CS 4120
Introduction to Compilers

Ross Tate
Cornell University

Lecture 26: Live-Variable Analysis

Problem

- Abstract assembly contains arbitrarily many registers t_i
- Want to replace all such nodes with register nodes for `e[a-d]x`, `e[sd]i`, `(ebp)`
- Local variables allocated to `TEMP`'s too
- Only 6-7 usable registers: need to allocate multiple t_i to each register
- For each statement, need to know which variables are **live** to reuse registers

Using scope

- Observation: temporaries, variables have bounded scope in program
- Simple idea: use information about program scope to decide which variables are live
- Problem: overestimates liveness

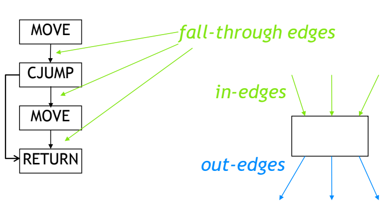
```
{  int b = a + 2;
                        ← b is live
   int c = b*b;
                        ← c is live, b is not
   int d = c + 1;
                        ← what is live here?
   return d; }
```


Live-variable analysis

- Goal: for each statement, identify which temporaries are live
- Analysis will be conservative (may over-estimate liveness, will never under-estimate)
- But more precise than simple scope analysis (will estimate fewer live temporaries)

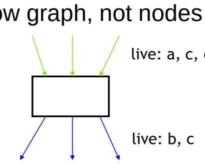
Control-Flow Graph

- Canonical IR forms *control-flow graph* (*CFG*)
  - statements are nodes; jumps/fall-throughs are edges



Liveness

- Liveness is associated with *edges* of control flow graph, not nodes (statements)



- Same register can be used for different temporaries manipulated by one statement

Example

a = b + 1

MOVE(TEMP(ta), TEMP(tb) + 1)

mov ta, tb
add ta, 1

Live: tb
mov ta, tb
add ta,1
Live: ta

Register allocation: ta ⇒ eax, tb ⇒ eax
~~mov eax, eax~~
add eax, 1

Use/Def

- Every statement uses some set of variables (reads from them) and defines some set of variables (writes to them)
- For statement s define:
  - $use[s]$: set of variables used by s
  - $def[s]$: set of variables defined by s
- Example:
  - a = b + c   *use* = b,c   *def* = a
  - a = a + 1   *use* = a   *def* = a

Liveness

- Variable v is *live* on edge e if there is
  - a node n in the CFG that uses it *and*
  - a directed path from e to n passing through no *def*
- How to compute efficiently?
- How to use?

Simple algorithm: Backtracing

- "variable v is live on edge e if there is a node n in the CFG that uses it and a directed path from e to n passing through no *def*"
- (Slow) algorithm: Try all paths "from" each *use* of a variable, tracing **backward** in the CFG until a *def* node or previously visited node is reached. Mark variable live on each edge traversed.

Dataflow Analysis

- Idea: compute liveness for all variables simultaneously
- Approach: define formulae that must be satisfied by any liveness determination
- Solve formulae by iteratively converging on solution
- Instance of general technique for computing program properties: data-flow analysis

Data-flow values

$use[n]$: set of variables used by n
$def[n]$: set of variables defined by n
$in[n]$: variables live on entry to n
$out[n]$: variables live on exit from n

Clearly: $in[n] \supseteq use[n]$

What other constraints are there?

Data-flow constraints

- $in[n] \supseteq use[n]$
  - A variable must be live on entry to n if it is used by the statement itself
- $in[n] \supseteq out[n] \setminus def[n]$
  - If a variable is live on output and the statement does not define it, it must be live on input too
- $out[n] \supseteq in[n']$ if $n' \in succ[n]$
  - if live on input to n', must be live on output from n

Iterative data-flow analysis

- Initial assignment to $in[n]$, $out[n]$ is empty set Ø
  - will not satisfy constraints

$$in[n] \supseteq use[n]$$
$$in[n] \supseteq out[n] \setminus def[n]$$
$$out[n] \supseteq in[n'] \quad \text{if} \quad n' \in succ[n]$$

- Idea: iteratively recompute in[n], out[n] when forced to by constraints. Live-variable sets will increase monotonically.
- Dataflow equations:

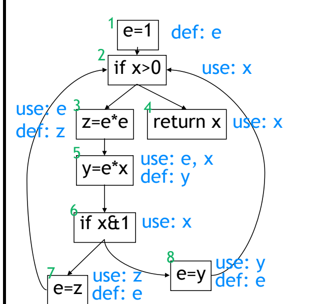
$$in'[n] = use[n] \cup (out[n] \setminus def[n])$$
$$out'[n] = \bigcup_{n' \in succ[n]} in[n']$$

Complete algorithm

```
for all n, in[n] = out[n] = Ø
repeat until no change
        for all n
                out[n] = ∪n' ∈ succ[n] in[n']
                in[n] = use[n] ∪ (out[n] \ def[n])
        end
end
```

- Finds *fixed point* of in/out equations
- Problem: does extra work recomputing in/out values when no change can happen

Example



2: in={x}
3: in={e}
4: in={x}
5: in={e,x}
6: in={x}
7: out={x}, in={x,z}
8: out={x}, in={x,y}
1: out={x}, in={x}
2: out={e,x}, in={e,x}
3: out={e,x}, in={e,x}
5: out={x}, in={e,x}
6: out={x,y,z}, in={x,y,z}
7: out={e,x}, in={x,z}
8: out={e,x}, in={x,y}
1: out={e,x}, in={x}
5: out={x,y,z}, in={e,x,z}
3: out={e,x,z}, in={e,x}
all equations satisfied

Faster algorithm

- Information only propagates between nodes because of this equation:

$$out[n] = \bigcup_{n' \in succ[n]} in[n']$$

- Node is updated from its successors
  - If successors haven't changed, no need to apply equation for node
  - Should start with nodes at "end" and work backward

Worklist algorithm

- Idea: keep track of nodes that might need to be updated in *worklist* **:** FIFO queue

```
for all n, in[n] = out[n] = Ø
w = { set of all nodes }
repeat until w empty
    n = w.pop( )
    out[n] = ∪n' ∈ succ [n] in[n']
    in[n] = use[n] ∪ (out[n] \ def [n])
    if change to in[n]
        for all predecessors m of n, w.add(m)
end
```