1. Let $M$ be an $n \times n$ elementary unit lower triangular matrix, that is, a matrix of the form $I - me_k^T$ where $m \in \mathbb{R}^n$ is a vector whose first $k$ entries are 0’s and $e_k$ is the $k$th column of the identity matrix. See p. 67 of the text for an example and more explanation. Let $P(i, j)$ be the permutation matrix that exchanges row $i$ with row $j$, but leaves other rows unchanged. Assume $i > k$ and $j > k$. Show that $P(i, j)M = NP(i, j)$, where $N$ is some other elementary lower triangular matrix. Exactly how is $N$ related to $M$?

2. Let $M$ be an $n \times n$ elementary unit lower triangular matrix $I - me_k^T$ such that all entries of $m$ have absolute value at most 1. Consider solving $Mx = b$ for $x$. Show that the absolute values of entries in $x$ are all no more than twice the maximum absolute value in $b$, i.e.,

$$\max_i |x(i)| \leq 2 \max_i |b(i)|.$$ 

3. In lecture, a Matlab fragment for GEPP was provided that computes an array $p$ to store information about the row exchanges. Write (on paper) a Matlab fragment that takes as input the array $p$ and produces as output the $n \times n$ corresponding permutation matrix $P$.

4. Show that the product of two $n \times n$ lower triangular matrices is itself lower triangular. Provide a Matlab fragment for multiplying two $n \times n$ lower triangular matrices that avoids unnecessary operations on zeros. The innermost loop should be vectorized. Analyze the number of flops required by your fragment, accurate to the leading term.