Problem 1: Ray-surface intersection

1. What are the $t$ values of the intersection points for the unit sphere and the ray $\mathbf{p} = (1, 1, 1), \mathbf{d} = (-1, -1, -1)/\sqrt{3}$?

2. Provide pseudocode to efficiently test whether a ray (with normalized direction vector) intersects a sphere with center $\mathbf{c}$ and radius $r$ (no need to determine the particular intersection points or $t$ values). Assume that multiplication is expensive, division and square roots are extremely expensive, and everything else is free. How many multiplications, divisions, and/or square roots are required?

3. How do the answers to part 2 change if the ray direction might not be normalized?

4. What are the $t$ value and barycentric coordinates $(\beta, \gamma)$ of the intersection point for the same ray with the triangle that has vertices $(1, 0, 0), (0, 1, 0)$, and $(0, 0, 1)$?

5. Write down the matrix system that would be used to solve directly for $t$, $\beta$, and $\gamma$.

6. Write pseudocode for the intersection of a ray with a cone. Let the cone’s base be the unit circle in the plane $y = -1$ and let the apex be at $(0, 1, 0)$.

Problem 2: Shading and Shadows

1. A rectangular solid (that is, a scaled cube) of dimensions 4 by 9 by 1 (in $x$, $y$, and $z$ respectively) is standing on the plane $y = 0$, centered on the origin. Draw the shape of its shadow on the plane (I mean draw the edge of the shadow as if your paper were the plane) when it is illuminated by:
   
   (a) A point source at $(20, 20, 20)$.
   
   (b) A directional source in the direction $(1, 1, 1)$.

2. Can you see a shadow edge cast on a polished metal surface? Why or why not?
Problem 3: Snell’s law

Consider a pencil standing on end in a rectangular aquarium, part above and part below the water:

1. From each of the three vantage points (a), (b), and (c) shown in the diagram, will the part of the pencil below the water appear in line with, to the left of, or to the right of the part above the water? Assume the glass wall of the aquarium is thin enough to be ignored for this purpose.

2. Assuming the viewpoints are far enough away to be treated as orthographic views, how far apart will the two parts of the pencil appear to be in each case? That is, how far would you have to move the top half of the pencil (moving it parallel to the image plane) to align it with the bottom half? Your answer will be a function of \( d \) and \( \theta \).

A diver is floating a meter or two below the surface of a lake. The water is deep and a bit murky, so that looking downward it is dark. Above the surface it is a bright overcast day with good visibility. Assume for the sake of this problem that the surface is perfectly planar, with no waves or ripples.

3. What will the diver see looking up? That is, in what directions does it look bright or dark? What does it look like from the diver’s point of view?

4. If there is a man standing on the shore 100 meters away, can the diver see him? If not, why not? If so, where will he appear in the diver’s view?
Problem 4: Color Science

Consider the spectral reflectance curves for two different paints, A and B:

1. Assuming the two paints are viewed under the same light source, which of the two colors has higher saturation? Which has higher luminance?

Here are the spectra \( s_r(\lambda) \), \( s_g(\lambda) \), and \( s_b(\lambda) \) for the three phosphors in a particular CRT.

2. What are the chromaticities of the three primaries? The numerical values for \( s_r(\lambda) \), \( s_g(\lambda) \), and \( s_b(\lambda) \), as well as \( \bar{x}(\lambda) \), \( \bar{y}(\lambda) \), and \( \bar{z}(\lambda) \), are available on the web site.

3. If the white point of the monitor is \((x = 0.310, y = 0.316)\), give the color conversion matrix that should be used to display an image represented in the sRGB color space on this monitor. The sRGB primary chromaticities are as follows:

   red: \((0.640, 0.330)\)
   green: \((0.300, 0.600)\)
   blue: \((0.150, 0.060)\)

   and the sRGB white point is D65 \((x = 0.313, y = 0.329)\).