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## Data Redundancy

| S | N | L | R | W | H |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 123-22-3666 | Attishoo | 48 | 8 | 10 | 40 |
| 231-31-5368 | Smiley | 22 | 8 | 10 | 30 |
| 131-24-3650 | Smethurst | 35 | 5 | 7 | 30 |
| 434-26-3751 | Guldu | 35 | 5 | 7 | 32 |
| 612-67-4134 | Madayan | 35 | 8 | 10 | 40 |

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- Application constraint: all sailors with the same rating have the same wage ( $\mathrm{R} \rightarrow \mathrm{W}$ )
- Problems due to data redundancy?

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## Problems due to Data Redundancy

$\hat{*}$ Problems due to $\mathrm{R} \rightarrow \mathrm{W}$ :

- Update anomaly: Can we change W in just the first tuple of SNLRWH?
- Insertion anomaly: What if we want to insert an employee and don't know the hourly wage for his rating?
- Deletion anomaly: If we delete all employees with rating 5, we lose the information about the wage for rating 5 !
- Solution?

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## Relation Decomposition

| S | N | L | R | W | H |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $123-22-3666$ | Attishoo | 48 | 8 | 10 | 40 |
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| S | N | L | R | H | Wages |  | Problem? |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 123-22-3666 | Attishoo | 48 | 8 | 40 |  |  |  |
| 231-31-5368 | Smiley | 22 | 8 | 30 |  | W |  |
| 131-24-3650 | Smethurst | 35 | 5 | 30 |  | 10 |  |
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## Normal Forms

- First question is to ask whether any schema refinement is needed
- If a relation is in a normal form (BCNF, 3NF etc.), certain anomalies are avoided/minimized
- If not, decompose relation to normal form
- Role of FDs in detecting redundancy:

Consider a relation R with 3 attributes, ABC .

- No FDs hold: There is no redundancy here.
- Given $\mathrm{A} \rightarrow \mathrm{B}$ : Several tuples could have the same A value, and if so, they'll all have the same B value!

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## Outline

- Functional Dependencies
- Decompositions
- Normal Forms

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## Functional Dependencies (FDs)

- A functional dependency $X \rightarrow Y$ holds over relation $R$ if, for every allowable instance $r$ of R:
- $t 1 \in r, t 2 \in r, \pi_{X}(t 1)=\pi_{X}(t 2)$ implies $\pi_{Y}(t 1)=\pi_{Y}(t 2)$
- i.e., given two tuples in $r$, if the $X$ values agree, then the $Y$ values must also agree. ( X and Y are sets of attributes.) $\qquad$
- An FD is a statement about all allowable relations.
$\qquad$
- Must be identified based on semantics of application.
- Given some allowable instance $r 1$ of R, we can check if it violates some $\operatorname{FD} f$, but we cannot tell if $f$ holds over R!
- $K$ is a candidate key for $R$ means that $K \rightarrow R$ $\qquad$
- However, $K \rightarrow R$ does not require $K$ to be minimal! Database Management Systems, $2^{\text {mid }}$ Edition. R. Ramakrishnan and J. Gehrke


## Reasoning About FDs

ث Given some FDs, we can usually infer additional FDs:

- ssn $\rightarrow$ did, did $\rightarrow$ lot implies ssn $\rightarrow$ lot

ज An FD $f$ is implied by a set of FDs $F$ if $f$ holds whenever all FDs in $F$ hold.

- $F^{+}=$closure of $F$ is the set of all FDs that are implied by $F$.
- Armstrong's Axioms ( $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ are sets of attributes):

Reflexivity: If $\mathrm{X} \subseteq \mathrm{Y}$, then $\mathrm{X} \rightarrow \mathrm{Y}$

- Augmentation: If $\mathrm{X} \rightarrow \mathrm{Y}$, then $\mathrm{XZ} \rightarrow \mathrm{YZ}$ for any Z

Transitivity: If $\mathrm{X} \rightarrow \mathrm{Y}$ and $\mathrm{Y} \rightarrow \mathrm{Z}$, then $\mathrm{X} \rightarrow \mathrm{Z}$

- These are sound and complete inference rules for FDs!

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## Reasoning About FDs (Contd.)

$\stackrel{\text { Couple of additional rules (that follow from AA): }}{\text { A }}$

- Union: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow Y Z$
- Decomposition: If $\mathrm{X} \rightarrow \mathrm{YZ}$, then $\mathrm{X} \rightarrow \mathrm{Y}$ and $\mathrm{X} \rightarrow \mathrm{Z}$
- Example: Contracts(cid,sid,jid,did,pid,qty,value), and:
- C is the key: $\mathrm{C} \rightarrow$ CSJDPQV
- Project purchases each part using single contract: JP $\rightarrow$ C Dept purchases at most one part from a supplier: SD $\rightarrow \mathrm{P}$
- Can you infer SDJ $\rightarrow$ CSJDPQV ?

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## Reasoning About FDs (Contd.)

- Computing the closure of a set of FDs can be expensive. (Size of closure is exponential in \# attrs!)
$\stackrel{\text { Typically, we just want to check if a given FD } X \rightarrow Y \text { is }}{ }$ in the closure of a set of FDs F. An efficient check:

Compute attribute closure of $\mathrm{X}\left(\right.$ denoted $\left.X^{+}\right)$wrt $F$ :

- Set of all attributes A such that $\mathrm{X} \rightarrow \mathrm{A}$ is in $F^{+}$
- There is a linear time algorithm to compute this. Check if Y is in $X^{+}$
$\hat{*}$ Does $\mathrm{F}=\{\mathrm{A} \rightarrow \mathrm{B}, \mathrm{B} \rightarrow \mathrm{C}, \mathrm{CD} \rightarrow \mathrm{E}\}$ imply $\mathrm{A}_{\rightarrow} \mathrm{E}$ ? - i.e, is $\mathrm{A} \rightarrow \mathrm{E}$ in the closure $F^{+}$? Equivalently, is E in $A^{+}$? - Can be used to find keys!!!

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## Decomposition of a Relation Scheme

- Suppose that relation R contains attributes $A 1$... $A n$. A decomposition of $R$ consists of replacing $R$ by two or more relations such that:
- Each new relation scheme contains a subset of the attributes of $R$ (and no attributes that do not appear in $R$ ), and
- Every attribute of $R$ appears as an attribute of one of the new relations.
- Intuitively, decomposing R means we will store instances of the relation schemes produced by the decomposition, instead of instances of R.
- E.g., Can decompose SNLRWH into SNLRH and RW. Database Management Systems, $2^{244}$ Edition. R. Ramakrishnan and J. Gehrke $\qquad$


## Example Decomposition

* Decompositions should be used only when needed. $\qquad$
- SNLRWH has FDs S $\rightarrow$ SNLRWH and $\mathrm{R} \rightarrow \mathrm{W}$
- Data duplication due to second FD
- Will make this more precise during the definition of normal forms
- Decompose to SNLRH and RW
- What should we be careful about?

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## Problems with Decompositions

- There are three potential problems to consider:
$\square$ Some queries become more expensive.
- e.g., How much did sailor Joe earn? (salary $=W * H$ )
$\square$ Given instances of the decomposed relations, we may not be able to reconstruct the corresponding instance of the original relation!
- Fortunately, not in the SNLRWH example.
$\square$ Checking some dependencies may require joining the instances of the decomposed relations.
- Fortunately, not in the SNLRWH example.
- Tradeoff: Must consider these issues vs. redundancy.

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## Lossless Join Decompositions

 a set of FDs F if, for every instance $r$ that satisfies F:

$$
\pi_{X}(r) \bowtie \pi_{Y}(r)=r
$$

$\stackrel{\text { It is always true that }}{ } r \subseteq \pi_{X}(r) \bowtie \pi_{Y}(r)$

- In general, the other direction does not hold! If it does, the decomposition is lossless-join.
- Definition extended to decomposition into 3 or more relations in a straightforward way.
- It is essential that all decompositions used to deal with redundancy be lossless! (Avoids Problem (2).)

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## More on Lossless Join

- The decomposition of R into $X$ and $Y$ is lossless-join wrt $F$ if and only if the closure of $F$ contains:

$$
\begin{aligned}
& -X \cap Y \rightarrow X, \text { or } \\
& -X \cap Y \rightarrow Y
\end{aligned}
$$

- In particular, the decomposition of $R$ into UV and $\mathrm{R}-\mathrm{V}$ is lossless-join if $\mathrm{U} \rightarrow \mathrm{V}$ holds over R .



## Dependency Preserving Decomposition



- Decomposition: CSJDQV and SDP
- (Is it lossless join?)
- Problem: Checking JP $\rightarrow$ C requires a join!
$\hat{\rightharpoonup}$ Dependency preserving decomposition (Intuitive):
- If R is decomposed into $\mathrm{X}, \mathrm{Y}$ and Z , and we enforce the FDs that hold on X , on Y and on Z , then all FDs that were given to hold on R must also hold. (Avoids Problem (3).)
 projection of $F$ onto $X$ (denoted $F_{X}$ ) is the set of FDs $\mathrm{U} \rightarrow \mathrm{V}$ in $\mathrm{F}^{+}$(closure of $F$ ) such that $\mathrm{U}, \mathrm{V}$ are in X .

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## Dependency Preserving Decompositions (Contd.)

- Decomposition of R into X and Y is dependency preserving if $\left(\mathrm{F}_{\mathrm{X}} \text { union } \mathrm{F}_{\mathrm{Y}}\right)^{+}=\mathrm{F}^{+}$
- i.e., if we consider only dependencies in the closure $\mathrm{F}^{+}$that can be checked in X without considering Y , and in Y without considering $X$, these imply all dependencies in $\mathrm{F}^{+}$.
- Important to consider $\mathrm{F}^{+}$, not F , in this definition: $-\mathrm{ABC}, \mathrm{A} \rightarrow \mathrm{B}, \mathrm{B} \rightarrow \mathrm{C}, \mathrm{C} \rightarrow \mathrm{A}$, decomposed into AB and BC . - Is this dependency preserving? Is $\mathrm{C} \rightarrow \mathrm{A}$ preserved?????
$\checkmark$ Dependency preserving does not imply lossless join: ABC, $\mathrm{A} \rightarrow \mathrm{B}$, decomposed into AB and BC .
- And vice-versa! (Example?)

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## Outline

- Functional Dependencies
- Decompositions
- Normal Forms


## Boyee-Codd Normal Form (BCNF)

$\hat{*}$ Reln R with FDs $F$ is in BCNF if, for all $\mathrm{X} \rightarrow \mathrm{A}$ in $F^{+}$

- $\mathrm{A} \in \mathrm{X}$ (called a trivial FD), or
- X contains a key for $R$.
- In other words, R is in BCNF if the only non-trivial FDs that hold over R are key constraints.

No dependency in R that can be predicted using FDs alone. \begin{tabular}{ll|l|l|}
If we are shown two tuples that agree upon \& X \& Y \& A <br>
\hline

 the X value, we cannot infer the A value in 

one tuple from the A value in the other. \& $x$ \& $y 1$ \& $a$ <br>
\hline

 If example relation is in BCNF, the 2 tuples 

x \& y 2 \& $?$
\end{tabular} must be identical (since X is a key).

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## Decomposition into BCNF

ث Consider relation R with FDs F . If $\mathrm{X} \rightarrow \mathrm{Y}$ violates $\qquad$ BCNF, decompose R into $\mathrm{R}-\mathrm{Y}$ and XY .

- Repeated application of this idea will give us a collection of relations that are in BCNF; lossless join decomposition, and guaranteed to terminate.
- e.g., CSJDPQV, key C, JP $\rightarrow$ C, SD $\rightarrow$ P, J $\rightarrow$ S
- To deal with SD $\rightarrow$ P, decompose into SDP, CSJDQV.
- To deal with J $\rightarrow$ S, decompose CSJDQV into JS and CJDQV
- In general, several dependencies may cause violation of BCNF. The order in which we "deal with" them could lead to very different sets of relations!
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## BCNF and Dependency Preservation

- In general, there may not be a dependency preserving decomposition into BCNF. $\qquad$
- e.g., CSZ, CS $\rightarrow \mathrm{Z}, \mathrm{Z} \rightarrow \mathrm{C}$
- Can't decompose while preserving 1st FD; not in BCNF.
- Similarly, decomposition of CSJDQV into SDP, JS and CJDQV is not dependency preserving (w.r.t. $\qquad$ the FDs JP $\rightarrow \mathrm{C}, \mathrm{SD} \rightarrow \mathrm{P}$ and $\mathrm{J} \rightarrow \mathrm{S}$ ).
$\qquad$

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## Third Normal Form (3NF)

$\hat{*}$ Reln $R$ with $F D s F$ is in $3 N F$ if, for all $X \rightarrow A$ in $F^{+}$

- $A \in X$ (called a trivial FD), or
- X contains a key for R, or
- A is part of some key for $R$.
- Minimality of a key is crucial in third condition above!
- If R is in BCNF, obviously in 3NF.
- If $R$ is in 3NF, some redundancy is possible. It is a compromise, used when BCNF not achievable (e.g., no "good" decomp, or performance considerations).
- Lossless-join, dependency-preserving decomposition of R into a collection of $3 N F$ relations always possible.
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## What Does 3NF Achieve?

- If 3NF violated by $\mathrm{X} \rightarrow \mathrm{A}$, one of the following holds: $\qquad$
- X is a subset of some key K
- We store ( $\mathrm{X}, \mathrm{A}$ ) pairs redundantly.
- X is not a proper subset of any key.
- There is a chain of FDs $K \rightarrow X \rightarrow A$, which means that we cannot associate an $X$ value with a $K$ value unless we also associate an $A$ value with an $X$ value.
* But: even if reln is in 3NF, these problems could arise. - e.g., Reserves SBDC, $S \rightarrow C, C \rightarrow S$ is in 3NF, but for each reservation of sailor $S$, same $(S, C)$ pair is stored.
$\stackrel{\text { Thus, }}{ }$ 3NF is indeed a compromise relative to BCNF.
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## Decomposition into 3NF

- Obviously, the algorithm for lossless join decomp into BCNF can be used to obtain a lossless join decomp into 3NF (typically, can stop earlier).
- To ensure dependency preservation, one idea:
- If $X \rightarrow Y$ is not preserved, add relation $X Y$.
- Problem is that $X Y$ may violate $3 N F$ ! e.g., consider the addition of CJP to 'preserve' JP $\rightarrow \mathrm{C}$. What if we also have $\mathrm{J} \rightarrow \mathrm{C}$ ?
ث Refinement: Instead of the given set of FDs F, use a minimal cover for $F$.


## Minimal Cover for a Set of FDs

- Minimal cover G for a set of FDs F: $\qquad$
- Closure of F = closure of G.
- Right hand side of each FD in $G$ is a single attribute. - If we modify G by deleting an FD or by deleting attributes from an FD in G , the closure changes.
- Intuitively, every FD in G is needed, and "as small as possible" in order to get the same closure as F .
ث e.g., $\mathrm{A} \rightarrow \mathrm{B}, \mathrm{ABCD} \rightarrow \mathrm{E}, \mathrm{EF} \rightarrow \mathrm{GH}, \mathrm{ACDF} \rightarrow \mathrm{EG}$ has the following minimal cover:
$-\mathrm{A} \rightarrow \mathrm{B}, \mathrm{ACD} \rightarrow \mathrm{E}, \mathrm{EF} \rightarrow \mathrm{G}$ and $\mathrm{EF} \rightarrow \mathrm{H}$
$\stackrel{\rightharpoonup}{\text { D }}$ M.C. $\rightarrow$ Lossless-Join, Dep. Pres. Decomp!!! (in book)
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## Summary of Schema Refinement

$\bullet$ BCNF implies free of redundancies due to FDs

- If a relation is not in BCNF, we can try to decompose it into a collection of BCNF relations.
* If a lossless-join, dependency preserving decomposition into BCNF is not possible, consider 3NF
- Decompositions should be carried out and/or re-examined keeping performance issues in mind

