Midterm

You should not consult with anyone inside or outside the class, except for the course staff. You may use textbooks or online sources with citation.

1: Harder, faster, better, stronger  Rewrite the following MATLAB code fragments to perform an equivalent computation with better efficiency. Assume a Cholesky factor $A = R^T R$ is precomputed. In each case, also state the complexity of the revised operation:

1. \[ x_1 = \text{diag}(d) \ast z; \; \% \; d \; \text{and} \; z \; \text{are vectors of length} \; n \]
2. \[ x_2 = \text{trace}(u \ast v'); \; \% \; u \; \text{and} \; v \; \text{are vectors of length} \; n \]
3. \[ B = \text{inv}(A); \; x_3 = B(1,1); \; \% \; \text{We only care about} \; x_3, \; \text{not} \; B \]
4. \[ x_4 = (\text{eye}(n) + u \ast v') \backslash z; \]

2: Norm!  Let $M$ be an invertible matrix, and define

\[ \|v\|_* = \|Mv\|_\infty. \]

Argue briefly that this is indeed a vector space norm, and write a MATLAB code to compute the associated operator norm.

```matlab
function [normA] = mt_norm(M, A)
```

3: Floating point fandango  For $x > 1$, the equation $x = \cosh(y)$ can be solved as

\[ y = -\log \left( x - \sqrt{x^2 - 1} \right). \]

Describe briefly why this formula fails to achieve good accuracy when $x = 10^8$ (for example). Write an alternate code that retains good accuracy for such large $x$ values:

```matlab
function [y] = my_acosh(z)
```

4: Bordered bonanza  Suppose $x = \text{solveA}(b)$ is a black-box routine that solves the (well-conditioned) linear system of equations $Ax = b$. Construct a new routine that satisfies all but the $k$th row of $Ax = b$, subject to the constraint that $x_k = 0$.

```matlab
function [x] = solveA_constrain(solveA, b, k, l)
```
5: Almost rank one  Given $A \in \mathbb{R}^{n \times m}$ and $b \in \mathbb{R}^n$, find $u \in \mathbb{R}^m$ so that $\|A - bu^T\|_F^2$ is as small as possible. Please use the fact that MATLAB backslash solves a least squares problem if the matrix involved is rectangular.

```matlab
function [u] = approx_rank1(A, b)
```