section
  - ☐ Wait if already locked
- Critical Section code
- Exit section
  - ☐ "Unlock" when leaving the critical section

- OO programming style
  - ☐ Associate a lock with each shared object
  - ☐ Methods that access shared objects are critical section
  - ☐ Acquire/release locks when entering/exiting a method that defines a critical section

Too Much Milk

Too Much Milk!

Jack	Jill
◻ Look in the fridge: out of milk	
◻ Leave for store	◻ Look in fridge: no milk
◻ Arrive at store	◻ Leave for store
◻ Buy milk	◻ Arrive at store
◻ Arrive at home: put milk away	◻ Buy milk
	◻ Arrive at home: put milk away
	◻ Oh no!

Formalizing "Too Much Milk"

- Shared variables
  - "Look in the fridge for milk" – check variable "milk"
  - "Put milk away" – increment "milk"
- Safety
  - At most one person buys milk
- Liveness
  - If milk is needed, eventually somebody buys milk

Solution #0: Taking Turns

Jack

```
procedure Check-Milk
while(turn ≠ Jack) relax;
while (Milk) relax;
buy milk;
turn := Jill
```

Jill

```
procedure Check-Milk
while(turn ≠ Jill) relax;
while (Milk) relax;
buy milk;
turn := Jack
```

Solution #0: Taking Turns

Jack

```
procedure Check-Milk
while(turn ≠ Jack) relax;
while (Milk) relax;
buy milk;
turn := Jill
```

Jill

```
procedure Check-Milk
while(turn ≠ Jill) relax;
while (Milk) relax;
buy milk;
turn := Jack
```

- Safe? Why?
  - True, False
- Live? Why?
  - True, False
- Bounded waiting?
  - True, false

Solution #0:
Taking Turns

```
procedure Check-Milk
while(turn ≠ Jack) relax;
while (Milk) relax;
buy milk;
turn := Jill
```

```
procedure Check-Milk
while(turn ≠ Jill) relax;
while (Milk) relax;
buy milk;
turn := Jack
```

- Safe? Yes!
  - it is either Jack's or Jill turn
- Live? No
  - what if the other guy stops checking milk?
- Bounded waiting? Yes
  - … and the bound is 1!

Solution #1:
Leave a note

- Leave note = lock
- Remove note = unlock
- If you find a note from your roommate- don't buy!

```
procedure Check-Milk
if (noMilk) {
    if (noNote) {
        leave Note;
        buy milk;
        remove Note
    }
}
```

- Safe? Live? Bounded waiting? Why?

Solution #1:
Leave a note

- Leave note = lock
- Remove note = unlock
- If you find a note from your roommate- don't buy!

```
procedure Check-Milk
if (noMilk) {
    if (noNote) {
        leave Note;
        buy milk;
        remove Note
    }
}
```

- Safe? Live? Bounded waiting? Why?

Solution #1:
Leave a note

- If you find a note from your roommate don't buy!
  - Leave note ≈ lock
  - Remove note ≈ unlock

Jack/Jill

```
if (noMilk) {
    if (noNote) {
        leave Note;
        buy milk;
        remove Note
    }
}
```

Solution #1:
Leave a note

- If you find a note from your roommate don't buy!
  - ☐ Leave note ≈ lock
  - ☐ Remove note ≈ unlock
- Safe?

Jack/Jill

```
if (milk==0) {
    if (!note) {
        note = True;
        milk++;
        note = False;
    }
}
```

T1

```
if (milk==0) {
```

S
w
i
t
c
h

T2

```
if (milk==0) {
    if (!note) {
        note = True;
        milk++;
        note = False;
    }
}
```

S
w
i
t
c
h

T1

```
if (!note) {
        note = True;
        milk++;
        note = False;
    }
}
```

Oh no!

Solution #1:
Leave a note

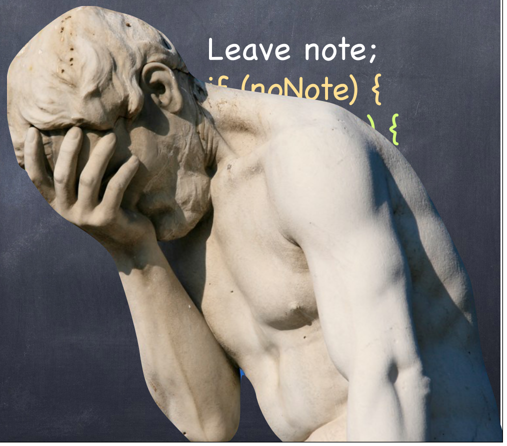
- If you find a note from your roommate don't buy!
  - ☐ Leave note ≈ lock
  - ☐ Remove note ≈ unlock
- Safe?
- This "solution" makes the problem worse!
  - ☐ sometime it works, sometime it doesn't

Jack/Jill

```
if (milk == 0) {
    if (note==0) {
        note = 1;
        milk++;
        note = 0;
    }
}
```

What if we leave
the note first?

```
if (noMilk) {
    if (noNote) {
        leave Note;
        buy milk;
        remove Note
    }
}
```



```
Leave note;
if (noNote) {
             ) {
```

Solution #2: Colored Notes

Jack

```
Leave Blue note
if (noPinknote) {
    if (noMilk) {
        buy milk;
    }
}
Remove Blue note
```

Jill

```
Leave Pink note
if (noBluenote) {
    if (noMilk) {
        buy milk;
    }
}
Remove Pink note
```

Solution #2: Colored Notes

```
        Jack                          Jill

        BlueNote = 1;                 PinkNote = 1;
A1      if (PinkNote == 0) {          if (BlueNote == 0) {   B1
A2        if (milk == 0) {              if (milk == 0) {     B2
A3          milk++;                       milk++;            B3
          }                             }
        }                             }
        BlueNote = 0;                 PinkNote = 0;
```

Proof of Safety

By contradiction:

Suppose Jack and Jill both buy milk

Consider state of variables (PinkNote,milk) at A_1

- Case 1: PinkNote == 1

  Impossible, since Jack ends up buying milk

- Case 2: PinkNote == 0, milk > 0

  Impossible. milk > 0 is a stable property, so Jack would fail test A_2 and never buy milk

- Case 3: PinkNote == 0, milk == 0

  Impossible. Jill cannot be executing in B_1-B_3 (PinkNote is not 1!)

  Since (BlueNote==1 or milk>0) is stable, then Jill will not pass B_1

Solution #2: Colored Notes

```
        Jack                          Jill

        BlueNote = 1;                 PinkNote = 1;
A1      if (PinkNote == 0) {          if (BlueNote == 0) {   B1
A2        if (milk == 0) {              if (milk == 0) {     B2
A3          milk++;                       milk++;            B3
          }                             }                    B4
        }                             }                      B5
        BlueNote = 0;                 PinkNote = 0;
```

Proof of Liveness

Not Live!

Solution #3

```
        Jack                          Jill

        BlueNote = 1;                 PinkNote = 1;
        while (PinkNote == 1) {       if (BlueNote == 0) {
              ;                         if (milk == 0) {
        }                                 milk++;
A1      if (milk == 0) {                }
          milk++;                     }
        }                             PinkNote = 0;
        BlueNote = 0;
```

Proof of Safety

Similar to previous case

Proof of Liveness

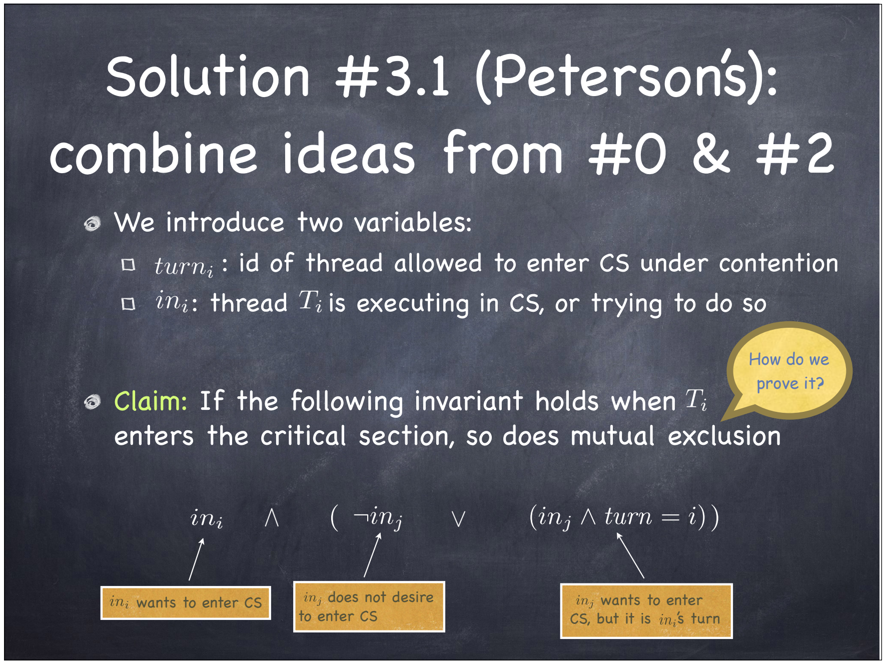
Jill will eventually set PinkNote = 0 (no loops)

Jack will then reach line A_1

- if Jack finds milk, done
- If still no milk, Jack will buy it

Too Much Milk: Lessons

- Last solution works, but it is really unsatisfactory:
  - Complicated; proving correctness is tricky even for the simple example
  - Inefficient: while thread is waiting, it is consuming CPU time
  - Asymmetric: hard to scale to many threads
  - Incorrect(?) : instruction reordering can produce surprising results



Solution #3.1 (Peterson's): combine ideas from #0 & #2

- We introduce two variables:
  - $turn_i$: id of thread allowed to enter CS under contention
  - in_i: thread T_i is executing in CS, or trying to do so
- **Claim:** If the following invariant holds when T_i enters the critical section, so does mutual exclusion

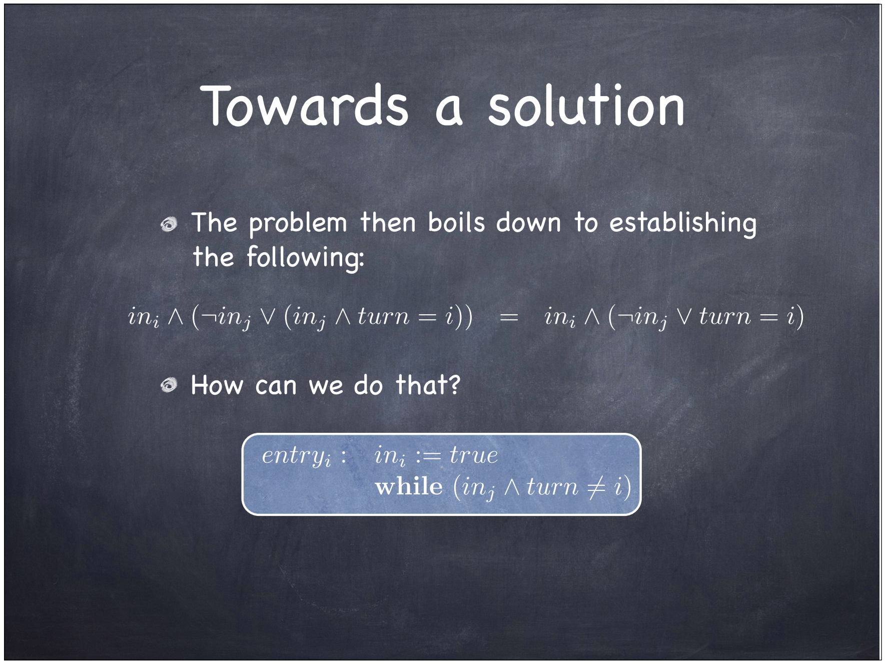
How do we prove it?

$$in_i \quad \land \quad (\; \neg in_j \quad \lor \quad (in_j \land turn = i)\,)$$

in_i wants to enter CS

in_j does not desire to enter CS

in_j wants to enter CS, but it is in_i's turn



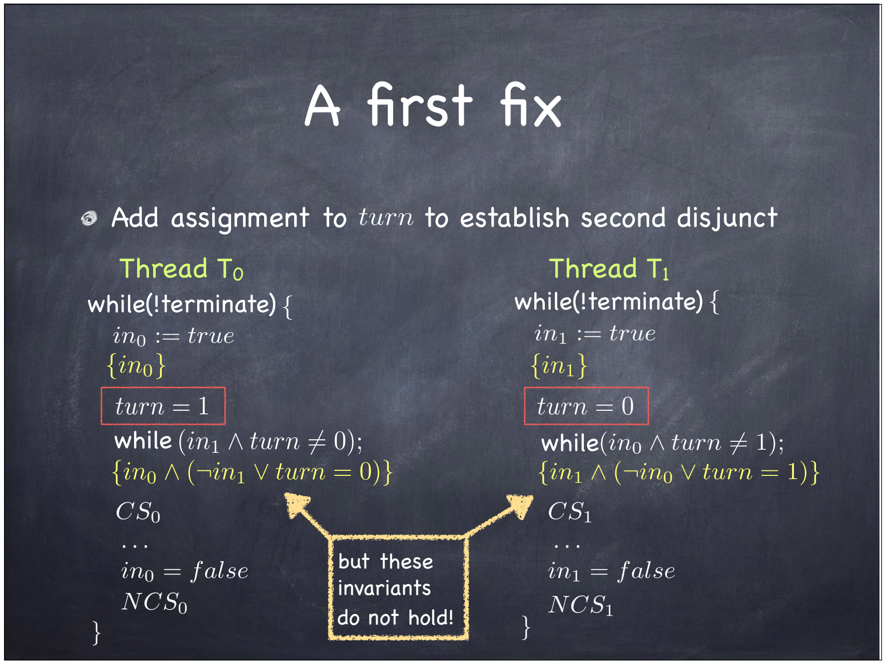
Towards a solution

- The problem then boils down to establishing the following:

$$in_i \land (\neg in_j \lor (in_j \land turn = i)) \quad = \quad in_i \land (\neg in_j \lor turn = i)$$

- How can we do that?

```
$entry_i :$  $in_i := true$
             while $(in_j \land turn \neq i)$
```



A first fix

- Add assignment to $turn$ to establish second disjunct

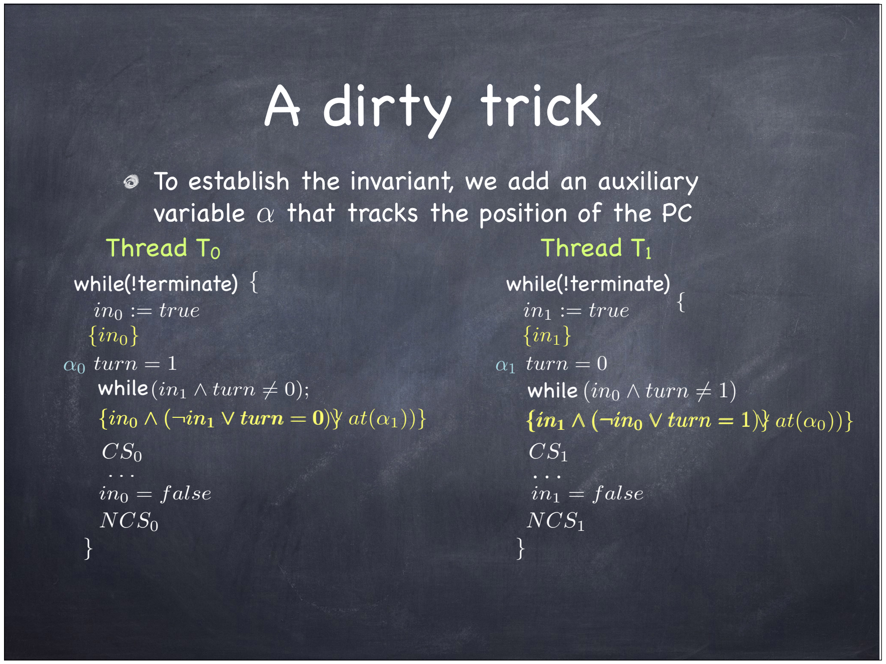
Thread T_0

```
while(!terminate) {
  $in_0 := true$
  $\{in_0\}$
  $turn = 1$
  while $(in_1 \land turn \neq 0)$;
  $\{in_0 \land (\neg in_1 \lor turn = 0)\}$
  $CS_0$
  $\dots$
  $in_0 = false$
  $NCS_0$
}
```

Thread T_1

```
while(!terminate) {
  $in_1 := true$
  $\{in_1\}$
  $turn = 0$
  while$(in_0 \land turn \neq 1)$;
  $\{in_1 \land (\neg in_0 \lor turn = 1)\}$
  $CS_1$
  $\dots$
  $in_1 = false$
  $NCS_1$
}
```

but these invariants do not hold!



A dirty trick

- To establish the invariant, we add an auxiliary variable α that tracks the position of the PC

Thread T_0

```
while(!terminate) {
  $in_0 := true$
  $\{in_0\}$
$\alpha_0$ $turn = 1$
  while$(in_1 \land turn \neq 0)$;
  $\{in_0 \land (\neg in_1 \lor turn = 0) \lor at(\alpha_1))\}$
  $CS_0$
  $\dots$
  $in_0 = false$
  $NCS_0$
}
```

Thread T_1

```
while(!terminate) {
  $in_1 := true$
  $\{in_1\}$
$\alpha_1$ $turn = 0$
  while $(in_0 \land turn \neq 1)$
  $\{in_1 \land (\neg in_0 \lor turn = 1) \lor at(\alpha_0))\}$
  $CS_1$
  $\dots$
  $in_1 = false$
  $NCS_1$
}
```

Is Peterson safe?

Thread T_0

```
while(!terminate) {
   $in_0 := true$
   $\{in_0\}$
$\alpha_0$ $turn = 1$
   while$(in_1 \wedge turn \neq 0)$;
   $\{in_0 \wedge (\neg in_1 \vee turn = 0 \vee at(\alpha_1))\}$
   $CS_0$
   $\dots$
   $in_0 = false$
   $NCS_0$
}
```

Thread T_1

```
while(!terminate) {
   $in_1 := true$
   $\{in_1\}$
$\alpha_1$ $turn = 0$
   while $(in_0 \wedge turn \neq 1)$
   $\{in_1 \wedge (\neg in_0 \vee turn = 1 \vee at(\alpha_0))\}$
   $CS_1$
   $\dots$
   $in_1 = false$
   $NCS_1$
}
```

If both in the critical section, then:

$$in_0 \wedge (\neg in_1 \vee turn = 0 \vee at(\alpha_1)) \wedge in_1 \wedge (\neg in_0 \vee turn = 1 \vee at(\alpha_0)) \wedge \neg at(\alpha_0) \wedge \neg at(\alpha_1) =$$
$$= (turn = 0) \wedge (turn = 1) = false$$

Live: Non-blocking

```
while(!terminate) {
   $\{R_1 : \neg in_0 \wedge (turn = 1 \vee turn = 0)\}$
   $in_0 = true$
   $\{R_2 : in_0 \wedge (turn = 1 \vee turn = 0)\}$
$\alpha_0$ $turn = 1$
   $\{R_2\}$
   while$(in_1 \wedge turn \neq 0)$;
   $\{R_3 : in_0 \wedge (\neg in_1 \vee turn = 0 \vee at(\alpha_1))\}$
   $CS_0$
   $\{R_3\}$
   $in_0 = false$
   $\{R_1\}$
   $NCS_0$
}
```

```
while(!terminate) {
   $\{S_1 : \neg in_1 \wedge (turn = 1 \vee turn = 0)\}$
   $in_1 := true$
   $\{S_2 : in_1 \wedge (turn = 1 \vee turn = 0)\}$
$\alpha_1$ $turn := 0$
   $\{S_2\}$
   while$(in_0 \wedge turn \neq 1)$;
   $\{S_3 : in_1 \wedge (\neg in_0 \vee turn = 1 \vee at(\alpha_0))\}$
   $CS_1$
   $\{S_3\}$
   $in_1 = false$
   $\{S_1\}$
   $NCS_1$
}
```

Blocking Scenario: T_0 before NCS_0, T_1 stuck at while loop

$$R_1 \wedge S_2 \wedge in_0 \wedge (turn = 0) = \neg in_0 \wedge in_1 \wedge in_0 \wedge (turn = 0) = false$$

Live: Deadlock-free

```
while(!terminate) {
   $\{R_1 : \neg in_0 \wedge (turn = 1 \vee turn = 0)\}$
   $in_0 = true$
   $\{R_2 : in_0 \wedge (turn = 1 \vee turn = 0)\}$
$\alpha_0$ $turn = 1$
   $\{R_2\}$
   while$(in_1 \wedge turn \neq 0)$;
   $\{R_3 : in_0 \wedge (\neg in_1 \vee turn = 0 \vee at(\alpha_1))\}$
   $CS_0$
   $\{R_3\}$
   $in_0 = false$
   $\{R_1\}$
   $NCS_0$
}
```

```
while(!terminate) {
   $\{S_1 : \neg in_1 \wedge (turn = 1 \vee turn = 0)\}$
   $in_1 := true$
   $\{S_2 : in_1 \wedge (turn = 1 \vee turn = 0)\}$
$\alpha_1$ $turn := 0$
   $\{S_2\}$
   while$(in_0 \wedge turn \neq 1)$;
   $\{S_3 : in_1 \wedge (\neg in_0 \vee turn = 1 \vee at(\alpha_0))\}$
   $CS_1$
   $\{S_3\}$
   $in_1 = false$
   $\{S_1\}$
   $NCS_1$
}
```

Blocking Scenario: T_0 and T_1 at the while loop, before entering critical section

$$R_2 \wedge S_2 \wedge in_1 \wedge (turn = 1) \wedge in_0 \wedge (turn = 0) \Rightarrow (turn = 0) \wedge (turn = 1) = false$$

A better way

- How can we do better?
- ☐ Define higher-level programming abstractions (shared objects, synchronization variables) to simplify concurrent programming
  - ☐ lock.acquire() - wait until lock is free, then grab it • atomic
  - ☐ lock.release() - unlock, waking up a waiter, if any • atomic

Jack/Jill/even Dame Dob!

```
Kitchen::buyIfNeeded() {
    lock.acquire():
    if (milk == 0) {
            milk++;
        }
    lock.release();
}
```

- ☐ Use hardware to support atomic operations beyond load and store