Formal Derivation

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Judgements

entails

context $\vdash$ property

$$
\begin{array}{ll}
\Psi \mid \Theta & \vdash \tau <: \tau' \\
\Psi & \vdash \nu\langle\Theta\rangle \ \textbf{extends}\ \tau \\
\Psi \mid \Theta & \vdash \tau.\nu : \sigma \\
\Psi & \vdash \nu\langle\Theta\rangle.\nu : \sigma \\
\Psi \mid \Theta & \vdash \tau \\
\Psi \mid \Theta & \vdash_{\hat{\tau}} \tau \\
\Psi \mid \Theta & \vdash \Gamma \\
\Psi \mid \Theta & \vdash \sigma \\
\Psi \mid \Theta \mid \Delta \mid \Gamma & \vdash e : \tau \\
\Psi \mid \Theta \mid \Delta \mid \Gamma \mid \hat{\Gamma} & \vdash s : \hat{\Gamma}' \\
\Psi \mid \Theta \mid \Delta \mid \Gamma \mid \hat{\Gamma} & \vdash^{b}_{\tau} s : \hat{\Gamma}' \\
\Psi \mid \Delta \mid \Gamma & \vdash i : \Psi' \\
\Psi \mid \Delta \mid \Gamma & \vdash c : \Psi' \mid \Delta' \\
\Psi \mid \Delta \mid \Gamma & \vdash p \\
 & \vdash p
\end{array}
$$

subtyping
inheritance
method lookup
"

Contexts in Cubex

- Ψ – Class/Interface context
  - Specifies methods, inheritance, and classiness
- Θ – Kind context
  - Indicates what type variables are in scope
- Δ – Function context
  - Indicates the type schemes of functions in scope
- Γ – Type context
  - Indicates the types of variables in scope

Inference Rules

$$
\frac{}{\Psi \mid \Theta \vdash \nu <: \nu}
\qquad
\frac{}{\Psi \mid \Theta \vdash \bot <: \tau}
\qquad
\frac{}{\Psi \mid \Theta \vdash \tau <: \top}
$$

existentially quantified

premises

$$
\frac{\Psi \mid \Theta \vdash \tau_i <: \tau}{\Psi \mid \Theta \vdash \tau_1 \cap \tau_2 <: \tau}
\qquad
\frac{\Psi \mid \Theta \vdash \tau <: \tau_1 \quad \Psi \mid \Theta \vdash \tau <: \tau_2}{\Psi \mid \Theta \vdash \tau <: \tau_1 \cap \tau_2}
$$

conclusion

Proof Derivations

$$
\frac{}{\Psi \mid \Theta \vdash \nu <: \nu}
\qquad
\frac{}{\Psi \mid \Theta \vdash \bot <: \tau}
\qquad
\frac{}{\Psi \mid \Theta \vdash \tau <: \top}
$$

$$
\frac{\Psi \mid \Theta \vdash \tau_i <: \tau}{\Psi \mid \Theta \vdash \tau_1 \cap \tau_2 <: \tau}
\qquad
\frac{\Psi \mid \Theta \vdash \tau <: \tau_1 \quad \Psi \mid \Theta \vdash \tau <: \tau_2}{\Psi \mid \Theta \vdash \tau <: \tau_1 \cap \tau_2}
$$

$$
\frac{\dfrac{}{\emptyset \mid A \vdash A <: A}}{\emptyset \mid A \vdash A \cap T <: A}
\qquad
\frac{\dfrac{}{\emptyset \mid A \vdash A <: A} \quad \dfrac{}{\emptyset \mid A \vdash A <: T}}{\emptyset \mid A \vdash A <: A \cap T}
$$

$$\dfrac{}{\Psi \mid \Theta \vdash \nu <: \nu} \qquad \dfrac{}{\Psi \mid \Theta \vdash \bot <: \tau} \qquad \dfrac{}{\Psi \mid \Theta \vdash \tau <: \top}$$

$$\dfrac{\Psi \mid \Theta \vdash \tau_i <: \tau}{\Psi \mid \Theta \vdash \tau_1 \cap \tau_2 <: \tau} \quad < \quad \dfrac{\Psi \mid \Theta \vdash \tau <: \tau_1 \qquad \Psi \mid \Theta \vdash \tau <: \tau_2}{\Psi \mid \Theta \vdash \tau <: \tau_1 \cap \tau_2}$$

$$\dfrac{\dfrac{\phi \mid A, B \vdash B <: B}{\emptyset \mid A, B \vdash A \cap B <: B} \qquad \dfrac{\emptyset \mid A, B \vdash A <: A}{\emptyset \mid A, B \vdash A \cap B <: A}}{\emptyset \mid A, B \vdash A \cap B <: B \cap A}$$

$$\dfrac{}{\Psi \mid \Theta \vdash \nu <: \nu} \qquad \dfrac{}{\Psi \mid \Theta \vdash \bot <: \tau} \qquad \dfrac{}{\Psi \mid \Theta \vdash \tau <: \top}$$

$$\dfrac{\Psi \mid \Theta \vdash \tau_i <: \tau}{\Psi \mid \Theta \vdash \tau_1 \cap \tau_2 <: \tau} \qquad \dfrac{\Psi \mid \Theta \vdash \tau <: \tau_1 \qquad \Psi \mid \Theta \vdash \tau <: \tau_2}{\Psi \mid \Theta \vdash \tau <: \tau_1 \cap \tau_2}$$

$$\dfrac{\Psi \mid \Theta \vdash \tau <: \tau'}{\Psi \mid \Theta \vdash \texttt{Iterable}\langle\tau\rangle <: \texttt{Iterable}\langle\tau'\rangle} \quad \text{Covariant}$$

$$\dfrac{\emptyset \mid \phi \vdash \bot <: \top}{\emptyset \mid \phi \vdash \text{Iter}\langle\bot\rangle <: \text{Iter}\langle\top\rangle}$$

Subtyping

$$\boxed{\Psi \mid \Theta \vdash \tau <: \tau \qquad \Psi \vdash \nu\langle\Theta\rangle \ \textbf{extends}\ \tau}$$

$$\dfrac{}{\Psi \mid \Theta \vdash \nu <: \nu} \qquad \dfrac{}{\Psi \mid \Theta \vdash \bot <: \tau} \qquad \dfrac{}{\Psi \mid \Theta \vdash \tau <: \top}$$

$$\dfrac{\Psi \mid \Theta \vdash \tau_i <: \tau}{\Psi \mid \Theta \vdash \tau_1 \cap \tau_2 <: \tau} \qquad \dfrac{\Psi \mid \Theta \vdash \tau <: \tau_1 \qquad \Psi \mid \Theta \vdash \tau <: \tau_2}{\Psi \mid \Theta \vdash \tau <: \tau_1 \cap \tau_2}$$

$$\dfrac{\text{for all } i, \quad \Psi \mid \Theta \vdash \tau_i <: \tau_i' \quad \text{and} \quad \Psi \mid \Theta \vdash \tau_i' <: \tau_i}{\Psi \mid \Theta \vdash \nu\langle\tau_1, \ldots, \tau_n\rangle <: \nu\langle\tau_1', \ldots, \tau_n'\rangle} \qquad \dfrac{\Psi \mid \Theta \vdash \tau <: \tau'}{\Psi \mid \Theta \vdash \texttt{Iterable}\langle\tau\rangle <: \texttt{Iterable}\langle\tau'\rangle}$$

$$\dfrac{\begin{array}{c}\Psi \vdash \nu\langle\nu_1, \ldots, \nu_n\rangle \ \textbf{extends}\ \tau' \\ \Psi \mid \Theta \vdash \tau'[\nu_1 \mapsto \tau_1, \ldots, \nu_n \mapsto \tau_n] <: \tau\end{array}}{\Psi \mid \Theta \vdash \nu\langle\tau_1, \ldots \tau_n\rangle <: \tau} \qquad \dfrac{\textbf{interface}\ \nu\langle\Theta\rangle\ \textbf{extends}\ \tau\ \{\ldots\} \text{ in } \Psi}{\Psi \vdash \nu\langle\Theta\rangle\ \textbf{extends}\ \tau} \qquad \dfrac{\textbf{class}\ \nu\langle\Theta\rangle\ \textbf{extends}\ \tau\ \{\ldots\} \text{ in } \Psi}{\Psi \vdash \nu\langle\Theta\rangle\ \textbf{extends}\ \tau}$$

Method Lookup

$$\boxed{\Psi \mid \Theta \vdash \tau.\nu : \sigma \qquad \Psi \vdash \nu\langle\Theta\rangle.\nu : \sigma}$$

$$\dfrac{\Psi \mid \Theta \vdash \tau'.\nu : \sigma \qquad \Psi \mid \Theta \vdash \tau <: \tau'}{\Psi \mid \Theta \vdash \tau.\nu : \sigma} \qquad \dfrac{}{\Psi \mid \Theta \vdash \bot.\nu : \sigma}$$

$$\dfrac{\Psi \vdash \hat{\nu}\langle\nu_1, \ldots, \nu_n\rangle.\nu : \sigma}{\Psi \mid \Theta \vdash \hat{\nu}\langle\tau_1, \ldots, \tau_n\rangle.\nu : \sigma[\nu_1 \mapsto \tau_1, \ldots, \nu_n \mapsto \tau_n]}$$

$$\dfrac{\textbf{interface}\ \hat{\nu}\langle\Theta\rangle\ \textbf{extends}\ \tau\ \{\ldots\nu\sigma;\ldots\} \text{ in } \Psi}{\Psi \vdash \hat{\nu}\langle\Theta\rangle.\nu : \sigma} \qquad \dfrac{\textbf{class}\ \hat{\nu}\langle\Theta\rangle\ \textbf{extends}\ \tau\ \{\ldots\nu\sigma;\ldots\} \text{ in } \Psi}{\Psi \vdash \hat{\nu}\langle\Theta\rangle.\nu : \sigma}$$

Types

$$\boxed{\Psi \mid \Theta \vdash \tau \qquad \Psi \mid \Theta \vdash_{\tau} \tau \qquad \Psi \mid \Theta \vdash \Gamma \qquad \Psi \mid \Theta \vdash \sigma}$$

$$\dfrac{\Psi \mid \Theta \vdash_{\tau} \tau}{\Psi \mid \Theta \vdash \tau} \qquad \dfrac{}{\Psi \mid \Theta \vdash_{\top} \top} \qquad \dfrac{}{\Psi \mid \Theta \vdash \bot} \qquad \dfrac{\nu \text{ in } \Theta}{\Psi \mid \Theta \vdash \nu}$$

$$\dfrac{\textbf{interface}\ \nu\langle\overset{\nu_1..\nu_n}{\Theta}\rangle\ \textbf{extends}\ \tau\ \{\ldots\} \text{ in } \Psi \qquad \text{for all } i, \quad \Psi \mid \Theta \vdash \tau_i}{\Psi \mid \Theta \vdash_{\top} \nu\langle\tau_1, \ldots, \tau_n\rangle}$$

$$\dfrac{\textbf{class}\ \nu\langle\overset{\nu_1..\nu_n}{\Theta}\rangle\ \textbf{extends}\ \tau\ \{\ldots\} \text{ in } \Psi \qquad \text{for all } i, \quad \Psi \mid \Theta \vdash \tau_i}{\Psi \mid \Theta \vdash_{\nu\langle\tau_1,\ldots,\tau_n\rangle} \nu\langle\tau_1, \ldots, \tau_n\rangle}$$

$$\dfrac{\begin{array}{c}\Psi \mid \Theta \vdash_{\tau} \tau \qquad\qquad \Psi \mid \Theta \vdash_{\top} \tau' \\ \text{for all } \nu, \left(\begin{array}{c}\Psi \mid \Theta \vdash \tau <: \nu\langle\tau_1, \ldots, \tau_n\rangle \\ \text{and} \\ \Psi \mid \Theta \vdash \tau' <: \nu\langle\tau_1', \ldots, \tau_n'\rangle\end{array}\right) \text{ implies } \left(\begin{array}{c}\Psi \mid \Theta \vdash \tau <: \nu\langle\tau_1', \ldots, \tau_n'\rangle \\ \text{and} \\ \Psi \mid \Theta \vdash \tau' <: \nu\langle\tau_1, \ldots, \tau_n\rangle\end{array}\right) \\ \text{for all } \nu, \quad \Psi \mid \Theta \vdash \tau.\nu : \sigma \quad \text{and} \quad \Psi \mid \Theta \vdash \tau'.\nu : \sigma' \quad \text{implies} \quad \sigma = \sigma'\end{array}}{\Psi \mid \Theta \vdash_{\tau} \tau \cap \tau'}$$

$$\dfrac{\text{for all } i, \quad \Psi \mid \Theta \vdash \tau_i}{\Psi \mid \Theta \vdash \nu_1 : \tau_1, \ldots, \nu_n : \tau_n} \qquad \dfrac{\Psi \mid \Theta, \hat{\Theta} \vdash \Gamma \qquad \Psi \mid \Theta, \hat{\Theta} \vdash \tau}{\Psi \mid \Theta \vdash \langle\hat{\Theta}\rangle(\Gamma) : \tau}$$